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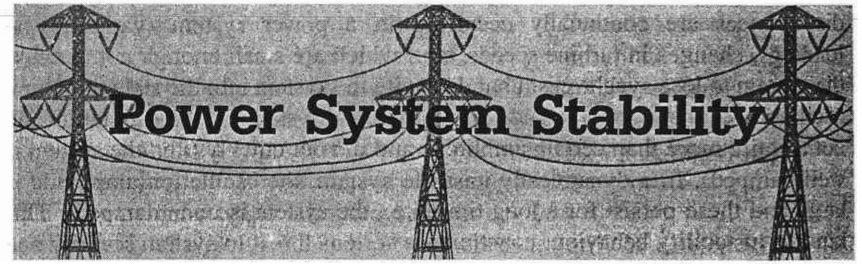


Power System Stability

12.1 INTRODUCTION

The stability of an interconnected power system is its ability to return to normal or stable operation after having been subjected to some form of disturbance. Conversely, instability means a condition denoting loss of synchronism or falling out of step. Stability considerations have been recognized as an essential part of power system planning for a long time. With interconnected systems continually growing in size and extending over vast geographical regions, it is becoming increasingly more difficult to maintain synchronism between various parts of a power system.

The dynamics of a power system are characterised by its basic features given below:

1. Synchronous tie exhibits the typical behaviour that as power transfer is gradually increased a maximum limit is reached beyond which the system cannot stay in synchronism, i.e., it falls out of step.

2. The system is basically a spring-inertia oscillatory system with inertia on the mechanical side and spring action provided by the synchronous tie wherein power transfer is proportional to sin δ or δ (for small δ; δ being the relative internal angle of machines).

3. Because of power transfer being proportional to sin δ, the equation determining system dynamics is nonlinear for disturbances causing large variations in angle δ. Stability phenomenon peculiar to non-linear systems as distinguished from linear systems is therefore exhibited by power systems (stable up to a certain magnitude of disturbance and unstable for larger disturbances).

Accordingly power system stability problems are classified into three basic types*—steady state, dynamic and transient.

*There are no universally accepted precise definitions of this terminology. For a definition of some important terms related to power system stability, refer to IEEE Standard Dictionary of Electrical and Electronic Terms, IEEE, New York, 1972.

The study of steady state stability is basically concerned with the determination of the upper limit of machine loadings before losing synchronism, provided the loading is increased gradually.

Dynamic instability is more probable than steady state instability. Small disturbances are continually occurring in a power system (variations in loadings, changes in turbine speeds, etc.) which are small enough not to cause the system to lose synchronism but do excite the system into the state of natural oscillations. The system is said to be dynamically stable if the oscillations do not acquire more than certain amplitude and die out quickly (i.e., the system is well-damped). In a dynamically unstable system, the oscillation amplitude is large and these persist for a long time (i.e., the system is underdamped). This kind of instability behaviour constitutes a serious threat to system security and creates very difficult operating conditions. Dynamic stability can be significantly improved through the use of power system stabilizers. Dynamic system study has to be carried out for 5–10 s and sometimes up to 30 s. Computer simulation is the only effective means of studying dynamic stability problems. The same simulation programmes are, of course, applicable to transient stability studies.

Following a sudden disturbance on a power system rotor speeds, rotor angular differences and power transfer undergo fast changes whose magnitudes are dependent upon the severity of disturbance. For a large disturbance, changes in angular differences may be so large as to cause the machines to fall out of step. This type of instability is known as transient instability and is a fast phenomenon usually occurring within 1 s for a generator close to the cause of disturbance. There is a large range of disturbances which may occur on a power system, but a fault on a heavily loaded line which requires opening the line to clear the fault is usually of greatest concern. The tripping of a loaded generator or the abrupt dropping of a large load may also cause instability.

The effect of short circuits (faults), the most severe type of disturbance to which a power system is subjected, must be determined in nearly all stability studies. During a fault, electrical power from nearby generators is reduced drastically, while power from remote generators is scarcely affected. In some cases, the system may be stable even with a sustained fault, whereas other systems will be stable only if the fault is cleared with sufficient rapidity. Whether the system is stable on occurrence of a fault depends not only on the system itself, but also on the type of fault, location of fault, rapidity of clearing and method of clearing, i.e., whether cleared by the sequential opening of two or more breakers or by simultaneous opening and whether or not the faulted line is reclosed. The transient stability limit is almost always lower than the steady state limit, but unlike the latter, it may exhibit different values depending on the nature, location and magnitude of disturbance.

Modern power systems have many interconnected generating stations, each with several generators and many loads. The machines located at any one point in a system normally act in unison. It is, therefore, common practice in stability studies to consider all the machines at one point as one large machine. Also machines which are not separated by lines of high reactance are lumped together and considered as one equivalent machine. Thus a multimachine system can often be reduced to an equivalent few machine system. If synchronism is lost, the machines of each group stay together although they go out of step with other groups. Qualitative behaviour of machines in an actual system is usually that of a two machine system. Because of its simplicity, the two machine system is extremely useful in describing the general concepts of power system stability and the influence of various factors on stability. It will be seen in this chapter that a two machine system can be regarded as a single machine system connected to infinite system.

Stability study of a multimachine system must necessarily be carried out on a digital computer.

12.2 DYNAMICS OF A SYNCHRONOUS MACHINE

The kinetic energy of the rotor at synchronous machine is

$$\text{KE} = \frac{1}{2} J\omega^2_{sm} \times 10^{-6} \text{ MJ}$$

where J = rotor moment of inertia in kg-m^2

ω_{sm} = synchronous speed in rad (mech)/s

But $\omega_s = \left(\frac{P}{2}\right) \omega_{sm}$ = rotor speed in rad (elect)/s

where P = number of machine poles

$$\therefore \quad \text{KE} = \frac{1}{2}\left(J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right)\omega_s$$

$$= \frac{1}{2} M \omega_s$$

where $M = J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}$

= moment of inertia in MJ-s/elect rad

We shall define the inertia constant H such that

$$GH = \text{KE} = \frac{1}{2} M \omega_s \text{ MJ}$$

where G = machine rating (base) in MVA (3-phase)

H = *inertia constant* in MJ/MVA or MW-s/MVA

It immediately follows that

$$M = \frac{2GH}{\omega_s} = \frac{GH}{\pi f} \text{ MJ-s/elect rad} \tag{12.1}$$

$$= \frac{GH}{180 f} \text{ MJ-s/elect degree}$$

M is also called the inertia constant.

Taking G as base, the inertia constant in pu is

$$M(\text{pu}) = \frac{H}{\pi f} \text{ s}^2\text{/elect rad} \tag{12.2}$$

$$= \frac{H}{180 f} \text{ s}^2\text{/elect degree}$$

The inertia constant H has a characteristic value or a range of values for each class of machines. Table 12.1 lists some typical inertia constants.

Table 12.1 Typical inertia constants of synchronous machines*

Type of Machine		*Intertia Constant H* *Stored Energy in MW Sec per MVA***
Turbine Generator		
Condensing	1,800 rpm	9-6
	3,000 rpm	7-4
Non-Condensing	3,000 rpm	4-3
Water wheel Generator		
Slow-speed (< 200 rpm)		2-3
High-speed (> 200 rpm)		2-4
Synchronous Condenser***		
Large		1.25
Small		1.00
Synchronous Motor with load varying from 1.0 to 5.0 and higher for heavy flywheels		2.00

It is observed from Table 12.1 that the value of H is considerably higher for steam turbogenerator than for water wheel generator. Thirty to sixty per cent of the total inertia of a steam turbogenerator unit is that of the prime mover, whereas only 4–15% of the inertia of a hydroelectric generating unit is that of the waterwheel, including water.

* Reprinted with permission of the Westinghous Electric Corporation from Electrical Transmission and Distribution Reference Book.

** Where range is given, the first figure applies to the smaller MVA sizes.

*** Hydrogen-Cooled, 25 per cent less.

e Swing Equation

ure 12.1 shows the torque, speed and flow of mechanical and electrical wers in a synchronous machine. It is assumed that the windage, friction and n-loss torque is negligible. The differential equation governing the rotor namics can then be written as

$$J \frac{d^2\theta_m}{dt^2} = T_m - T_e \text{ Nm} \tag{12.3}$$

here

θ_m = angle in rad (mech)

T_m = turbine torque in Nm; it acquires a negative value for a motoring machine

T_e = electromagnetic torque developed in Nm; it acquires negative value for a motoring machine

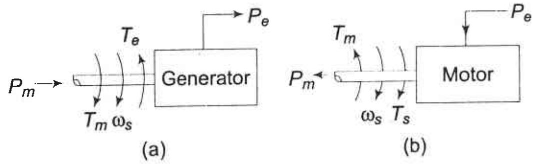


Fig. 12.1 Flow of mechanical and electrical powers in a synchronous machine

While the rotor undergoes dynamics as per Eq. (12.3), the rotor speed changes by insignificant magnitude for the time period of interest (1s) [Sec. 12.1]. Equation (12.3) can therefore be converted into its more convenient ower form by assuming the rotor speed to remain constant at the synchronous peed (ω_{sm}). Multiplying both sides of Eq. (12.3) by ω_{sm}' we can write

$$J\omega_{sm} \frac{d^2\theta_m}{dt^2} \times 10^{-6} = P_m - P_e \text{ MW} \tag{12.4}$$

where

P_m = mechanical power input in MW

P_e = electrical power output in MW; stator copper loss is assumed negligible.

Rewriting Eq. (12.4)

$$\left(J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right) \frac{d^2\theta_e}{dt^2} = P_m - P_e \text{ MW}$$

where θ_e = angle in rad (elect)

or

$$M\frac{d^2\theta_e}{dt^2} = P_m - P_e \tag{12.5}$$

It is more convenient to measure the angular position of the rotor with respect to a synchronously rotating frame of reference. Let

$$\delta = \theta_e - \omega_s t; \text{ rotor angular displacement from synchronously rotating reference frame (called } \textit{torque angle/power angle}) \tag{12.6}$$

From Eq. (12.6)

$$\frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2} \tag{12.7}$$

Hence Eq. (12.5) can be written in terms of δ as

$$M\frac{d^2\delta}{dt^2} = P_m - P_e \text{ MW} \tag{12.8}$$

With M as defined in Eq. (12.1), we can write

$$\frac{GH}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ MW} \tag{12.9}$$

Dividing throughout by G, the MVA rating of the machine,

$$M(\text{pu})\frac{d^2\delta}{dt^2} = P_m - P_e;$$

$$\text{in pu of machine rating as base} \tag{12.10}$$

where

$$M(\text{pu}) = \frac{H}{\pi f}$$

or

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu} \tag{12.11}$$

This equation (Eq. (12.10)/Eq. (12.11)), is called the *swing equation* and it describes the rotor dynamics for a synchronous machine (generating/motoring). It is a second-order differential equation where the damping term (proportional to $d\delta/dt$) is absent because of the assumption of a lossless machine and the fact that the torque of *damper winding* has been ignored. This assumption leads to pessimistic results in transient stability analysis—damping helps to stabilize the system. Damping must of course be considered in a dynamic stability study. Since the electrical power P_e depends upon the sine of angle δ (see Eq. (12.29)), the swing equation is a non-linear second-order differential equation.

Multimachine System

In a multimachine system a common system base must be chosen. Let

$$G_{mach} = \text{machine rating (base)}$$
$$G_{system} = \text{system base}$$

Equation (12.11) can then be written as

$$\frac{G_{mach}}{G_{system}}\left(\frac{H_{mach}}{f}\frac{d^2\delta}{dt^2}\right) = (P_m - P_e)\frac{G_{mach}}{G_{system}}$$

or

$$\frac{H_{system}}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu in system base} \tag{12.12}$$

where

$$H_{system} = H_{mach}\left(\frac{G_{mach}}{G_{system}}\right) \tag{12.13}$$

$$= \text{machine inertia constant in system base}$$

Machines Swinging Coherently

Consider the swing equations of two machines on a *common system base*.

$$\frac{H_1}{\pi f}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \text{ pu} \tag{12.14}$$

$$\frac{H_2}{\pi f}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \text{ pu} \tag{12.15}$$

Since the machine rotors swing together (coherently or in unison)

$$\delta_1 = \delta_2 = \delta$$

Adding Eqs (12.14) and (12.15)

$$\frac{H_{eq}}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \tag{12.16}$$

where

$$\begin{aligned} P_m &= P_{m1} + P_{m2} \\ P_e &= P_{e1} + P_{e2} \\ H_{eq} &= H_1 + H_2 \end{aligned} \tag{12.17}$$

The two machines swinging coherently are thus reduced to a single machine as in Eq. (12.16). The equivalent inertia in Eq. (12.17) can be written as

$$H_{eq} = H_{1\,mach}\,G_{1\,mach}/G_{system} + H_{2\,mach}\,G_{2\,mach}/G_{system} \tag{12.18}$$

The above results are easily extendable to any number of machines swinging coherently.

Example 12.1

A 50 Hz, four pole turbogenerator rated 100 MVA, 11 kV has an inertia constant of 8.0 MJ/MVA

(a) Find the stored energy in the rotor at synchronous speed.

(b) If the mechanical input is suddenly raised to 80 MW for an electrical load of 50 MW, find rotor acceleration, neglecting mechanical and electrical losses.

(c) If the acceleration calculated in part (b) is maintained for 10 cycles, find the change in torque angle and rotor speed in revolutions per minute at the end of this period.

Solution

(a) Stored energy = $GH = 100 \times 8 = 800$ MJ

(b) $P_a = 80 - 50 = 30 \text{ MW} = M \dfrac{d^2\delta}{dt^2}$

$$M = \frac{GH}{180f} = \frac{800}{180 \times 50} = \frac{4}{45} \text{ MJ-s/elect deg}$$

$$\therefore \quad \frac{4}{45}\frac{d^2\delta}{dt^2} = 30$$

or

$$\alpha = \frac{d^2\delta}{dt^2} = 337.5 \text{ elect deg/s}^2$$

(c) 10 cycles = 0.2 s

Change in $\delta = \frac{1}{2}(337.5) \times (0.2)^2 = 6.75$ elect degrees

$$= 60 \times \frac{337.5}{2 \times 360°} = 28.125 \text{ rpm/s}$$

$\therefore$ Rotor speed at the end of 10 cycles

$$= \frac{120 \times 50}{4} + 28.125 \times 0.2$$

$$= 1505.625 \text{ rpm}$$

12.3 POWER ANGLE EQUATION

In solving the swing equation (Eq. (12.10)), certain simplifying assumptions are usually made. These are:

1. Mechanical power input to the machine (P_m) remains constant during the period of electromechanical transient of interest. In other words, it means that the effect of the turbine governing loop is ignored being much slower than the speed of the transient. This assumption leads to pessimistic result—governing loop helps to stabilize the system.

2. Rotor speed changes are insignificant—these have already been ignored in formulating the swing equation.

3. Effect of voltage regulating loop during the transient is ignored, as a consequence the generated machine emf remains constant. This assumption also leads to pessimistic results—voltage regulator helps to stabilize the system.

Before the swing equation can be solved, it is necessary to determine the dependence of the electrical power output (P_e) upon the rotor angle.

Simplified Machine Model

For a nonsalient pole machine, the per phase induced emf-terminal voltage equation under steady conditions is

$$E = V + jX_dI_d + jX_qI_q;\ X_d > X_q \tag{12.19}$$

where

$$I = I_d + I_g \tag{12.20}$$

and usual symbols are used.

Under transient condition

$$X_d \to X'_d < X_d$$

but

$$X'_q = X_q \text{ since the main field is on the d-axis}$$

$$X'_d < X_q\text{; but the difference is less than in Eq. (12.19)}$$

Equation (12.19) during the transient modifies to

$$E' = V + jX'_dI_d + jX_qI_q \tag{12.21}$$

$$= V + jX_q(I - I_d) + jX'_dI_d$$

$$= (V + jX_qI) + j(X'_d - X_q)I_d \tag{12.22}$$

The phasor diagram corresponding to Eqs. (12.21) and (12.22) is drawn in Fig. 12.2.

Since under transient condition, $X'_d < X_d$ but X_q remains almost unaffected, it is fairly valid to assume that

$$X'_d \approx X_q \tag{12.23}$$

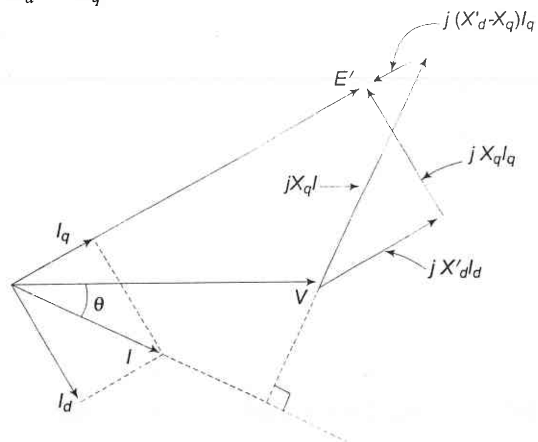


Fig. 12.2 Phasor diagram—salient pole machine

Equation (12.22) now becomes

$$E' = V + jX_q I$$

$$= V + jX_d' I \tag{12.24}$$

The machine model corresponding to Eq. (12.24) is drawn in Fig. 12.3 which also applies to a cylindrical rotor machine where $X_d' = X_q' = X_s'$(transient synchronous reactance)

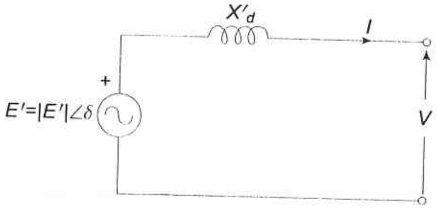


Fig. 12.3 Simplified machine model

The simplified machine of Fig. 12.3 will be used in all stability studies.

Power Angle Curve

For the purposes of stability studies $|E'|$, transient emf of generator motor, remains constant or is the independent variable determined by the voltage regulating loop but V, the generator determined terminal voltage is a dependent variable. Therefore, the nodes (buses) of the stability study network pertain to the emf terminal in the machine model as shown in Fig. 12.4, while the machine reactance (X_d') is absorbed in the system network as different from a load flow study. Further, the loads (other than large synchronous motors) will be replaced by equivalent static admittances (connected in shunt between transmission network buses and the reference bus). This is so because load voltages vary during a stability study (in a load flow study, these remain constant within a narrow band).

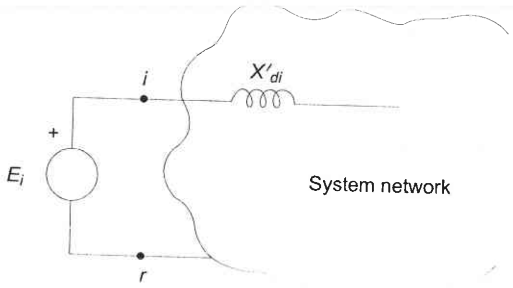


Fig. 12.4

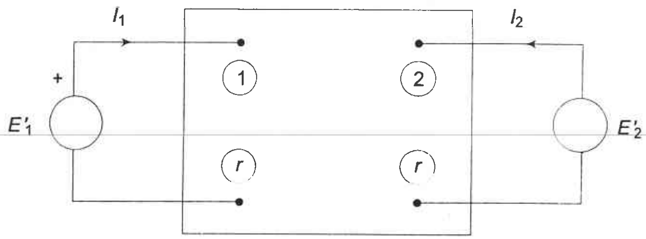


Fig. 12.5 Two-bus stability study network

For the 2-bus system of Fig. 12.5

$$\mathbf{Y}_{\mathbf{BUS}} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}; \; Y_{12} = Y_{21} \tag{12.25}$$

Complex power into bus is given by

$$P_i + jQ_i = E_i I_i^*$$

At bus 1

$$P_1 + jQ_1 = E_1' (Y_{11}E_1')^* + E_1 (Y_{12}E_2')^* \tag{12.26}$$

But

$$E_1' = |E_1'| \angle \delta_1; \; E_2' = |E_2'| \angle \delta_2$$

$$Y_{11} = G_{11} + jB_{11}; \; Y_{12} = |Y_{12}| \angle \theta_{12}$$

Since in solution of the swing equation only real power is involved, we have from Eq. (12.26)

$$P_1 = |E_1'|^2 G_{11} + |E_1'|\,|E_2'|\,|Y_{12}| \cos (\delta_1 - \delta_2 - \theta_{12}) \tag{12.27}$$

A similar equation will hold at bus 2.

Let

$$|E_1'|^2 G_{11} = P_c$$

$$|E_1'|\,|E_2'|\,|Y_{12}| = P_{max}$$

$$\delta_1 - \delta_2 = \delta$$

and

$$\phi_{12} = \pi/2 - \gamma$$

Then Eq. (12.27) can be written as

$$P_1 = P_c + P_{max} \sin (\delta - \gamma); \; \textit{Power Angle Equation} \tag{12.28}$$

For a purely reactive network

$$G_{11} = 0 \; (\therefore P_c = 0); \text{ lossless network}$$

$$\theta_{12} = \pi/2, \; \therefore \gamma = 0$$

Hence

$$P_e = P_{max} \sin \delta \tag{12.29a}$$

where $$P_{\max} = \frac{|E_1'||E_2'|}{X};$$

simplified *power angle equation* (12.29b)

where X = transfer reactance between nodes (i.e., between E_1' and E_2')

The graphical plot of power angle equation (Eq.(12.29)) is shown in Fig. 12.6.

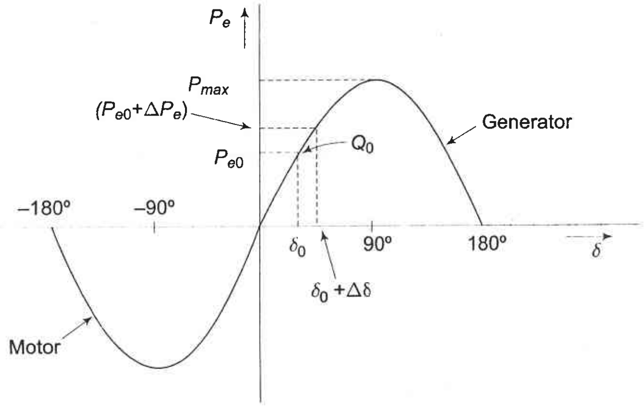


Fig. 12.6 Power angle curve

The swing equation (Eq. (12.10)) can now be written as

$$\frac{\mathrm{H}}{\pi f}\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_m - P_{\max}\sin\delta\,\mathrm{pu} \tag{12.30}$$

which, as already stated, is a non-linear second-order differential equation with no damping.

12.4 NODE ELIMINATION TECHNIQUE

In stability studies, it has been indicated that the buses to be considered are those which are excited by the internal machine voltages (transient emf's) and not the load buses which are excited by the terminal voltages of the generators. Therefore, in Y_{BUS} formulation for the stability study, the load buses must be eliminated. Three methods are available for bus elimination. These are illustrated by the simple system of Fig. 12.7(a) whose reactance diagram is drawn in Fig. 12.7(b). In this simple situation, bus 3 gets easily eliminated by parallel combination of the lines. Thus

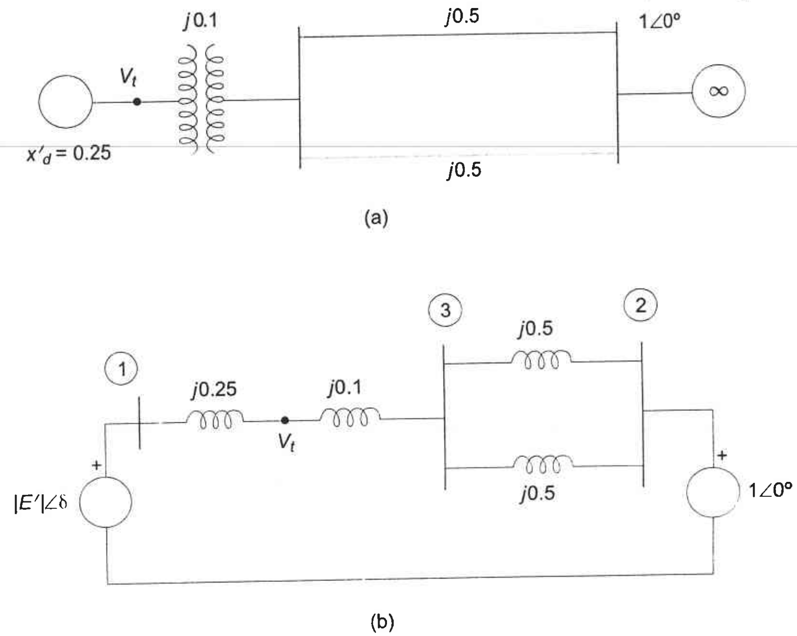


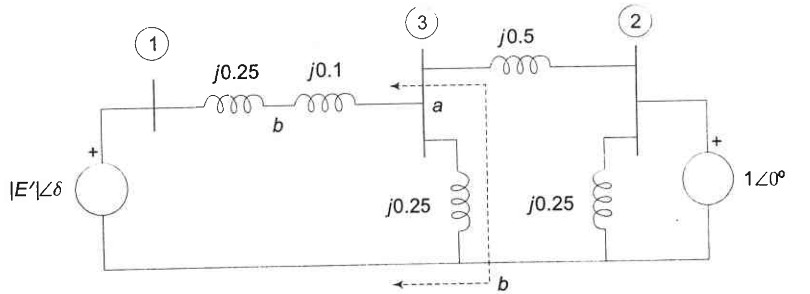
Fig. 12.7 A simple system with its reactance diagram

$$X_{12} = 0.25 + 0.1 + \frac{0.5}{2}$$
$$= 0.6$$

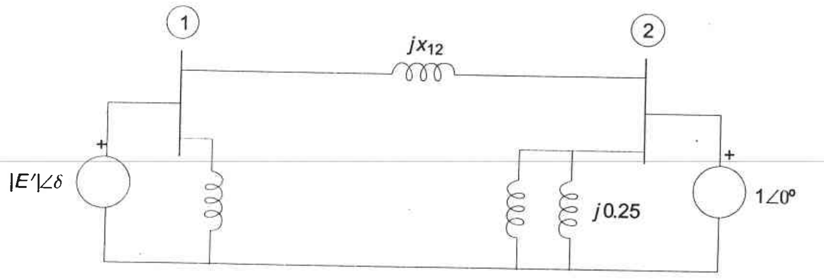
Consider now a more complicated case wherein a 3-phase fault occurs at the midpoint of one of the lines in which case the reactance diagram becomes that of Fig. 12.8 (a).

Star-Delta Conversion

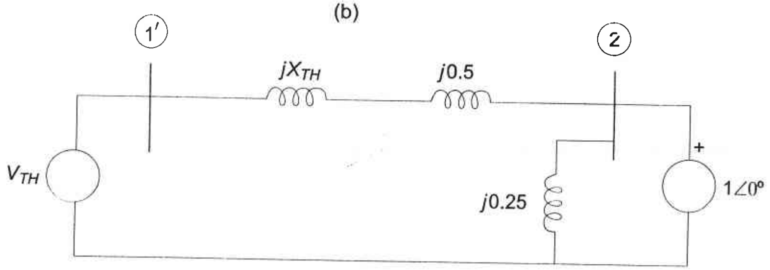
Converting the star at the bus 3 to delta, the network transforms to that of Fig. 12.8(b) wherein



(a)



(b)



(c)

Fig. 12.8

$$X_{12} = \frac{0.25 \times 0.35 + 0.35 \times 0.5 + 0.5 \times 0.25}{0.25}$$

$$= 1.55$$

This method for a complex network, however, cannot be mechainzed for preparing a computer programme.

Thevenin's Equivalent

With reference to Fig. 12.8(a), the Thevenin's equivalent for the network portion to the left of terminals *a b* as drawn in Fig. 12.8(c) wherein bus 1 has been modified to 1′.

$$V_{Th} = \frac{0.25}{0.25 + 0.35} \, |E'| \angle \delta$$

$$= 0.417 \, |E'| \angle \delta$$

$$X_{Th} = \frac{0.35 \times 0.25}{0.35 + 0.25} = 0.146$$

Now

$$X_{12} = 0.146 + 0.5 = 0.646^*$$

*This value is different from that obtained by star delta transformation as V_{Th} is no longer $|E'| \angle \delta$; in fact it is $0.417 \, |E'| \angle \delta$.

This method obviously is cumbersome to apply for a network of even small complexity and cannot be computerized.

Node Elimination Technique

Formulate the bus admittances for the 3-bus system of Fig. 12.8(a). This network is redrawn in Fig. 12.9 wherein instead of reactance branch, admittances are shown. For this network,

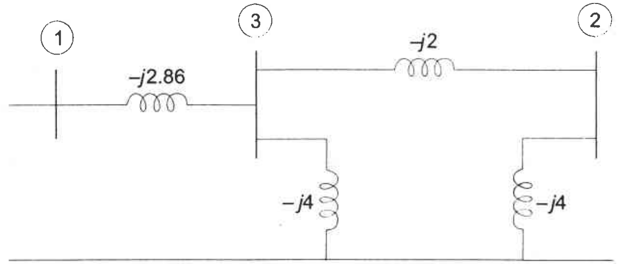


Fig. 12.9

$$Y_{BUS} = j \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} -2.86 & 0 & 2.86 \\ 0 & -6 & 2 \\ 2.86 & 2 & -8.86 \end{bmatrix}$$

The bus 3 is to be eliminated.

In general for a 3-bus system

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{12.31}$$

Since no source is connected at the bus 3

$$I_3 = 0$$

or

$$Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 = 0$$

or

$$V_3 = -\frac{Y_{31}}{Y_{33}} V_1 - \frac{Y_{32}}{Y_{33}} V_2 \tag{12.32}$$

Substituting this value of V_3 in the remaining two equations of Eq. (12.31), thereby eliminating V_3,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3$$

$$= \left(Y_{11} - \frac{Y_{13}Y_{31}}{Y_{33}} \right) V_1 + \left(Y_{12} - \frac{Y_{13}Y_{32}}{Y_{33}} \right) V_2$$

In compact form

$$\mathbf{Y}_{\text{BUS}}\ \text{(reduced)} = \begin{bmatrix} Y'_{11} & Y'_{12} \\ Y'_{21} & Y'_{22} \end{bmatrix} \tag{12.33}$$

where

$$Y'_{11} = Y_{11} - \frac{Y_{13}Y_{31}}{Y_{33}} \tag{12.34a}$$

$$Y'_{12} = Y'_{21} = Y_{12} - \frac{Y_{13}Y_{32}}{Y_{33}} \tag{12.34b}$$

$$Y'_{22} = Y_{22} - \frac{Y_{23}Y_{32}}{Y_{33}} \tag{12.34c}$$

In general, in eliminating node n

$$Y_{kj}\ \text{(new)} = Y_{kj}\ \text{(old)} - \frac{Y_{kn}\text{(old)}Y_{nj}\text{(old)}}{Y_{nn}\text{(old)}} \tag{12.35}$$

Applying Eq. (12.34) to the example in hand

$$Y_{\text{BUS}}\ \text{(reduced)} = j\begin{bmatrix} -1.937 & 0.646 \\ 0.646 & -5.549 \end{bmatrix}$$

It then follows that

$$X_{12} = \frac{1}{0.646} = 1.548\ (\approx 1.55)$$

Example 12.2

In the system shown in Fig. 12.10, a three-phase static capacitive reactor of reactance 1 pu per phase is connected through a switch at motor bus bar. Calculate the limit of steady state power with and without reactor switch closed. Recalculate the power limit with capacitive reactor replaced by an inductive reactor of the same value.

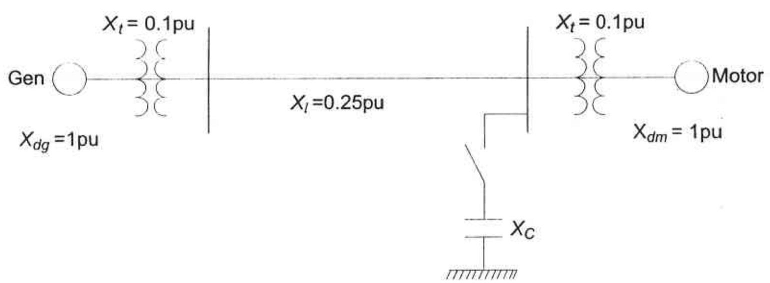


Fig. 12.10

Assume the internal voltage of the generator to be 1.2 pu and that of the motor to be 1.0 pu.

Solution

(1) Steady state power limit without reactor

$$= \frac{|E_g||E_m|}{X\text{(total)}} = \frac{1.2\times 1}{1+0.1+0.25+0.1+1} = 0.49 \text{ pu}$$

(2) Equivalent circuit with capacitive reactor is shown in Fig. 12.11 (a).

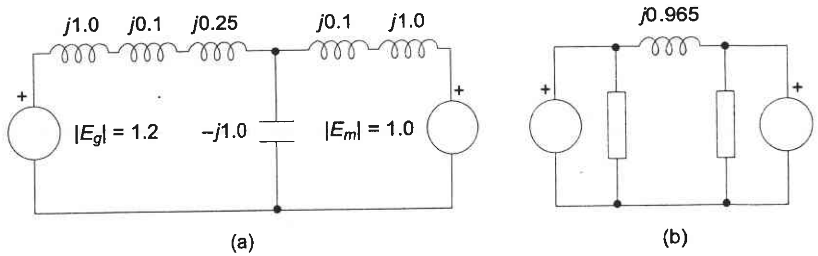


Fig. 12.11

Converting star to delta, the network of Fig. 12.11(a) is reduced to that of Fig. 12.11(b) where

$$jX\text{(transfer)} = \frac{j1.35\times j1.1 + j1.1\times(-j1.0)+(-j1.0)\times j1.35}{-j1.0}$$

$$= j0.965$$

$$\text{Steady state power limit} = \frac{1.2\times 1}{0.965} = 1.244 \text{ pu}$$

(3) With capacitive reactance replaced by inductive reactance, we get the equivalent circuit of Fig. 12.12. Converting star to delta, we have the trasfer reactance of

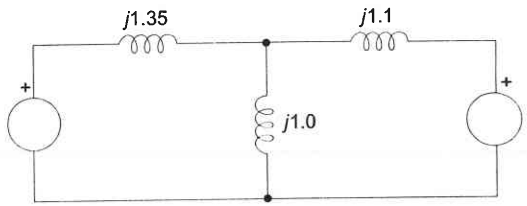


Fig. 12.12

$$jX\text{(transfer)} = \frac{j1.35\times j1.1 + j1.1\times j1.0 + j1.0\times j1.35}{j1.0}$$

$$= j3.935$$

Steady state power limit $= \frac{1.2 \times 1}{3.935} = 0.304$ pu

Example 12.3

The generator of Fig. 12.7(a) is delivering 1.0 pu power to the infinite bus ($|V|$ = 1.0 pu), with the generator terminal voltage of $|V_t| = 1.0$ pu. Calculate the generator emf behind transient reactance. Find the maximum power that can be transferred under the following conditions:

(a) System healthy
(b) One line shorted (3-phase) in the middle
(c) One line open.

Plot all the three power angle curves.

Solution

Let $V_t = |V_t| \angle\alpha = 1\angle\alpha$

From power angle equation

$$\frac{|V_t||V|}{X} \sin\alpha = P_e$$

or $$\left(\frac{1\times 1}{0.25+0.1}\right)\sin\alpha = 1$$

or $$\alpha = 20.5°$$

Current into infinite bus,

$$I = \frac{|V_t|\angle\alpha - |V|\angle 0°}{jX}$$

$$= \frac{1\angle 20.5° - 1\angle 0°}{j0.35}$$

$$= 1 + j0.18 = 1.016 \angle 10.3°$$

Voltage behind transient reactance,

$$E' = 1\angle 0° + j0.6 \times (1 + j0.18)$$

$$= 0.892 + j0.6 = 1.075 \angle 33.9°$$

(a) System healthy

$$P_{max} = \frac{|V||E'|}{X_{12}} = \frac{1\times 1.075}{0.6} = 1.79 \text{ pu}$$

$$P_e = 1.79 \sin\delta \qquad \text{(i)}$$

(b) One line shorted in the middle:

As already calculated in this section,

$$X_{12} = 1.55$$

$$\therefore \quad P_{max} = \frac{1\times 1.075}{1.55} = 0.694 \text{ pu}$$

or $$P_e = 0.694 \sin\delta \qquad \text{(ii)}$$

(c) One line open:

It easily follows from Fig. 12.7(b) that

$$X_{12} = 0.25 + 0.1 + 0.5 = 0.85$$

$$\therefore \quad P_{max} = \frac{1\times 1.075}{0.85} = 1.265$$

or $$P_e = 1.265 \sin\delta \qquad \text{(iii)}$$

The plot of the three power angle curves (Eqs. (i), (ii) and (iii)) is drawn in Fig. 12.13. Under healthy condition, the system is operated with $P_m = P_e = 1.0$ pu and $\delta_0 = 33.9°$, i.e., at the point P on the power angle curve 1.79 sin δ. As one line is shorted in the middle, P_m remains fixed at 1.0 pu (governing system act instantaneously) and is further assumed to remain fixed throughout the transient (governing action is slow), while the operating point instantly shifts to Q on the curve 0.694 sin δ at $\delta = 33.9°$. Notice that because of machine inertia, the rotor angle can not change suddenly.

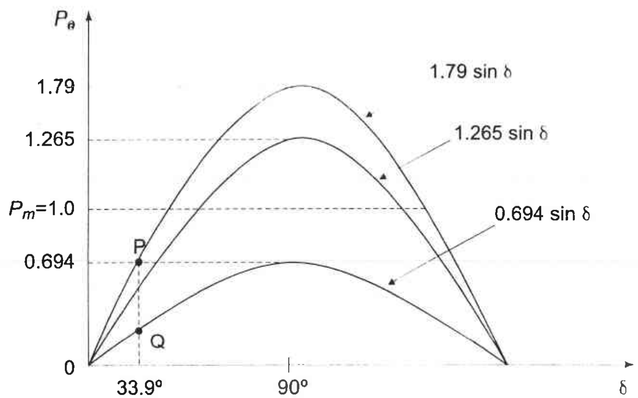


Fig. 12.13 Power angle curves

12.5 SIMPLE SYSTEMS

Machine Connected to Infinite Bus

Figure 12.14 is the circuit model of a single machine connected to infinite bus through a line of reactance X_e. In this simple case

$$X_{\text{transfer}} = X'_d + X_e$$

From Eq. (12.29b)

$$P_e = \frac{|E'||V|}{X_{\text{transfer}}} \sin \delta = P_{\max} \sin \delta \qquad (12.36)$$

The dynamics of this system are described in Eq. (12.11) as

$$\frac{\mathrm{H}}{\pi f} \frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_m - P_e \text{ pu} \qquad (12.37)$$

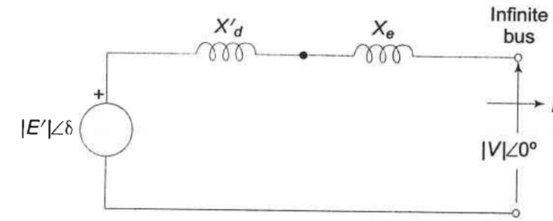


Fig. 12.14 Machine connected to infinite bus

Two Machine System

The case of two finite machines connected through a line (X_e) is illustrated in Fig. 12.15 where one of the machines must be generating and the other must be motoring. Under steady condition, before the system goes into dynamics and

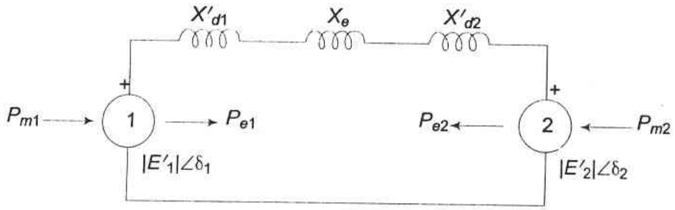


Fig. 12.15 Two-machine system

$$P_{m1} = -P_{m2} = P_m \qquad (12.38a)$$

the mechanical input/output of the two machines is assumed to remain constant at these values throughout the dynamics (governor action assumed slow). During steady state or in dynamic condition, the electrical power output of the generator must be absorbed by the motor (network being lossless). Thus at all time

$$P_{e1} = -P_{e2} = P_e \qquad (12.38b)$$

The swing equations for the two machines can now be written as

$$\frac{\mathrm{d}^2\delta_1}{\mathrm{d}t^2} = \pi f \left(\frac{P_{m1} - P_{e1}}{H_1} \right) = \pi f \left(\frac{P_m - P_e}{H_1} \right) \qquad (12.39a)$$

and

$$\frac{\mathrm{d}^2\delta_2}{\mathrm{d}t^2} = \pi f \left(\frac{P_{m2} - P_{e2}}{H_2} \right) = \pi f \left(\frac{P_e - P_m}{H_2} \right) \qquad (12.39b)$$

Subtracting Eq. (12.39b) from Eq. (12.39a)

$$\frac{\mathrm{d}^2(\delta_1 - \delta_2)}{\mathrm{d}t^2} = \pi f \left(\frac{H_1 + H_2}{H_1 H_2} \right) (P_m - P_e) \qquad (12.40)$$

or

$$\frac{H_{\text{eq}}}{\pi \mathrm{f}} \frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_m - P_e \qquad (12.41)$$

where

$$\delta = \delta_1 - \delta_2 \qquad (12.42)$$

$$H_{\text{eq}} = \frac{H_1 H_2}{H_1 + H_2} \qquad (12.43)$$

The electrical power interchange is given by expression

$$P_e = \frac{|E'_1||E'_2|}{X'_{d1} + X_e + X'_{d2}} \sin \delta \qquad (12.44)$$

The swing equation Eq. (12.41) and the power angle equation Eq. (12.44) have the same form as for a single machine connected to infinite bus. Thus a two-machine system is equivalent to a single machine connected to infinite bus. Because of this, the single-machine (connected to infinite bus) system would be studied extensively in this chapter.

Example 12.4

In the system of Example 12.3, the generator has an inertia constant of 4 MJ/MVA, write the swing equation upon occurrence of the fault. What is the initial angular acceleration? If this acceleration can be assumed to remain constant for $\Delta t = 0.05$s, find the rotor angle at the end of this time interval and the new acceleration.

Solution

Swing equation upon occurrence of fault

$$\frac{\mathrm{H}}{180 f} \frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = P_m - P_e$$

$$\frac{4}{180 \times 50} \frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = 1 - 0.694 \sin \delta$$

or

$$\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} = 2250\ (1 - 0.694 \sin \delta).$$

Initial rotor angle δ_0 = 33.9° (calculated in Example 12.3)

$$\left.\frac{d^2\delta}{dt^2}\right|_{t=0^+} = 2250\,(1 - 0.694 \sin 33.9°)$$

$$= 1379 \text{ elect deg/s}^2$$

$$\left.\frac{d\delta}{dt}\right|_{t=0^+} = 0; \text{ rotor speed cannot change suddenly}$$

$$\Delta\delta \text{ (in } \Delta t = 0.05\text{s)} = \frac{1}{2} \times 1379 \times (0.05)^2$$

$$= 1.7°$$

$$\delta_1 = \delta_0 + \Delta\delta = 33.9 + 1.7° = 35.6°$$

$$\left.\frac{d^2\delta}{dt^2}\right|_{t=0.05s} = 2250\,(1 - 0.694 \sin 35.6°)$$

$$= 1341 \text{ elect deg/s}^2$$

Observe that as the rotor angle increases, the electrical power output of the generator increases and so the acceleration of the rotor reduces.

12.6 STEADY STATE STABILITY

The steady state stability limit of a particular circuit of a power system is defined as the maximum power that can be transmitted to the receiving end without loss of synchronism.

Consider the simple system of Fig. 12.14 whose dynamics is described by equations

$$M\frac{d^2\delta}{dt^2} = P_m - P_e \text{ MW; Eq. (12.8)}$$

$$M = \frac{H}{\pi f} \text{ in pu system} \tag{12.45}$$

and

$$P_e = \frac{|E||V|}{X_d} \sin \delta = P_{max} \sin \delta \tag{12.46}$$

For determination of steady state stability, the direct axis reactance (X_d) and voltage behind X_d are used in the above equations.

The plot of Eq. (12.46) is given in Fig. 12.6. Let the system be operating with steady power transfer of $P_{e0} = P_m$ with torque angle δ_0 as indicated in the figure. Assume a small increment ΔP in the electric power with the input from the prime mover remaining fixed at P_m (governor response is slow compared to

the speed of energy dynamics), causing the torque angle to change to ($\delta_0 + \Delta\delta$). Linearizing about the operating point Q_0 (P_{e0}, δ_0) we can write

$$\Delta P_e = \left(\frac{\partial P_e}{\partial \delta}\right)_0 \Delta\delta$$

The excursions of $\Delta\delta$ are then described by

$$M\frac{d^2\Delta\delta}{dt^2} = P_m - (P_{e0} + \Delta P_e) = -\Delta P_e$$

or

$$M\frac{d^2\Delta\delta}{dt^2} + \left[\frac{\partial P_e}{\partial \delta}\right]_0 \Delta\delta = 0 \tag{12.47}$$

or

$$\left[Mp^2 + \left(\frac{\partial P_e}{\partial \delta}\right)_0\right]\Delta\delta = 0$$

where

$$p = \frac{d}{dt}$$

The system stability to small changes is determined from the characteristic equation

$$Mp^2 + \left[\frac{\partial P_e}{\partial \delta}\right]_0 = 0$$

whose two roots are

$$p = \pm\left[\frac{-(\partial P_e/\partial\delta)_0}{M}\right]^{\frac{1}{2}}$$

As long as $(\partial P_e/\partial \delta)_0$ is positive, the roots are purely imaginary and conjugate and the system behaviour is oscillatory about δ_0. Line resistance and damper windings of machine, which have been ignored in the above modelling, cause the system oscillations to decay. The system is therefore stable for a small increment in power so long as

$$(\partial P_e/\partial\delta)_0 > 0 \tag{12.48}$$

When $(\partial P_e/\partial\delta)_0$ is negative, the roots are real, one positive and the other negative but of equal magnitude. The torque angle therefore increases without bound upon occurrence of a small power increment (disturbance) and the synchronism is soon lost. The system is therefore unstable for

$$(\partial P_e/\partial\delta)_0 < 0$$

$(\partial P_e/\partial\delta)_0$ is known as *synchronizing coefficient*. This is also called *stiffness* (electrical) of synchronous machine.

Assuming $|E|$ and $|V|$ to remain constant, the system is unstable, if

$$\frac{|E||V|}{X}\cos\delta_0 < 0$$

or

$$\delta_0 > 90° \qquad (12.49)$$

The maximum power that can be transmitted without loss of stability (steady state) occurs for

$$\delta_0 = 90° \qquad (12.50)$$

and is given by

$$P_{max} = \frac{|E||V|}{X} \qquad (12.51)$$

If the system is operating below the limit of steady stability condition (Eq. 12.48), it may continue to oscillate for a long time if the damping is low. Persistent oscillations are a threat to system security. The study of system damping is the study of dynamical stability.

The above procedure is also applicable for complex systems wherein governor action and excitation control are also accounted for. The describing differential equation is linearized about the operating point. Condition for steady state stability is then determined from the corresponding characteristic equation (which now is of order higher than two).

It was assumed in the above account that the internal machine voltage $|E|$ remains constant (i.e., excitation is held constant). The result is that as loading increases, the terminal voltage $|V_t|$ dips heavily which cannot be tolerated in practice. Therefore, we must consider the steady state stability limit by assuming that excitation is adjusted for every load increase to keep $|V_t|$ constant. This is how the system will be operated practically. It may be understood that we are still not considering the effect of automatic excitation control.

Steady state stability limit with $|V_t|$ and $|V|$ constant is considered in Example 12.6.

Example 12.5

A synchronous generator of reactance 1.20 pu is connected to an infinite bus bar ($|V|$ = 1.0 pu) through transformers and a line of total reactance of 0.60 pu. The generator no load voltage is 1.20 pu and its inertia constant is H = 4 MW-s/MVA. The resistance and machine damping may be assumed negligible. The system frequency is 50 Hz.

Calculate the frequency of natural oscillations if the generator is loaded to (i) 50% and (ii) 80% of its maximum power limit.

Solution

(i) For 50% loading

$$\sin\delta_0 = \frac{P_e}{P_{max}} = 0.5 \text{ or } \delta_0 = 30°$$

$$\left[\frac{\partial P_e}{\partial \delta}\right]_{30°} = \frac{1.2\times 1}{1.8}\cos 30°$$

$$= 0.577 \text{ MW (pu)/elect rad}$$

$$M(\text{pu}) = \frac{H}{\pi\times 50} = \frac{4}{\pi\times 50} \text{ s}^2\text{/elect rad}$$

From characteristic equation

$$p = \pm j\left[\left(\frac{\partial P_e}{\partial \delta}\right)_{30°}\Big/ M\right]^{\frac{1}{2}}$$

$$= \pm j\left(\frac{0.577\times 50\,\pi}{4}\right)^{\frac{1}{2}} = \pm j4.76$$

$$\text{Frequency of oscillations} = 4.76 \text{ rad/sec}$$

$$= \frac{4.76}{2\pi} = 0.758 \text{ Hz}$$

(ii) For 80% loading

$$\sin\delta_0 = \frac{P_e}{P_{max}} = 0.8 \text{ or } \delta_0 = 53.1°$$

$$\left(\frac{\partial P_e}{\partial \delta}\right)_{53.1°} = \frac{1.2\times 1}{1.8}\cos 53.1°$$

$$= 0.4 \text{ MW (pu)/elect rad}$$

$$p = \pm j\left(\frac{0.4\times 50\,\pi}{4}\right)^{\frac{1}{2}} = \pm j3.96$$

$$\text{Frequency of oscillations} = 3.96 \text{ rad/sec}$$

$$= \frac{3.96}{2\pi} = 0.63 \text{ Hz}$$

Example 12.6

Find the steady state power limit of a system consisting of a generator equivalent reactance 0.50 pu connected to an infinite bus through a series reactance of 1.0 pu. The terminal voltage of the generator is held at 1.20 pu and the voltage of the infinite bus is 1.0 pu.

Solution

The system is shown in Fig. 12.16. Let the voltage of the infinite bus be taken as reference.

Then $V = 1.0 \angle 0°,\ V_t = 1.2 \angle \theta$

Now $$I = \frac{V_t - V}{jX} = \frac{1.2 \angle \theta - 1.0}{j1}$$

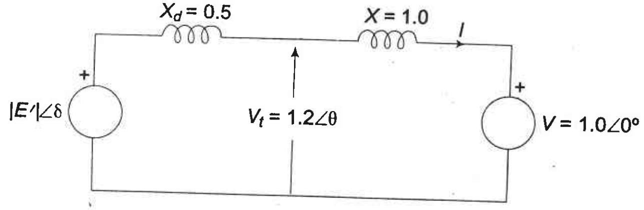


Fig. 12.16

$$E = V_t + jX_d I = 1.2 \angle \theta + j0.5 \left[\frac{1.2 \angle \theta - 1.0}{j1} \right]$$

or

$$E = 1.8 \angle \theta - 0.5 = (1.8 \cos \theta - 0.5) + j1.8 \sin \theta$$

Steady state power limit is reached when E has an angle of $\delta = 90°$, i.e., its real part is zero. Thus,

$$1.8 \cos \theta - 0.5 = 0$$

or $$\theta = 73.87°$$

Now $$V_t = 1.2 \angle 73.87° = 0.332 + j1.152$$

$$I = \frac{0.332 + j1.152 - 1}{j1} = 1.152 + j0.668$$

$$E = 0.332 + j1.152 + j0.5\,(1.152 + j0.668)$$

$$= -0.002 + j1.728 \simeq 1.728 \angle 90°$$

Steady state power limit is given by

$$P_{max} = \frac{|E|\,|V|}{X_d + X} = \frac{1.728 \times 1}{1.5} = 1.152 \text{ pu}$$

If instead, the generator emf is held fixed at a value of 1.2 pu, the steady state power limit would be

$$P_{max} = \frac{1.2 \times 1}{1.5} = 0.8 \text{ pu}$$

It is observed that regulating the generator emf to hold the terminal generator voltage at 1.2 pu raises the power limit from 0.8 pu to 1.152 pu; this is how the voltage regulating loop helps in power system stability.

Some Comments on Steady State Stability

A knowledge of steady state stability limit is important for various reasons. A system can be operated above its transient stability limit but not above its steady state limit. Now, with increased fault clearing speeds, it is possible to make the transient limit closely approach the steady state limit.

As is clear from Eq. (12.51), the methods of improving steady state stability limit of a system are to reduce X and increase either or both $|E|$ and $|V|$. If the transmission lines are of sufficiently high reactance, the stability limit can be raised by using two parallel lines which incidently also increases the reliability of the system. Series capacitors are sometimes employed in lines to get better voltage regulation and to raise the stability limit by decreasing the line reactance. Higher excitation voltages and quick excitation system are also employed to improve the stability limit.

12.7 TRANSIENT STABILITY

It has been shown in Sec. 12.4 that the dynamics of a single synchronous machine connected to infinite bus bars is governed by the nonlinear differential equation

$$M \frac{d^2\delta}{dt^2} = P_m - P_e$$

where $$P_e = P_{max} \sin \delta$$

or $$M \frac{d^2\delta}{dt^2} = P_m - P_{max} \sin \delta \tag{12.52}$$

As said earlier, this equation is known as the *swing equation*. No closed form solution exists for swing equation except for the simple case $P_m = 0$ (not a practical case) which involves elliptical integrals. For small disturbance (say, gradual loading), the equation can be linearized (see Sec. 12.6) leading to the concept of steady state stability where a unique criterion of stability ($\partial P_e / \partial \delta > 0$) could be established. No generalized criteria are available* for determining system stability with large disturbances (called transient stability). The practical approach to the transient stability problem is therefore to list all important severe disturbances along with their possible locations to which the system is likely to be subjected according to the experience and judgement of the power system analyst. Numerical solution of the swing equation (or equations for a multimachine case) is then obtained in the presence of such disturbances giving a plot of δ vs. t called the *swing curve*. If δ starts to decrease after reaching a maximum value, it is normally assumed that the system is stable and the oscillation of δ around the equilibrium point will decay

*Recent literature gives methods of determining transient stability through Liapunov and Popov's stability criteria, but these have not been of partical use so far.

and finally die out. As already pointed out in the introduction, important severe disturbances are a short circuit or a sudden loss of load.

For ease of analysis certain assumptions and simplifications are always made (some of these have already been made in arriving at the swing equation (Eq. (12.52)). All the assumptions are listed, below along with their justification and consequences upon accuracy of results.

1. Transmission line as well as synchronous machine resistance are ignored. This leads to pessimistic result as resistance introduces damping term in the swing equation which helps stability. In Example 12.11, line resistance has been taken into account.
2. Damping term contributed by synchronous machine damper windings is ignored. This also leads to pessimistic results for the transient stability limit.
3. Rotor speed is assumed to be synchronous. In fact it varies insignificantly during the course of the stability transient.
4. Mechanical input to machine is assumed to remain constant during the transient, i.e., regulating action of the generator loop is ignored. This leads to pessimistic results.
5. Voltage behind transient reactance is assumed to remain constant, i.e., action of voltage regulating loop is ignored. It also leads to pessimistic results.
6. Shunt capacitances are not difficult to account for in a stability study. Where ignored, no greatly significant error is caused.
7. Loads are modelled as constant admittances. This is a reasonably accurate representation.

Note: Since rotor speed and hence frequency vary insignificantly, the network parameters remain fixed during a stability study.

A digital computer programme to compute the transient following sudden disturbance can be suitably modified to include the effect of governor action and excitation control.

Present day power systems are so large that even after lumping of machines (Eq. (12.17)), the system remains a multimachine one. Even then, a simple two-machine system greatly aids the understanding of the transient stability problem. It has been shown in Section 12.4 that an equivalent single-machine infinite bus system can be found for a two-machine system (Eqs. (12.41) to (12.43)).

Upon occurrence of a severe disturbance, say a short circuit, the power transfer between machines is greatly reduced, causing the machine torque angles to swing relatively. The circuit breakers near the fault disconnect the unhealthy part of the system so that power transfer can be partially restored, improving the chances of the system remaining stable. The shorter the time to breaker operating, called *clearing time*, the higher is the probability of the system being stable. Most of the line faults are transient in nature and get cleared on opening the line. Therefore, it is common practice now to employ *autoreclose breakers* which automatically close rapidly after each of the two

sequential openings. If the fault still persists, the circuit breakers open and lock permanently till cleared manually. Since in the majority of faults the first reclosure will be successful, the chances of system stability are greatly enhanced by using autoreclose breakers.

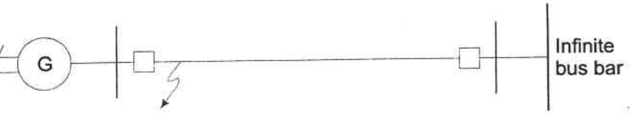


Fig. 12.17

The procedure of determining the stability of a system upon occurrence of a disturbance followed by various switching off and switching on actions is called a *stability study*. Steps to be followed in a stability study are outlined below for a single-machine infinite bus bar system shown in Fig. 12.17. The fault is assumed to be a transient one which is cleared by the time of first reclosure. In the case of a permanent fault, this system completely falls apart. This will not be the case in a multimachine system. The steps listed, in fact, apply to a system of any size.

1. From prefault loading, determine the voltage behind transient reactance and the torque angle δ_0 of the machine with reference to the infinite bus.
2. For the specified fault, determine the power transfer equation $P_e(\delta)$ during fault. In this system $P_e = 0$ for a three-phase fault.
3. From the swing equation starting with δ_0 as obtained in step 1, calculate δ as a function of time using a numerical technique of solving the non-linear differential equation.
4. After clearance of the fault, once again determine $P_e(\delta)$ and solve further for $\delta(t)$. In this case, $P_e(\delta) = 0$ as when the fault is cleared, the system gets disconnected.
5. After the transmission line is switched on, again find $P_e(\delta)$ and continue to calculate $\delta(t)$.
6. If $\delta(t)$ goes through a maximum value and starts to reduce, the system is regarded as stable. It is unstable if $\delta(t)$ continues to increase. Calculation is ceased after a suitable length of time.

An important numerical method of calculating $\delta(t)$ from the swing equation will be given in Section 12.9. For the single machine infinite bus bar system, stability can be conveniently determined by the equal area criterion presented in the following section.

12.8 EQUAL AREA CRITERION

In a system where one machine is swinging with respect to an infinite bus, it is possible to study transient stability by means of a simple criterion, without resorting to the numerical solution of a swing equation.

Consider the swing equation

$$\frac{d^2\delta}{dt^2} = \frac{1}{M}(P_m - P_e) = \frac{P_a}{M};\ P_a = \text{accelerating power}$$

$$M = \frac{H}{\pi f} \text{ in pu system} \tag{12.53}$$

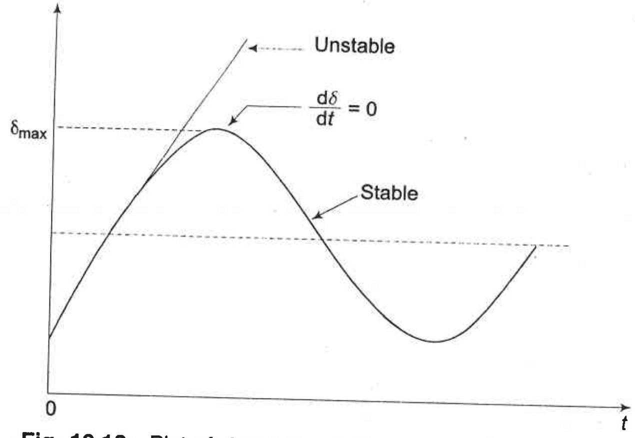


Fig. 12.18 Plot of δ vs t for stable and unstable systems

If the system is unstable δ continues to increase indefinitely with time and the machine loses synchronism. On the other hand, if the system is stable, $\delta(t)$ performs oscillations (nonsinusoidal) whose amplitude decreases in actual practice because of damping terms (not included in the swing equation). These two situations are shown in Fig. 12.18. Since the system is non-linear, the nature of its response [$\delta(t)$] is not unique and it may exhibit instability in a fashion different from that indicated in Fig. 12.18, depending upon the nature and severity of disturbance. However, experience indicates that the response $\delta(t)$ in a power system generally falls in the two broad categories as shown in the figure. It can easily be visualized now (this has also been stated earlier) that for a stable system, indication of stability will be given by observation of the first swing where δ will go to a maximum and will start to reduce. This fact can be stated as a stability criterion, that the system is stable if at some time

$$\frac{d\delta}{dt} = 0 \tag{12.54}$$

and is unstable, if

$$\frac{d\delta}{dt} > 0 \tag{12.55}$$

for a sufficiently long time (more than 1 s will generally do).

The stability criterion for power systems stated above can be converted into a simple and easily applicable form for a single machine infinite bus system.

Multiplying both sides of the swing equation by $\left(2\frac{d\delta}{dt}\right)$, we get

$$2\,\frac{d\delta}{dt}\cdot\frac{d^2\delta}{dt^2} = \frac{2P_a}{M}\frac{d\delta}{dt}$$

Integrating, we have

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a\, d\delta$$

or

$$\frac{d\delta}{dt} = \left(\frac{2}{M}\int_{\delta_0}^{\delta} P_a\, d\delta\right)^{\frac{1}{2}} \tag{12.56}$$

where δ_0 is the initial rotor angle before it begins to swing due to disturbance.

From Eqs. (12.55) and (12.56), the condition for stability can be written as

$$\left(\frac{2}{M}\int_{\delta_0}^{\delta} P_a d\,\delta\right)^{\frac{1}{2}} = 0$$

or

$$\int_{\delta_0}^{\delta} P_a\, d\delta = 0 \tag{12.57}$$

The condition of stability can therefore be stated as: the system is stable if the area under P_a (accelerating power) – δ curve reduces to zero at some value of δ. In other words, the positive (accelerating) area under $P_a - \delta$ curve must equal the negative (decelerating) area and hence the name 'equal area' criterion of stability.

To illustrate the equal area criterion of stability, we now consider several types of disturbances that may occur in a single machine infinite bus bar system.

Sudden Change in Mechanical Input

Figure 12.19 shows the transient model of a single machine tied to infinite bus bar. The electrical power transmitted is given by

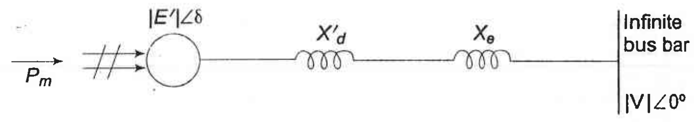


Fig. 12.19

$$P_e = \frac{|E'||V|}{X'_d + X_e} \sin \delta = P_{max} \sin \delta$$

Under steady operating condition

$$P_{m0} = P_{e0} = P_{max} \sin \delta_0$$

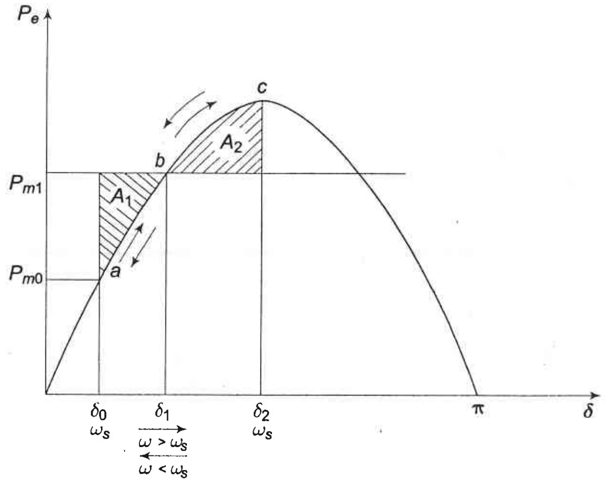


Fig. 12.20 $P_e - \delta$ diagram for sudden increase in mechanical input to generator of Fig. 12.19

This is indicated by the point a in the $P_e - \delta$ diagram of Fig. 12.20.

Let the mechanical input to the rotor be suddenly increased to P_{m1} (by opening the steam valve). The accelerating power $P_a = P_{m1} - P_e$ causes the rotor speed to increase ($\omega > \omega_s$) and so does the rotor angle. At angle δ_1, $P_a = P_{m1} - P_e$ $(= P_{max} \sin \delta_1) = 0$ (state point at b) but the rotor angle continues to increase as $\omega > \omega_s$. P_a now becomes negative (decelerating), the rotor speed begins to reduce but the angle continues to increase till at angle δ_2, $\omega = \omega_s$ once again (state point at c. At c), the decelerating area A_2 equals the accelerating area A_1 (areas are shaded), i.e., $\int_{\delta_0}^{\delta_2} P_a \, d\delta = 0$. Since the rotor is decelerating, the speed reduces below ω_s and the rotor angle begins to reduce. The state point now traverses the $P_e - \delta$ curve in the opposite direction as indicated by arrows in Fig. 12.20. It is easily seen that the system oscillates about the new steady state point b ($\delta = \delta_1$) with angle excursion up to δ_0 and δ_2 on the two sides. These oscillations are similar to the simple harmonic motion of an inertia-spring system except that these are not sinusoidal.

As the oscillations decay out because of inherent system damping (not modelled), the system settles to the new steady state where

$$P_{m1} = P_e = P_{max} \sin \delta_1$$

From Fig. 12.20, areas A_1 and A_2 are given by

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e) \, d\delta$$

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{m1}) \, d\delta$$

For the system to be stable, it should be possible to find angle δ_2 such that $A_1 = A_2$. As P_{m1} is increased, a limiting condition is finally reached when A_1 equals the area above the P_{m1} line as shown in Fig. 12.21. Under this condition, δ_2 acquires the maximum value such that

$$\delta_2 = \delta_{max} = \pi - \delta_1 = \pi - \sin^{-1} \frac{P_{m1}}{P_{max}} \tag{12.58}$$

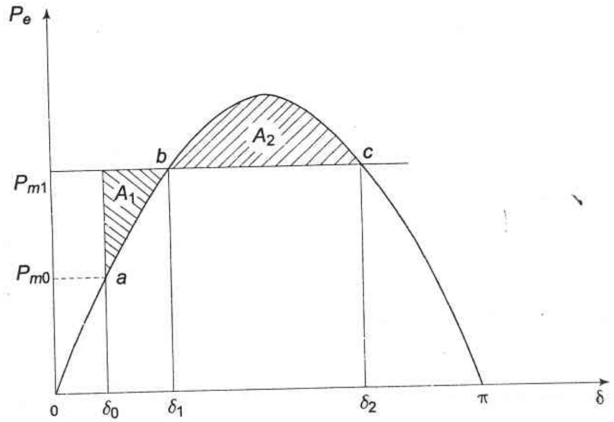


Fig. 12.21 Limiting case of transient stability with mechanical input suddenly increased

Any further increase in P_{m1} means that the area available for A_2 is less than A_1, so that the excess kinetic energy causes δ to increase beyond point c and the decelerating power changes over to accelerating power, with the system consequently becoming unstable. It has thus been shown by use of the equal area criterion that there is an upper limit to sudden increase in mechanical input ($P_{m1} - P_{m0}$), for the system in question to remain stable.

It may also be noted from Fig. 12.21 that the system will remain stable even though the rotor may oscillate beyond $\delta = 90°$, so long as the equal area criterion is met. The condition of $\delta = 90°$ is meant for use in steady state stability only and does not apply to the transient stability case.

Effect of Clearing Time on Stability

Let the system of Fig. 12.22 be operating with mechanical input P_m at a steady angle of δ_0 ($P_m = P_e$) as shown by the point a on the $P_e - \delta$ diagram of Fig. 12.23. If a 3-phase fault occurs at the point P of the outgoing radial line, the electrical output of the generator instantly reduces to zero, i.e., $P_e = 0$ and the state point drops to b. The acceleration area A_1 begins to increase and so does the rotor angle while the state point moves along bc. At time t_c corresponding to angle δ_c the faulted line is cleared by the opening of the line circuit breaker. The values of t_c and δ_c are respectively known as *clearing time* and *clearing angle*. The system once again becomes healthy and transmits $P_e = P_{max} \sin \delta$ i.e. the state point shifts to d on the original $P_e - \delta$ curve. The rotor now decelerates and the decelerating area A_2 begins while the state point moves along de.

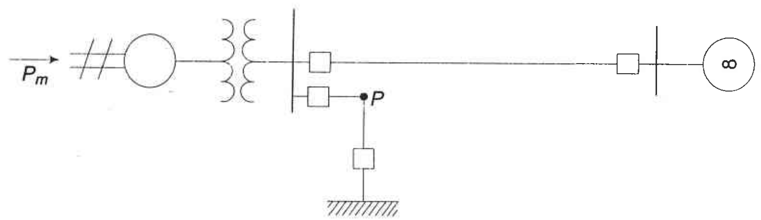


Fig. 12.22

If an angle δ_1 can be found such that $A_2 = A_1$, the system is found to be stable. The system finally settles down to the steady operating point a in an oscillatory manner because of inherent damping.

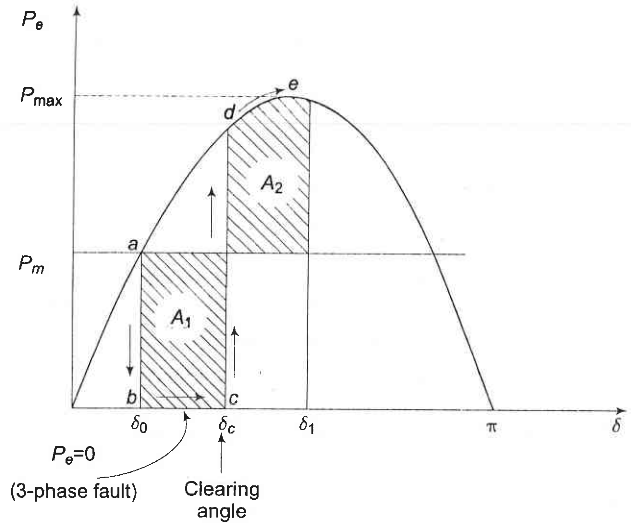


Fig. 12.23

The value of clearing time corresponding to a clearing angle can be established only by numerical integration except in this simple case. The equal area criterion therefore gives only qualitative answer to system stability as the time when the breaker should be opened is hard to establish.

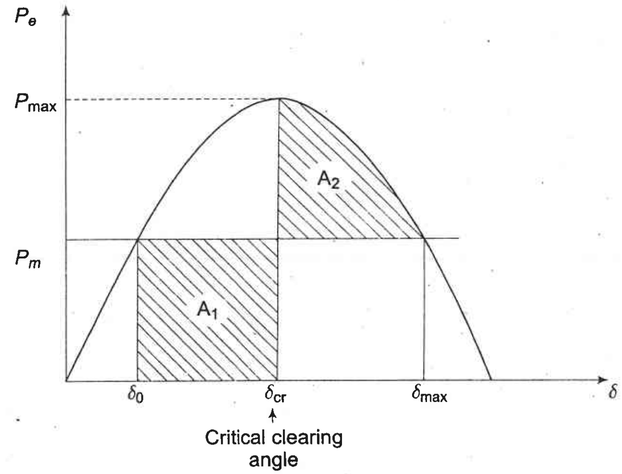


Fig. 12.24 Critical clearing angle

As the clearing of the faulty line is delayed, A_1 increases and so does δ_1 to find $A_2 = A_1$ till $\delta_1 = \delta_{max}$ as shown in Fig. 12.24. For a clearing time (or angle) larger than this value, the system would be unstable as $A_2 < A_1$. The maximum allowable value of the clearing time and angle for the system to remain stable are known respectively as *critical clearing time* and *angle*.

For this simple case (P_e = 0 during fault), explicit relationships for δ_c (critical) and t_c (critical) are established below. All angles are in *radians*.

It is easily seen from Fig. 12.24 that

$$\delta_{max} = \pi - \delta_0 \tag{12.59}$$

and

$$P_m = P_{max} \sin \delta_0 \tag{12.60}$$

Now

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - 0)\, d\delta = P_m (\delta_{cr} - \delta_0)$$

and

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin \delta - P_m)\, d\delta$$

$$= P_{max} (\cos \delta_{cr} - \cos \delta_{max}) - P_m (\delta_{max} - \delta_{cr})$$

For the system to be stable, $A_2 = A_1$, which yields

$$\cos \delta_{cr} = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos \delta_{max} \tag{12.61}$$

where δ_{cr} = critical clearing angle

Substituting Eqs. (12.59) and (12.60) in Eq. (12.61), we get

$$\delta_{cr} = \cos^{-1} [(\pi - 2\delta_0) \sin \delta_0 - \cos \delta_0] \tag{12.62}$$

During the period the fault is persisting, the swing equation is

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_m;\ P_e = 0 \tag{12.63}$$

Integrating twice

$$\delta = \frac{\pi f}{2H} P_m t^2 + \delta_0$$

or

$$\delta_{cr} = \frac{\pi f}{2H} P_m t^2_{cr} + \delta_0 \tag{12.64}$$

where

t_{cr} = critical clearing time

δ_{cr} = critical clearing angle

From Eq. (12.64)

$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m}} \tag{12.65}$$

where δ_{cr} is given by the expression of Eq. (12.62)

An explicit relationship for determining t_{cr} is possible in this case as during the faulted condition $P_e = 0$ and so the swing equation can be integrated in closed form. This will not be the case in most other situations.

Sudden Loss of One of Parallel Lines

Consider now a single machine tied to infinite bus through two parallel lines as in Fig. 12.25a. Circuit model of the system is given in Fig. 12.25b.

Let us study the transient stability of the system when one of the lines is suddenly switched off with the system operating at a steady load. Before switching off, power angle curve is given by

$$P_{eI} = \frac{|E'||V|}{X_d + X_1 \| X_2} \sin \delta = P_{maxI} \sin \delta$$

Immediately on switching off line 2, power angle curve is given by

$$P_{eII} = \frac{|E'||V|}{X'_d + X_1} \sin \delta = P_{maxII} \sin \delta$$

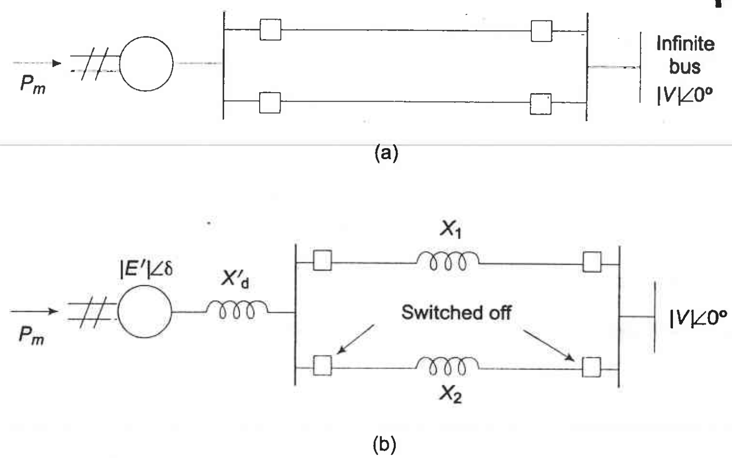


Fig. 12.25 Single machine tied to infinite bus through two parallel lines

Both these curves are plotted in Fig. 12.26, wherein $P_{maxII} < P_{maxI}$ as $(X'_d + X_1) > (X'_d + X_1 \| X_2)$. The system is operating initially with a steady power transfer $P_e = P_m$ at a torque angle δ_0 on curve I.

Immediately on switching off line 2, the electrical operating point shifts to curve II (point b). Accelerating energy corresponding to area A_1 is put into rotor followed by decelerating energy for $\delta > \delta_1$. Assuming that an area A_2 corresponding to decelerating energy (energy out of rotor) can be found such that $A_1 = A_2$, the system will be stable and will finally operate at c corresponding to a new rotor angle $\delta_1 > \delta_0$. This is so because a single line offers larger reactance and larger rotor angle is needed to transfer the same steady power.

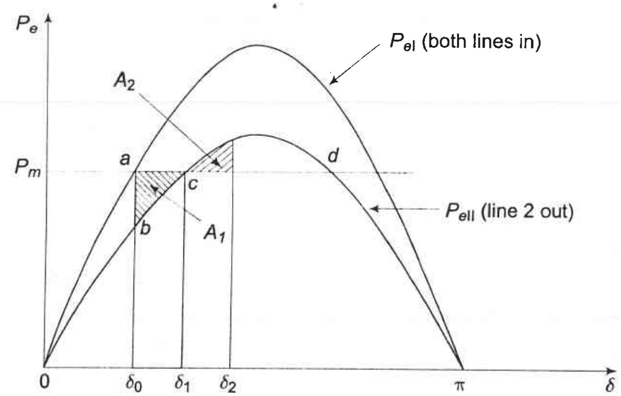


Fig. 12.26 Equal area criterion applied to the opening of one of the two lines in parallel

It is also easy to see that if the steady load is increased (line P_m is shifted upwards in Fig. 12.26), a limit is finally reached beyond which decelerating area equal to A_1 cannot be found and therefore, the system behaves as an unstable one. For the limiting case of stability, δ_1 has a maximum value given by

$$\delta_1 = \delta_{max} = \pi - \delta_c$$

which is the same condition as in the previous example.

Sudden Short Circuit on One of Parallel Lines

Case a: Short circuit at one end of line

Let us now assume the disturbance to be a short circuit at the generator end of line 2 of a double circuit line as shown in Fig. 12.27a. We shall assume the fault to be a three-phase one.

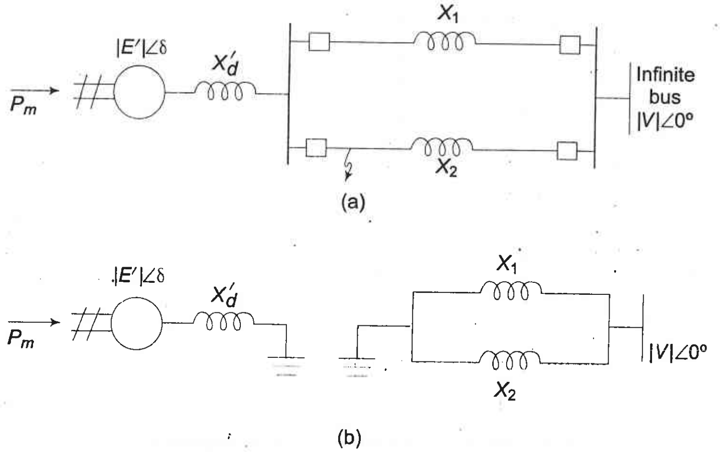


Fig. 12.27 Short circuit at one end of the line

Before the occurrence of a fault, the power angle curve is given by

$$P_{eI} = \frac{|E'||V|}{X'_d + X_1 \| X_2} \sin \delta = P_{maxI} \sin \delta$$

which is plotted in Fig. 12.25.

Upon occurrence of a three-phase fault at the generator end of line 2 (see Fig. 12.24a), the generator gets isolated from the power system for purposes of power flow as shown by Fig. 12.27b. Thus during the period the fault lasts,

$$P_{eII} = 0$$

The rotor therefore accelerates and angles δ increases. Synchronism will be lost unless the fault is cleared in time.

The circuit breakers at the two ends of the faulted line open at time t_c (corresponding to angle δ_c), the clearing time, disconnecting the faulted line.

The power flow is now restored via the healthy line (through higher line reactance X_2 in place of $X_1 \| X_2$), with power angle curve

$$P_{eIII} = \frac{|E'||V|}{X'_d + X_1} \sin \delta = P_{max\,II} \sin \delta$$

Obviously, $P_{maxII} < P_{maxI}$. The rotor now starts to decelerate as shown in Fig. 12.28. The system will be stable if a decelerating area A_2 can be found equal to accelerating area A_1 before δ reaches the maximum allowable value δ_{max}. As area A_1 depends upon clearing time t_c (corresponding to clearing angle δ_c), clearing time must be less than a certain value (critical clearing time) for the system to be stable. It is to be observed that the equal area criterion helps to determine critical clearing angle and not critical clearing time. Critical clearing time can be obtained by numerical solution of the swing equation (discussed in Section 12.8).

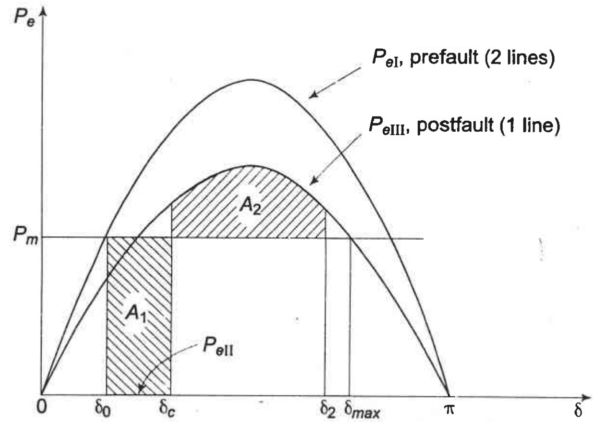


Fig. 12.28 Equal area criterion applied to the system of Fig. 12.24a, I system normal, II fault applied, III faulted line isolated.

It also easily follows that larger initial loading (P_m) increases A_1 for a given clearing angle (and time) and therefore quicker fault clearing would be needed to maintain stable operation.

Case b: Short circuit away from line ends

When the fault occurs away from line ends (say in the middle of a line), there is some power flow during the fault though considerably reduced, as different from case *a* where $P_{eII} = 0$. Circuit model of the system during fault is now shown in Fig. 12.29a. This circuit reduces to that of Fig. 12.29c through one delta-star and one star-delta conversion. Instead, node elimination technique of Section 12.3 could be employed profitably. The power angle curve during fault is therefore given by

$$P_{eII} = \frac{|E'||V|}{X_{II}} \sin \delta = P_{maxII} \sin \delta$$

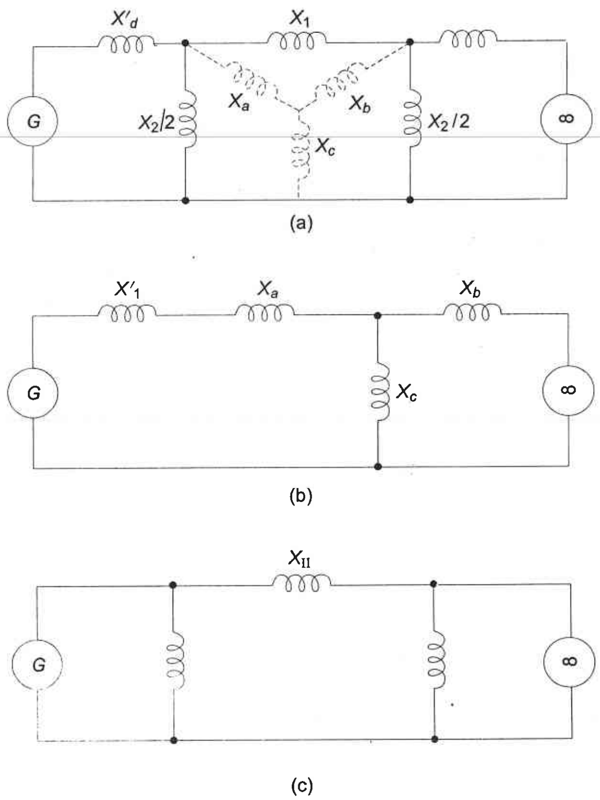


Fig. 12.29

P_{eI} and P_{eIII} as in Fig. 12.28 and $P_{e\,II}$ as obtained above are all plotted in Fig. 12.30. Accelerating area A_1 corresponding to a given clearing angle δ_c is less

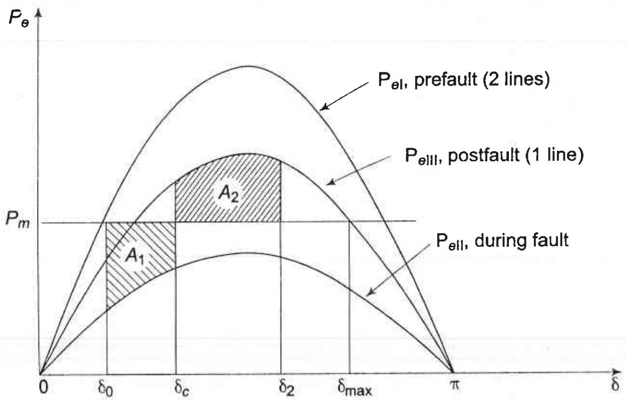


Fig. 12.30 Fault on middle of one line of the system of Fig. 12.24a with $\delta_c < \delta_{cr}$

in this case, than in case *a*, giving a better chance for stable operation. Stable system operation is shown in Fig. 12.30, wherein it is possible to find an area A_2 equal to A_1 for $\delta_2 < \delta_{max}$. As the clearing angle δ_c is increased, area A_1 increases and to find $A_2 = A_1$, δ_2 increases till it has a value δ_{max}, the maximum allowable for stability. This case of critical clearing angle is shown in Fig. 12.31.

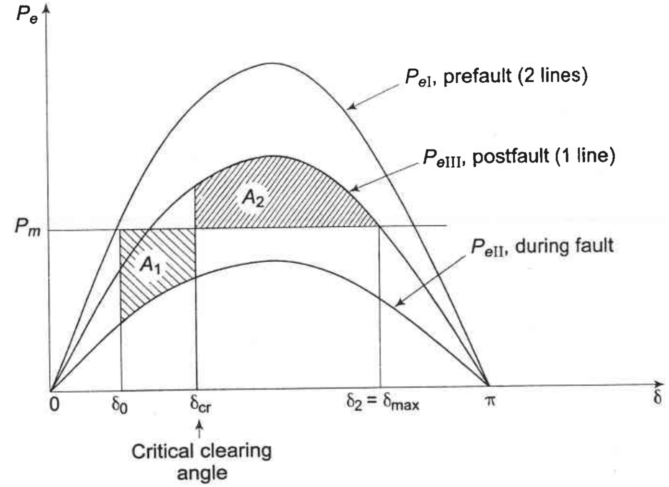


Fig. 12.31 Fault on middle of one line of the system of Fig. 12.24a, case of critical clearing angle

Applying equal area criterion to the case of critical clearing angle of Fig. 12.31, we can write

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{maxII}\sin\delta)\,d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{maxIII}\sin\delta - P_m)\,d\delta$$

where

$$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{max\,III}}\right) \tag{12.66}$$

Integrating, we get

$$(P_{m\delta} + P_{maxII}\cos\delta)\Big|_{\delta_o}^{\delta_{cr}} + (P_{maxIII}\cos\delta + P_m\delta)\Big|_{\delta_{cr}}^{\delta_{max}} = 0$$

or

$$P_m(\delta_{cr} - \delta_0) + P_{maxII}(\cos\delta_{cr} - \cos\delta_0)$$
$$+ P_m(\delta_{max} - \delta_{cr}) + P_{maxIII}(\cos\delta_{max} - \cos\delta_{cr}) = 0$$

or

$$\cos\,\delta_{cr} = \frac{P_m(\delta_{max}-\delta_0)-P_{max\,II}\cos\delta_0+P_{max\,III}\cos\delta_{max}}{P_{max\,III}-P_{max\,II}} \qquad (12.67)$$

Critical clearing angle can be calculated from Eq. (12.67) above. The angles in this equation are in radians. The equation modifies as below if the angles are in degrees.

$$\cos\,\delta_{cr} = \frac{\frac{\pi}{180}P_m(\delta_{max}-\delta_0)-P_{max\,II}\cos\delta_0+P_{max\,III}\cos\delta_{max}}{P_{max\,III}-P_{max\,II}}$$

Case c: Reclosure

If the circuit breakers of line 2 are reclosed successfully (i.e., the fault was a transient one and therefore vanished on clearing the faulty line), the power transfer once again becomes

$$P_{eIV} = P_{eI} = P_{maxI}\sin\,\delta$$

Since reclosure restores power transfer, the chances of stable operation improve. A case of stable operation is indicated by Fig. 12.32.

For critical clearing angle

$$\delta_1 = \delta_{max} = \pi - \sin^{-1}(P_m/P_{maxI})$$

$$\int_{\delta_0}^{\delta_{cr}}(P_m - P_{maxII}\sin\,\delta)\,d\delta = \int_{\delta_{cr}}^{\delta_{rc}}(P_{maxIII}\sin\,\delta - P_m)\,d\delta$$

$$+\int_{\delta_{rc}}^{\delta_{max}}(P_{maxI}\sin\,\delta - P_m)\,d\delta$$

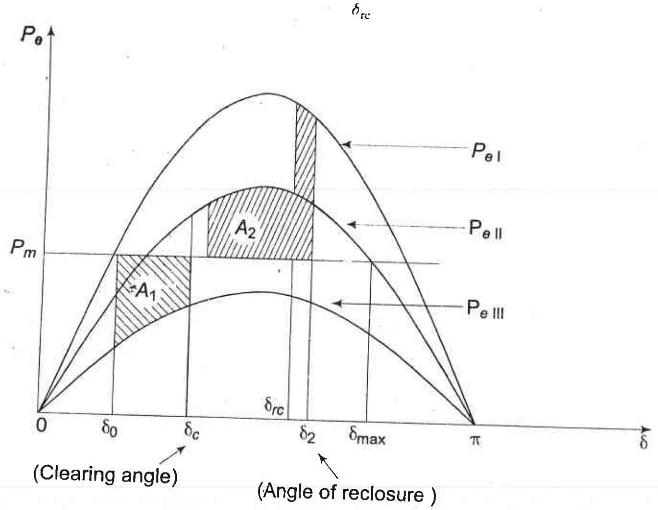


Fig. 12.32 Fault in middle of a line of the system of Fig. 12.27a

where $t_{rc} = t_{cr} + \tau$; τ = time between clearing and reclosure.

Example 12.7

Give the system of Fig. 12.33 where a three-phase fault is applied at the point P as shown.

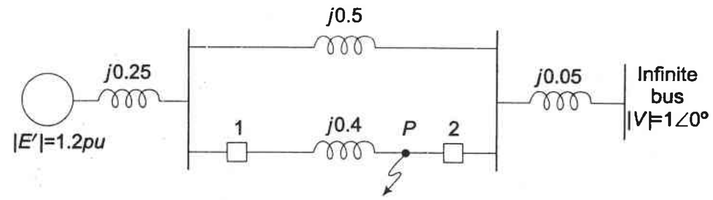


Fig. 12.33

Find the critical clearing angle for clearing the fault with simultaneous opening of the breakers 1 and 2. The reactance values of various components are indicated on the diagram. The generator is delivering 1.0 pu power at the instant preceding the fault.

Solution

With reference to Fig. 12.31, three separate power angle curves are involved.

I. Normal operation (prefault)

$$X_I = 0.25 + \frac{0.5\times0.4}{0.5+0.4} + 0.05$$

$$= 0.522 \text{ pu}$$

$$P_{eI} = \frac{|E'|\,|V|}{X_I}\sin\delta = \frac{1.2\times1}{0.522}\sin\,\delta$$

$$= 2.3\,\sin\,\delta \qquad \text{(i)}$$

Prefault operating power angle is given by

$$1.0 = 2.3\,\sin\,\delta_0$$

or $\delta_0 = 25.8° = 0.45$ radians

II. During fault

It is clear from Fig. 12.31 that no power is transferred during fault, i.e.,

$$P_{eII} = 0$$

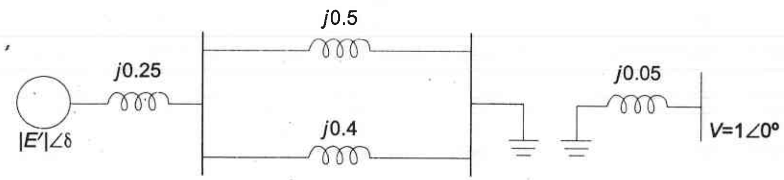


Fig. 12.34

III. Post fault operation (fault cleared by opening the faulted line)

$$X_{III} = 0.25 + 0.5 + 0.05 = 0.8$$

$$P_{eIII} = \frac{1.2 \times 1.0}{0.8} \sin \delta = 1.5 \sin \delta \qquad \text{(iii)}$$

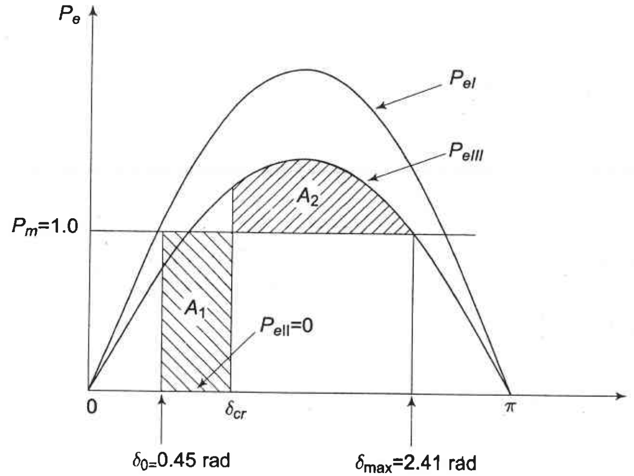


Fig. 12.35

The maximum permissible angle δ_{max} for area $A_1 = A_2$ (see Fig. 12.35) is given by

$$\delta_{max} = \pi - \sin^{-1} \frac{1}{1.5} = 2.41 \text{ radians}$$

Applying equal area criterion for critical clearing angle δ_c

$$A_1 = P_m (\delta_{cr} - \delta_0)$$

$$= 1.0 (\delta_{cr} - 0.45) = \delta_{cr} - 0.45$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{eIII} - P_m)\, d\delta$$

$$= \int_{\delta_{cr}}^{2.41} (1.5 \sin \delta - 1)\, d\delta$$

$$= - 1.5 \cos \delta - \delta \Big|_{\delta_{cr}}^{2.41}$$

$$= - 1.5 (\cos 2.41 - \cos \delta_{cr}) - (2.41 - \delta_{cr})$$

$$= 1.5 \cos \delta_{cr} + \delta_{cr} - 1.293$$

Setting $A_1 = A_2$ and solving

$$\delta_{cr} - 0.45 = 1.5 \cos \delta_{cr} + \delta_{cr} - 1.293$$

or $\qquad \cos \delta_{cr} = 0.843/1.5 = 0.562$

or $\qquad \delta_{cr} = 55.8°$

The corresponding power angle diagrams are shown in Fig. 12.35.

Example 12.8

Find the critical clearing angle for the system shown in Fig. 12.36 for a three-phase fault at the point P. The generator is delivering 1.0 pu power under prefault conditions.

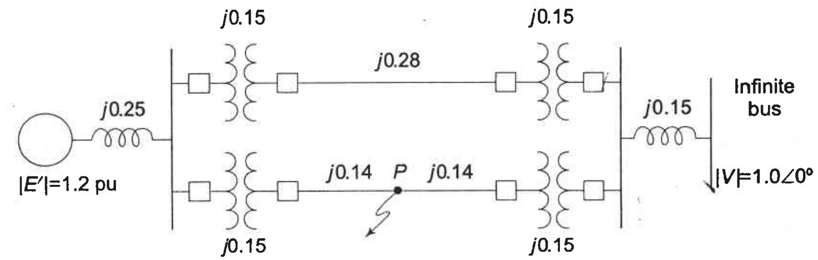


Fig. 12.36

Solution

I. Prefault operation Transfer reactance between generator and infinite bus is

$$X_I = 0.25 + 0.17 + \frac{0.15 + 0.28 + 0.15}{2} = 0.71$$

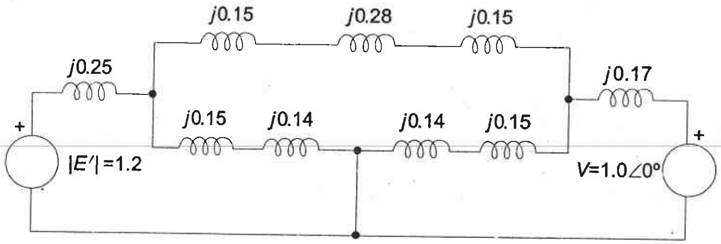
$$\therefore \qquad P_{eI} = \frac{1.2 \times 1}{0.71} \sin \delta = 1.69 \sin \delta \qquad \text{(i)}$$

The operating power angle is given by

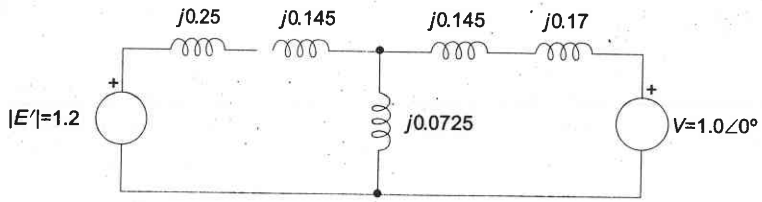
$$1.0 = 1.69 \sin \delta_0$$

or $\qquad \delta_0 = 0.633$ rad

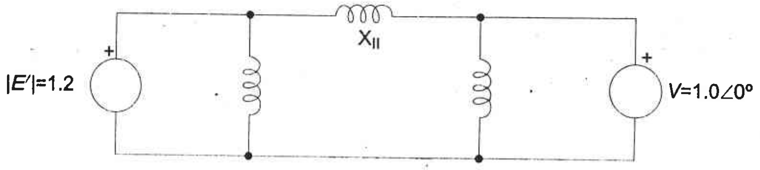
II. During fault The positive sequence reactance diagram during fault is presented in Fig. 12.37a.



(a) Positive sequence reactance diagram during fault



(b) Network after delta-star conversion



(c) Network after star-delta conversion

Fig. 12.37

Converting delta to star*, the reactance network is changed to that of Fig. 12.37(b). Further, upon converting star to delta, we obtain the reactance network of Fig. 12.37(c). The transfer reactance is given by

$$X_{II} = \frac{\dfrac{(0.25+0.145)\,0.0725+(0.145+0.17)\,0.0725+(0.25+0.145)\,(0.145+0.17)}{}}{0.075}$$

$$= 2.424$$

$$\therefore \quad P_{eII} = \frac{1.2\times 1}{2.424}\sin\delta = 0.495\sin\delta \qquad \text{(ii)}$$

III. Postfault operation (faulty line switched off)

$$X_{III} = 0.25 + 0.15 + 0.28 + 0.15 + 0.17 = 1.0$$

*Node elimination technique would be used for complex network.

$$P_{eIII} = \frac{1.2\times 1}{1}\sin\delta = 1.2\sin\delta \qquad \text{(iii)}$$

With reference to Fig. 12.30 and Eq. (12.66), we have

$$\delta_{max} = \pi - \sin^{-1}\frac{1}{1.2} = 2.155 \text{ rad}$$

To find the critical clearing angle, areas A_1 and A_2 are to be equated.

$$A_1 = 1.0\,(\delta_{cr} - 0.633) - \int_{\delta_0}^{\delta_{cr}} 0.495\sin\delta\, d\delta$$

and

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} 1.2\sin\delta\, d\delta - 1.0\,(2.155 - \delta_c)$$

Now

$$A_1 = A_2$$

or

$$\delta_{cr} = 0.633 - \int_{0.633}^{\delta_{cr}} 0.495\sin\delta\, d\delta$$

$$= \int_{\delta_{cr}}^{2.155} 1.2\sin\delta\, d\delta - 2.155 + \delta_{cr}$$

or $-0.633 + 0.495\cos\delta\Big|_{0.633}^{\delta_{cr}} = -1.2\cos\delta\Big|_{\delta_{cr}}^{2.155} - 2.155$

or $-0.633 + 0.495\cos\delta_{cr} - 0.399 = 0.661 + 1.2\cos\delta_{cr} - 2.155$

or $\cos\delta_{cr} = 0.655$

or $\delta_{cr} = 49.1°$

Example 12.9

A generator operating at 50 Hz delivers 1 pu power to an infinite bus through a transmission circuit in which resistance is ignored. A fault takes place reducing the maximum power transferable to 0.5 pu whereas before the fault, this power was 2.0 pu and after the clearance of the fault, it is 1.5 pu. By the use of equal area criterion, determine the critical clearing angle.

Solution

All the three power angle curves are shown in Fig. 12.30.

Here $\quad P_{\max I} = 2.0$ pu, $P_{\max II} = 0.5$ pu and $P_{\max III} = 1.5$ pu

Initial loading $P_m = 1.0$ pu

$$\delta_0 = \sin^{-1}\left(\frac{P_m}{P_{\max I}}\right) = \sin^{-1}\frac{1}{2} = 0.523 \text{ rad}$$

$$\delta_{\max} = \pi \sin^{-1}\left(\frac{P_m}{P_{\max III}}\right)$$

$$\doteq \pi - \sin^{-1}\frac{1}{1.5} = 2.41 \text{ rad}$$

Applying Eq. (12.67)

$$\cos \delta_{cr} = \frac{1.0(2.41 - 0.523) - 0.5 \cos 0.523 + 1.5 \cos 2.41}{1.5 - 0.5} = 0.337$$

or $\quad \delta_{cr} = 70.3°$

12.9 NUMERICAL SOLUTION OF SWING EQUATION

In most practical systems, after machine lumping has been done, there are still more than two machines to be considered from the point of view of system stability. Therefore, there is no choice but to solve the swing equation of each machine by a numerical technique on the digital computer. Even in the case of a single machine tied to infinite bus bar, the critical clearing time cannot be obtained from equal area criterion and we have to make this calculation numerically through swing equation. There are several sophisticated methods now available for the solution of the swing equation including the powerful Runge-Kutta method. Here we shall treat the point-by-point method of solution which is a conventional, approximate method like all numerical methods but a well tried and proven one. We shall illustrate the point-by-point method for one machine tied to infinite bus bar. The procedure is, however, general and can be applied to every machine of a multimachine system.

Consider the swing equation

$$\frac{d^2\delta}{dt^2} = \frac{1}{M}(P_m - P_{\max}\sin\delta) = P_a / M;$$

$$\left(M = \frac{GH}{\pi} \text{ or in pu system } M = \frac{H}{\pi f}\right)$$

The solution $\delta(t)$ is obtained at discrete intervals of time with interval spread of Δt uniform throughout. Accelerating power and change in speed which are continuous functions of time are discretized as below:

1. The accelerating power P_a computed at the beginning of an interval is assumed to remain constant from the middle of the preceding interval to the middle of the interval being considered as shown in Fig. 12.38.

2. The angular rotor velocity $\omega = d\delta/dt$ (over and above synchronous velocity ω_s) is assumed constant throughout any interval, at the value computed for the middle of the interval as shown in Fig. 12.38.

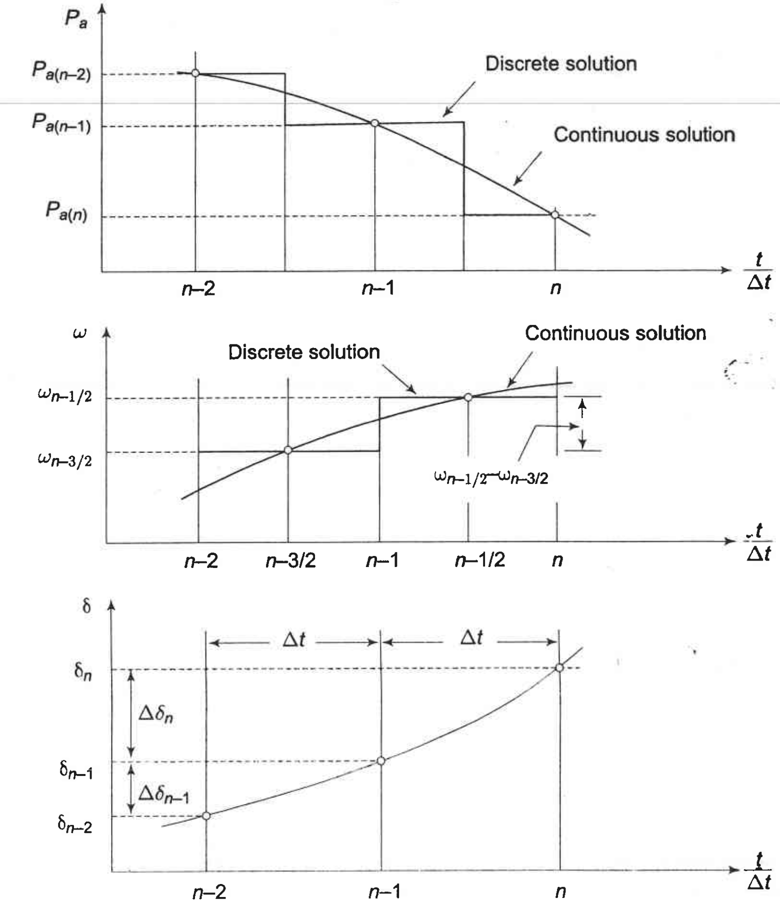


Fig. 12.38 Point-by-point solution of swing equation

In Fig. 12.38, the numbering on $t/\Delta t$ axis pertains to the end of intervals. At the end of the $(n-1)$th interval, the acceleration power is

$$P_{a\,(n-1)} = P_m - P_{\max} \sin \delta_{n-1} \tag{12.68}$$

where δ_{n-1} has been previously calculated. The change in velocity ($\omega = d\delta/dt$), caused by the $P_{a(n-1)}$, assumed constant over Δt from $(n-3/2)$ to $(n-1/2)$ is

$$w_{n-1/2} - w_{n-3/2} = (\Delta t/M)\, P_{a\,(n-1)} \tag{12.69}$$

The change in δ during the $(n-1)$th interval is

$$\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \Delta t \omega_{n-3/2} \tag{12.70a}$$

and during the nth interval

$$\Delta\delta_n = \delta_n - \delta_{n-1} = \Delta t \omega_{n-1/2} \tag{12.70b}$$

Subtracting Eq. (12.70a) from Eq. (12.70b) and using Eq. (12.69), we get

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \tag{12.71}$$

Using this, we can write

$$\delta_n = \delta_{n-1} + \Delta\delta_n \tag{12.72}$$

The process of computation is now repeated to obtain $P_{a(n)}$, $\Delta\delta_{n+1}$ and δ_{n+1}. The time solution in discrete form is thus carried out over the desired length of time, normally 0.5 s. Continuous form of solution is obtained by drawing a smooth curve through discrete values as shown in Fig. 12.38. Greater accuracy of solution can be achieved by reducing the time duration of intervals.

The occurrence or removal of a fault or initiation of any switching event causes a discontinuity in accelerating power P_a. If such a discontinuity occurs at the beginning of an interval, then the average of the values of P_a before and after the discontinuity must be used. Thus, in computing the increment of angle occurring during the first interval after a fault is applied at $t = 0$, Eq. (12.71) becomes

$$\Delta\delta_1 = \frac{(\Delta t)^2}{M} + \frac{P_{a0+}}{2}$$

where P_{a0+} is the accelerating power immediately after occurrence of fault. Immediately before the fault the system is in steady state, so that $P_{a0^-} = 0$ and δ_0 is a known value. If the fault is cleared at the beginning of the nth interval, in calculation for this interval one should use for $P_{a(n-1)}$ the value $\frac{1}{2}[P_{a(n-1)^-} + P_{a(n-1)+}]$, where $P_{a(n-1)^-}$ is the accelerating power immediately before clearing and $P_{a(n-1)+}$ is that immediately after clearing the fault. If the discontinuity occurs at the middle of an interval, no special procedure is needed. The increment of angle during such an interval is calculated, as usual, from the value of P_a at the beginning of the interval.

The procedure of calculating solution of swing equation is illustrated in the following example.

Example 12.10

A 20 MVA, 50 Hz generator delivers 18 MW over a double circuit line to an infinite bus. The generator has kinetic energy of 2.52 MJ/MVA at rated speed. The generator transient reactance is $X'_d = 0.35$ pu. Each transmission circuit has $R = 0$ and a reactance of 0.2 pu on a 20 MVA base. $|E'| = 1.1$ pu and infinite bus voltage $V = 1.0 \angle 0°$. A three-phase short circuit occurs at the mid point of one of the transmission lines. Plot swing curves with fault cleared by simultaneous opening of breakers at both ends of the line at 2.5 cycles and 6.25 cycles after the occurrence of fault. Also plot the swing curve over the period of 0.5 s if the fault is sustained.

Solution Before we can apply the step-by-step method, we need to calculate the inertia constant M and the power angle equations under prefault and postfault conditions.

Base MVA = 20

$$\text{Inertia constant, } M(\text{pu}) = \frac{H}{180 f} = \frac{1.0 \times 2.52}{180 \times 50}$$

$$= 2.8 \times 10^{-4}\ s^2/\text{elect degree}$$

I *Prefault*

$$X_{\text{I}} = 0.35 + \frac{0.2}{2} = 0.45$$

$$\therefore \quad P_{e\text{I}} = P_{\max\text{I}} \sin\delta$$

$$= \frac{1.1 \times 1}{0.45} \sin\delta = 2.44 \sin\delta \tag{i}$$

$$\text{Prefault power transfer} = \frac{18}{20} = 0.9 \text{ pu}$$

Initial power angle is given by

$$2.44 \sin\delta_0 = 0.9$$

or $\delta_0 = 21.64°$

II *During fault* A positive sequence reactance diagram is shown in Fig. 12.39a. Converting star to delta, we obtain the network of Fig. 12.39b, in which

$$X_{\text{II}} = \frac{0.35 \times 0.1 + 0.2 \times 0.1 + 0.35 \times 0.2}{0.1} = 1.25 \text{ pu}$$

$$\therefore \quad P_{e\text{II}} = P_{\max\text{II}} \sin\delta$$

$$= \frac{1.1 \times 1}{1.25} \sin\delta = 0.88 \sin\delta \tag{ii}$$

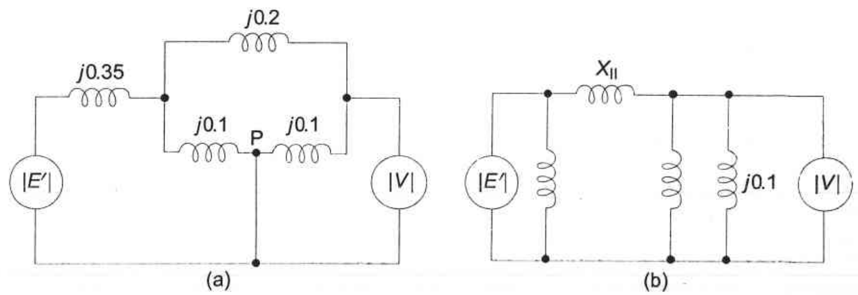


Fig. 12.39

III *Postfault* With the faulted line switched off,

$$X_{III} = 0.35 + 0.2 = 0.55$$

$$\therefore \quad P_{eIII} = P_{maxIII} \sin \delta$$

$$= \frac{1.1 \times 1}{0.55} \sin \delta = 2.0 \sin \delta \tag{iii}$$

Let us choose $\Delta t = 0.05$ s

The recursive relationships for step-by-step swing curve calculation are reproduced below.

$$P_{a(n-1)} = P_m - P_{max} \sin \delta_{n-1} \tag{iv}$$

$$\Delta \delta_n = \Delta \delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \tag{v}$$

$$\delta_n = \delta_{n-1} + \Delta \delta_n \tag{vi}$$

Since there is a discontinuity in P_e and hence in P_a, the average value of P_a must be used for the first interval.

$$P_a(0_-) = 0 \text{ pu and } P_a(0_+) = 0.9 - 0.88 \sin 21.64° = 0.576 \text{ pu}$$

$$P_a(0_{average}) = \frac{0 + 0.576}{2} = 0.288 \text{ pu}$$

Sustained Fault

Calculations are carried out in Table 12.2 in accordance with the recursive relationship (iv), (v) and (vi) above. The second column of the table shows P_{max} the maximum power that can be transferred at time t given in the first column. P_{max} in the case of a sustained fault undergoes a sudden change at $t = 0_+$ and remains constant thereafter. The procedure of calculations is illustrated below by calculating the row corresponding to $t = 0.15$ s.

$$(0.1 \text{ sec}) = 31.59°$$

$$P_{max} = 0.88$$

$$\sin \delta (0.1 \text{ s}) = 0.524$$

$$P_e (0.1 \text{ s}) = P_{max} \sin \delta (0.1 \text{ s}) = 0.88 \times 0.524 = 0.461$$

$$P_a (0.1 \text{ s}) = 0.9 - 0.461 = 0.439$$

$$\frac{(\Delta t)^2}{M} P_a (0.1 \text{ s}) = 8.929 \times 0.439 = 3.92°$$

$$\delta (0.15 \text{ s}) = \Delta \delta (0.1 \text{ s}) + \frac{(\Delta t)^2}{M} P_a (0.1 \text{ s})$$

$$= 7.38° + 3.92° = 11.33°$$

$$\delta (0.15 \text{ s}) = \delta (0.1 \text{ s}) + \Delta \delta (0.15 \text{ s})$$

$$= 31.59° + 11.30° = 42.89°$$

$\delta(t)$ for sustained fault as calculated in Table 12.2 is plotted in Fig. 12.40 from which it is obvious that the system is unstable.

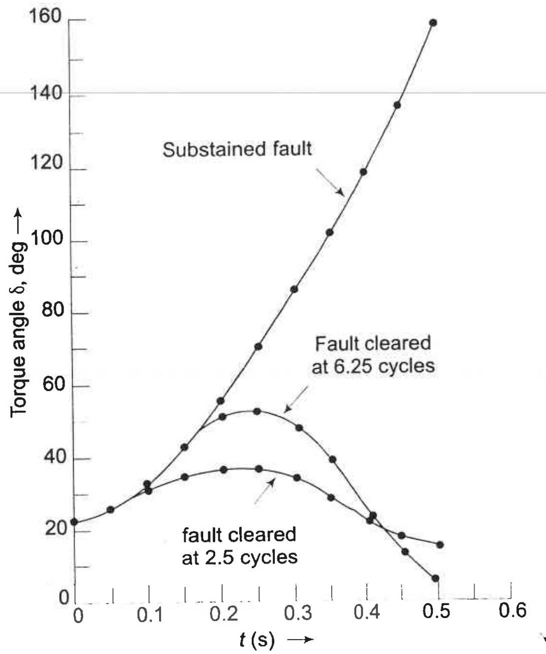


Fig. 12.40 Swing curves for Example 12.10 for a sustained fault and for clearing in 2.5 and 6.25 cycles

Table 12.2 Point-by-point computations of swing curve for sustained fault, $\Delta t = 0.05$ s

t sec	P_{max} pu	$\sin \delta$	$P_e = P_{max} \sin\delta$ pu	$P_a = 0.9 - P_e$ pu	$\frac{(\Delta t)^2}{M} P_a = 8.929 P_a$ deg	$\Delta\delta$ deg	δ deg
0_-	2.44	0.368	0.9	0.0	—	—	21.64
0_+	0.88	0.368	0.324	0.576	—	—	21.64
0_{avg}	—	0.368	—	0.288	2.57	2.57	21.64
0.05	0.88	0.41	0.361	0.539	4.81	7.38	24.21
0.10	0.88	0.524	0.461	0.439	3.92	11.30	31.59
0.15	0.88	0.680	0.598	0.301	2.68	13.98	42.89
0.20	0.88	0.837	0.736	0.163	1.45	15.43	56.87
0.25	0.88	0.953	0.838	0.06	0.55	15.98	72.30
0.30	0.88	0.999	0.879	0.021	0.18	16.16	88.28
0.35	0.88	0.968	0.852	0.048	0.426	16.58	104.44
0.40	0.88	0.856	0.754	0.145	1.30	17.88	121.02
0.45	0.88	0.657	0.578	0.321	2.87	20.75	138.90
0.50	0.88	—	—	—	—	—	159.65

Fault Cleared in 2.5 Cycles

$$\text{Time to clear fault} = \frac{2.5}{50} = 0.05 \text{ s}$$

P_{max} suddenly changes from 0.88 at $t = 0.05$ to 2.0 at $t = 0.05_+$. Since the discontinuity occurs at the beginning of an interval, the average value of P_a will be assumed to remain constant from 0.025 s to 0.075 s. The rest of the procedure is the same and complete calculations are shown in Table 12.3. The swing curve is plotted in Fig. 12.40 from which we find that the generator undergoes a maximum swing of 37.5° but is stable as δ finally begins to decrease.

Table 12.3 Computations of swing curves for fault cleared at 2.5 cycles (0.05 s), Δt = 0.05 s

t sec	P_{max} pu	$\sin\delta$	$P_e = P_{max}\sin\delta$ pu	$P_a = 0.9 - P_e$ pu	$\frac{(\Delta t)^2}{M}P_a = 8.929\,P_a$ deg	$\Delta\delta$ deg	δ deg
0_-	2.44	0.368	0.9	0.0	—	—	21.64
0_+	0.88	0.368	0.324	0.576	—	—	21.64
0_{avg}	—	0.368	—	0.288	2.57	2.57	21.64
0.05_-	0.88	0.41	0.36	0.54	—	—	24.21
0.05_+	2.00	0.41	0.82	0.08	—	—	24.21
0.05_{avg}				0.31	2.767	5.33	24.21
0.10	2.00	0.493	0.986	– 0.086	– 0.767	4.56	29.54
0.15	2.00	0.56	1.12	– 0.22	– 1.96	2.60	34.10
0.20	2.00	0.597	1.19	– 0.29	– 2.58	0.02	36.70
0.25	2.00	0.597	1.19	– 0.29	– 2.58	– 2.56	37.72
0.30	2.00	0.561	1.12	– 0.22	– 1.96	– 4.52	34.16
0.35	2.00	0.494	0.989	– 0.089	– 0.79	– 5.31	29.64
0.40	2.00	0.41	0.82	0.08	0.71	– 4.60	24.33
0.45	2.00	0.337	0.675	0.225	2.0	– 2.6	19.73
0.50							17.13

Fault Cleared in 6.25 Cycles

$$\text{Time to clear fault} = \frac{6.25}{50} = 0.125 \text{ s}$$

Since the discontinuity now lies in the middle of an interval, no special procedure is necessary, as in deriving Eqs. (iv) – (vi) discontinuity is assumed to occur in the middle of the time interval. The swing curve as calculated in Table 12.4 is also plotted in Fig. 12.40. It is observed that the system is stable with a maximum swing of 52.5° which is much larger than that in the case of 2.5 cycle clearing time.

To find the critical clearing time, swing curves can be obtained, similarly, for progressively greater clearing time till the torque angle δ increases without bound. In this example, however, we can first find the critical clearing angle using Eq. (12.67) and then read the critical clearing time from the swing curve corresponding to the sustained fault case. The values obtained are:

Critical clearing angle = 118.62°

Critical clearing time = 0.38 s

Table 12.4 Computations of swing curve for fault cleared at 6.25 cycles (0.125s), Δt = 0.05 s

t sec	P_{max} pu	$\sin\delta$	$P_e = P_{max}\sin\delta$ pu	$P_a = 0.9 - P_e$ pu	$\frac{(\Delta t)^2}{M}P_a = 8.929\,P_a$ deg	$\Delta\delta$ deg	δ deg
0_-	2.44	0.368	0.9	0.0	–	–	21.64
0_+	0.88	0.368	0.324	0.576	–	–	21.64
0_{avg}	–	0.368	–	0.288	2.57	2.57	21.64
0.05	0.88	0.41	0.361	0.539	4.81	7.38	24.21
0.10	0.88	0.524	0.461	0.439	3.92	11.30	31.59
0.15	2.00	0.680	1.36	– 4.46	– 4.10	7.20	42.89
0.20	2.00	0.767	1.53	– 0.63	– 5.66	1.54	50.09
0.25	2.00	0.78	1.56	– 0.66	– 5.89	– 4.35	51.63
0.30	2.00	0.734	1.46	– 0.56	– 5.08	– 9.43	47.28
0.35	2.00	0.613	1.22	– 0.327	– 2.92	– 12.35	37.85
0.40	2.00	0.430	0.86	0.04	0.35	– 12.00	25.50
0.45	2.00	0.233	0.466	0.434	3.87	– 8.13	13.50
0.50	2.00						5.37

12.10 MULTIMACHINE STABILITY

From what has been discussed so far, the following steps easily follow for determining multimachine stability.

1. From the prefault load flow data determine E'_k voltage behind transient reactance for all generators. This establishes generator emf magnitudes $|E_k|$ which remain constant during the study and initial rotor angle $\delta_k^o = \angle E_k$. Also record prime mover inputs to generators, $P_{mk} = P^o_{Gk}$.
2. Augment the load flow network by the generator transient reactances. Shift network buses behind the transient reactances.
3. Find Y_{BUS} for various network conditions—during fault, post fault (faulted line cleared), after line reclosure.
4. For faulted mode, find generator outputs from power angle equations (generalized forms of Eq. (12.27)) and solve swing equations step by step (point-by-point method).

5. Keep repeating the above step for post fault mode and after line reclosure mode.
6. Examine $\delta(t)$ plots of all generators and establish the answer to the stability question.

The above steps are illustrated in the following example.

Example 12.11

A 50 Hz, 220 kV transmission line has two generators and an infinite bus as shown in Fig. 12.41. The transformer and line data are given in Table 12.5. A three-phase fault occurs as shown. The prefault load flow solution is presented in Table 12.6. Find the swing equation for each generator during the fault period.

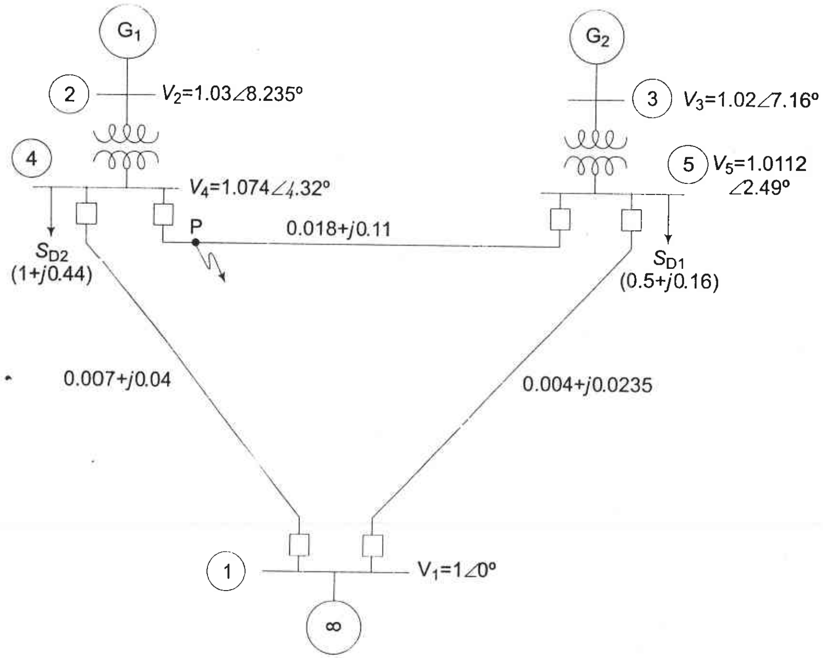


Fig. 12.41

Data are given below for the two generators on a 100 MVA base.

Gen 1 500 MVA, 25 kV, $X_d' = 0.067$ pu, H = 12 MJ/MVA

Gen 2 300 MVA, 20 kV, $X_d' = 0.10$ pu, H = 9 MJ/MVA

Plot the swing curves for the machines at buses 2 and 3 for the above fault which is cleared by simultaneous opening of the circuit breakers at the ends of the faulted line at (i) 0.275 s and (ii) 0.08 s.

Table 12.5 Line and transformer data for Ex. 12.11. All values are in pu on 220 kV, 100 MVA base

Bus to bus	*Series Z*		*Half line charging*
	R	*X*	
Line 4-5	0.018	0.11	0.113
Line 5-1	0.004	0.0235	0.098
Line 4-1	0.007	0.04	0.041
Trans: 2-4	—	0.022	—
Trans: 3-5	—	0.04	—

Table 12.6 Bus data and prefault load-flow values in pu on 220 kV, 100 MVA base

S.No. and Bus No.	*Voltage Polar Form*	*Bus type*	*Voltage*		*Generation*		*Load*	
			Real e	*Imaginary f*	*P*	*Q*	*P*	*Q*
1	1.0∠0°	Slack	1.00	0.0	– 3.8083	–0.2799	0	0
2	1.03∠8.35°	PV	1.0194	0.1475	3.25	0.6986	0	0
3	1.02∠7.16°	PV	1.0121	0.1271	2.10	0.3110	0	0
4	1.0174∠4.32°	PQ	1.0146	0.767	0	1.0	1.0	0.44
5	1.0112∠2.69°	PQ	1.0102	0.0439	0	0	0.5	0.16

Solution Before determining swing equations, we have to find transient internal voltages.

The current into the network at bus 2 based on the data in Table 12.6 is

$$I_2 = \frac{P_2 - jQ_2}{V_2^*} = \frac{3.25 - j0.6986}{1.03\angle -8.23519°}$$

$$E_2' = (1.0194 + j0.1475) + \frac{3.25 - j0.6986}{1.03\angle -8.23519°} \times 0.067\angle 90°$$

$$= 1.0340929 + j0.3632368$$

$$= 1.0960333\ \angle 19.354398° = 1.0960\ \angle 0.3377 \text{ rad}$$

$$E_1' = 1.0\ \angle 0° \text{ (slack bus)}$$

$$E_3' = (1.0121 + j0.1271) + \frac{2.1 - j0.311}{1.02\angle -7.15811°} \times 0.1\angle 90°$$

$$= 1.0166979 + j0.335177 = 1.0705\ \angle 18.2459°$$

$$= 1.071\ \angle 0.31845 \text{ rad}$$

The loads at buses 4 and 5 are represented by the admittances calculated as follows:

$$Y_{L4} = \frac{1.0 - j0.44}{(1.0174)^2} (0.9661 - j0.4251)$$

$$Y_{L5} = \frac{0.5 - j0.16}{(1.0112)^2} \; (0.4889 - j0.15647)$$

Prefault Bus Matrix

Load admittances, along with the transient reactances, are used with the line and transformer admittances to form the prefault augmented bus admittance matrix which contains the transient reactances of the machines. We will, therefore, now designate as buses 2 and 3, the fictitious internal nodes between the internal voltages and the transient reactances of the machines. Thus we get

$$Y_{22} = \frac{1}{(j0.067 + j0.022)} = -j11.236$$

$$Y_{24} = j11.236 = Y_{42}$$

$$Y_{33} = \frac{1}{j0.04 + j0.1} = -j7.143$$

$$Y_{35} = j7.143 = Y_{53}$$

$$Y_{44} = Y_{L4} + Y_{41} + Y_{45} + \frac{B_{41}}{2} + \frac{B_{45}}{2} + Y_{24}$$

$$= 0.9660877 - j0.4250785 + 4.245 - j24.2571 + 1.4488 - j8.8538 + j0.041 + j0.113 - j11.2359$$

$$= 6.6598977 - j44.6179$$

$$Y_{55} = Y_{L5} + Y_{54} + Y_{51} + \frac{B_{54}}{2} + \frac{B_{51}}{2} + Y_{35}$$

$$= 0.4889 - j0.1565 + 1.4488 - j8.8538 + 7.0391 - j41.335 + j0.113 + j0.098 - j7.1428$$

$$= 8.976955 - j57.297202$$

The complete augmented prefault Y_{BUS} matrix is shown in Table 12.7.

Table 12.7 The augmented prefault bus admittance matrix for Ex. 12.11, admittances in pu

Bus	1	2	3	4	5
1	11.284–j65.473	0	0	–4.245 + j24.257	–7.039 + j41.355
2	0	–j11.2359	0	j11.2359	0
3	0	0	– j7.1428	0	j7.1428
4	–4.245 + j24.257	j11.2359	0	6.6598–j44.617	–1.4488 +j8.8538
5	–7.039 + j41.355	0	0 + j7.1428	–1.4488 + j8.8538	8.9769 + j57.2972

During Fault Bus Matrix

Since the fault is near bus 4, it must be short circuited to ground. The Y_{BUS} during the fault conditions would, therefore, be obtained by deleting 4th row and 4th column from the above augmented prefault Y_{BUS} matrix. Reduced fault matrix (to the generator internal nodes) is obtained by eliminating the new 4th row and column (node 5) using the relationship

$$Y_{kj(\text{new})} = Y_{kj(\text{old})} - Y_{kn\,(\text{old})}\, Y_{nj(\text{Old})}/Y_{nn\,(\text{old})}$$

The reduced faulted matrix (Y_{BUS} during fault) (3 × 3) is given in Table 12.8, which clearly depicts that bus 2 decouples from the other buses during the fault and that bus 3 is directly connected to bus 1, showing that the fault at bus 4 reduces to zero the power pumped into the system from the generator at bus 2 and renders the second generator at bus 3 to give its power radially to bus 1.

Table 12.8 Elements of Y_{BUS} (during fault) and Y_{BUS} (post fault) for Ex. 12.11, admittances in pu.

	Reduced during fault Y_{BUS}		
Bus	1	2	3
1	5.7986–j35.6301	0	– 0.0681 + j5.1661
2	0	– j11.236	0
3	– 0.0681+ j5.1661	0	0.1362 – j6.2737
	Reduced post fault Y_{BUS}		
1	1.3932 – j13.8731	– 0.2214 + j7.6289	– 0.0901 + j6.0975
2	– 0.2214 + j7.6289	0.5 – j7.7898	0
3	– 0.0901 + j6.0975	0	0.1591 – j6.1168

Post Fault Bus Matrix

Once the fault is cleared by removing the line, simultaneously opening the circuit breakers at the either ends of the line between buses 4 and 5, the prefault Y_{BUS} has to be modified again. This is done by substituting $Y_{45} = Y_{54} = 0$ and subtracting the series admittance of line 4–5 and the capacitive susceptance of half the line from elements Y_{44} and Y_{55}.

$$Y_{44(\text{post fault})} = Y_{44(\text{prefault})} - Y_{45} - B_{45}/2$$

$$= 6.65989 - j44.6179 - 1.448 + j8.853 - j0.113$$

$$= 5.2111 - j35.8771$$

Similarly, $Y_{55(\text{post fault})} = 7.5281 - j48.5563$

The reduced post fault Y_{BUS} is shown in the lower half to Table 12.8. It may be noted that 0 element appears in 2nd and 3rd rows. This shows that, physically, the generators 1 and 2 are not interconnected when line 4–5 is removed.

During Fault Power Angle Equation

$$P_{e2} = 0$$

$$P_{e3} = Re\ [Y_{33}E_3'E_3'^{*} + E_3'^{*}\ Y_{31}E_1']; \text{ since } Y_{32} = 0$$

$$= |E_3'|^2\ G_{33} + |E_1'|\ |E_3'|\ |Y_{31}| \cos(\delta_{31} - \theta_{31})$$

$$= (1.071)^2\ (0.1362) + 1 \times 1.071 \times 5.1665 \cos(\delta_3 - 90.755°)$$

$$P_{e3} = 0.1561 + 5.531 \sin(\delta_3 - 0.755°)$$

Postfault Power Angle Equations

$$P_{e2} = |E_2'|^2\ G_{22} + |E_1'|\ |E_2'|\ |Y_{21}| \cos(\delta_{21} - \theta_{21})$$

$$= 1.096^2 \times 0.5005 + 1 \times 1.096 \times 7.6321 \cos(\delta_2 - 91.662°)$$

$$= 0.6012 + 8.365 \sin(\delta_2 - 1.662°)$$

$$P_{e3} = |E_3'|^{\,2}G_{33} + |E_1'|\ |E_3'|\ |Y_{31}| \cos(\delta_{31} - \theta_{31})$$

$$= 1.071^2 \times 0.1591 + 1 \times 1.071 \times 6.098 \cos(\delta_3 - 90.8466°)$$

$$= 0.1823 + 6.5282 \sin(\delta_3 - 0.8466°)$$

Swing Equations—During Fault

$$\frac{d^2\delta_2}{dt^2} = \frac{180f}{H_2}(P_{m2} - P_{e2}) = \frac{180f}{H_2} P_{a_2}$$

$$= \frac{180f}{12}(3.25 - 0) \text{ elect deg/s}^2$$

$$\frac{d^2\delta_3}{dt^2} = \frac{180f}{H_3}(P_{m3} - P_{e3})$$

$$= \frac{180f}{9}[2.1 - \{0.1561 + 5.531 \sin(\delta_3 - 0.755°)\}]$$

$$= \frac{180f}{9}[1.9439 - 5.531 \sin(\delta_3 - 0.755°)] \text{ elect deg/s}^2$$

Swing Equations—Postfault

$$\frac{d^2\delta_2}{dt^2} = \frac{180f}{11}[3.25 - \{0.6012 + 8.365 \sin(\delta_2 - 1.662°)\}] \text{ elect deg/s}^2$$

$$\frac{d^2\delta_3}{dt^2} = \frac{180f}{9}[2.10 - \{0.1823 + 6.5282 \sin(\delta_3 - 0.8466°)\}] \text{elect deg/s}^2$$

It may be noted that in the above swing equations, P_a may be written in general as follows:

$$P_a = P_m - P_c - P_{\max} \sin(\delta - \gamma)$$

Digital Computer Solution of Swing Equation

The above swing equations (during fault followed by post fault) can be solved by the point-by-point method presented earlier or by the Euler's method presented in the later part of this section. The plots of δ_2 and δ_3 are given in Fig. 12.42 for a clearing time of 0.275 s and in Fig. 12.43 for a clearing time of 0.08 s. For the case (i), the machine 2 is unstable, while the machine 3 is stable but it oscillates wherein the oscillations are expected to decay if effect of damper winding is considered. For the case (ii), both machines are stable but the machine 2 has large angular swings.

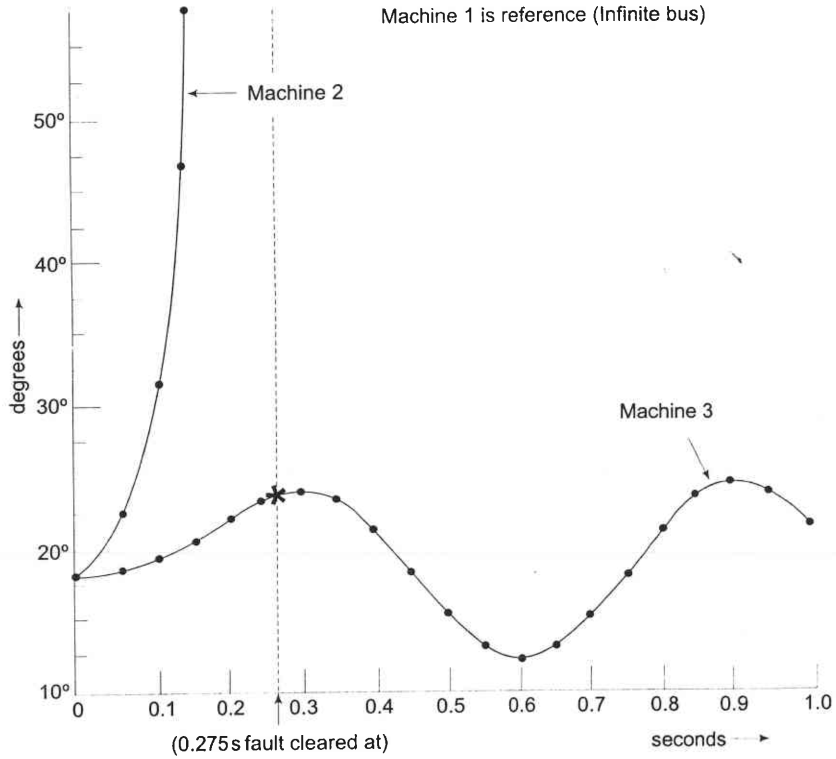


Fig. 12.42 Swing curves for machines 2 and 3 of Example 12.1 for clearing at 0.275 s.

If the fault is a transient one and the line is reclosed, power angle and swing equations are needed for the period after reclosure. These can be computed from the reduced Y_{BUS} matrix after line reclosure.

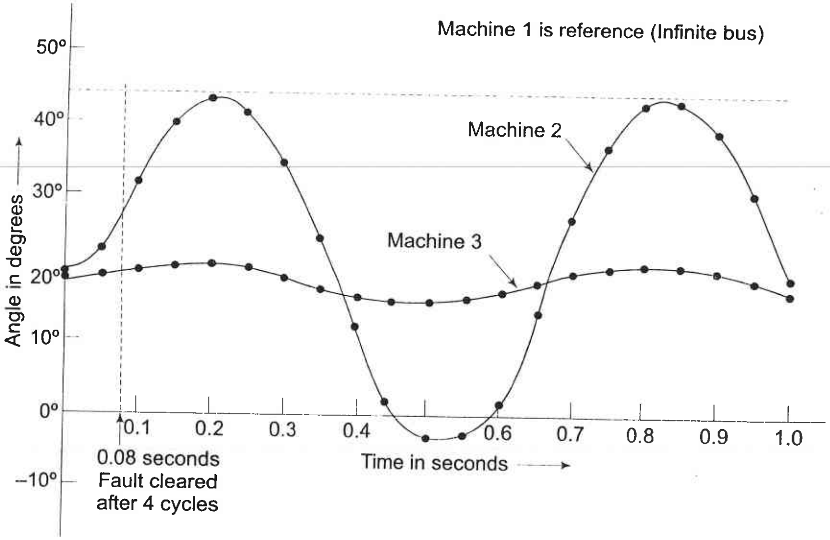


Fig. 12.43 Swing curves for machines 2 and 3 of Example 12.11 for clearing at 0.08 s

Consideration of Automatic Voltage Regulator (AVR) and Speed Governor Loops

This requires modelling of these two control loops in form of differential equations. At the end of every step in the stability algorithm, the programme computes the modified values of E_k' and P_{mk} and then proceeds to compute the next step. This considerably adds to the dimensionality and complexity of stability calculations. To reduce the computational effort, speed control can continue to be ignored without loss of accuracy of results.

State Variable Formulation of Swing Equations

The swing equation for the kth generator is

$$\frac{d^2\delta_k}{dt^2} = \frac{\pi f}{H_k}(P^0_{Gk} - P_{Gk});\ k = 1, 2, \ldots, m \tag{12.73}$$

For the multimachine case, it is more convenient to organise Eq. (12.73) is state variable form. Define

$$x_{1k} = \delta_k = \angle E_k'$$

$$x_{2k} = \dot{\delta}_k$$

Then

$$\dot{x}_{1k} = x_{2k}$$

$$\dot{x}_2k = \frac{\pi f}{H_k}(P^0_{Gk} - P_{Gk}),\ k = 1, 2, \ldots, m \tag{12.74}$$

Initial state vector (upon occurrence of fault) is

$$x^0_{1k} = \delta^0_k = \angle E^0_k$$

$$x^0_{2k} = 0 \tag{12.75}$$

The state form of swing equations (Eq. (12.74)) can be solved by the many available integration algorithms (modified Euler's method is a convenient choice).

Computational Algorithm for Obtaining Swing Curves Using Modified Euler's Method

1. Carry out a load flow study (prior to disturbance) using specified voltages and powers.
2. Compute voltage behind transient reactances of generators (E^0_k) using Eq. (9.31). This fixes generator emf magnitudes and initial rotor angle (reference slack bus voltage V^0_1).
3. Compute, Y_{BUS} (during fault, post fault, line reclosed).
4. Set time count $r = 0$.
5. Compute generator power outputs using appropriate Y_{BUS} with the help of the general form of Eq. (12.27). This gives $P^{(r)}_{Gk}$ for $t = t^{(r)}$.

   Note: After the occurrence of the fault, the period is divided into uniform discrete time intervals (Δt) so that time is counted as $t^{(0)}, t^{(1)}, \ldots\ldots$ A typical value of Δt is 0.05 s.
6. Compute $[(\dot{x}^{(r)}_{1k}, \dot{x}^{(r)}_{2k}), k = 1, 2, \ldots, m]$ from Eqs. (12.74).
7. Compute the first state estimates for $t = t^{(r+1)}$ as

$$x^{(r+1)}_{1k} = x^{(r)}_{1k} + \dot{x}^{(r)}_{1k}\Delta t \qquad k = 1, 2, \ldots, m$$

$$x^{(r+1)}_{2k} = x^{(r)}_{2k} + \dot{x}^{(r)}_{2k}\Delta t$$

8. Compute the first estimates of $E_k^{(r+1)}$

$$E^{(r+1)}_k = E^0_k\,(\cos x^{(r+1)}_{1k} + j \sin x^{(r+1)}_{1k})$$

9. Compute $P^{(r+1)}_{Gk}$; (appropriate Y_{BUS} and Eq. (12.72)).
10. Compute $\left[(\dot{x}^{(r+1)}_{1k}, \dot{x}^{(r+1)}_{2k}),\ k = 1, 2, \ldots, m\right]$ from Eqs. (12.74).
11. Compute the average values of state derivatives

$$\dot{x}^{(r)}_{1k,\,avg} = \tfrac{1}{2}[\dot{x}_{1k}^{(r)} + \dot{x}^{(r+1)}_{1k}] \qquad k = 1, 2, \ldots, m$$

$$\dot{x}^{(r)}_{2k,avg} = \tfrac{1}{2}\left[\dot{x}^{(r)}_{2k} + \dot{x}^{(r+1)}_{2k}\right]$$

12. Compute the final state estimates for $t = t^{(r+1)}$.

$$x_{1k}^{(r+1)} = x_{1k}^{(r)} + \dot{x}_{1k,\text{avg}}^{(r)}\,\Delta t$$

$$k = 1,2, \ldots, m$$

$$x_{2k}^{(r+1)} = x_{2k}^{(r)} + \dot{x}_{2k,\text{avg}}^{(r)}\,\Delta t$$

13. Compute the final estimate for E_k at $t = t^{(r+1)}$ using

$$E_k^{(r+1)} = |E_k^0| \cos x_{1k}^{(r+1)} + j \sin x_{1k}^{(r+1)}$$

14. Print $(x_{1k}^{(r+1)}, x_{2k}^{(r+1)})$; $k = 1, 2, \ldots, m$
15. Test for time limit (time for which swing curve is to be plotted), i.e., check if $r > r_{\text{final}}$. If not, $r = r + 1$ and repeat from step 5 above. Otherwise print results and stop.

The swing curves of all the machines are plotted. If the rotor angle of a machine (or a group of machines) with respect to other machines increases without bound, such a machine (or group of machines) is unstable and eventually falls out of step.

The computational algorithm given above can be easily modified to include simulation of voltage regulator, field excitation response, saturation of flux paths and governor action.

Stability Study of Large Systems

To limit the computer memory and the time requirements and for the sake of computational efficiency, a large multi-machine system is divided into a study subsystem and an external system. The study subsystem is modelled in detail whereas approximate modelling is carried out for the external subsystem. The total study is rendered by the modern technique of dynamic equivalencing. In the external subsystem, number of machines is drastically reduced using various methods—coherency based methods being most popular and widely used by various power utilities in the world.

12.11 SOME FACTORS AFFECTING TRANSIENT STABILITY

We have seen in this chapter that the two-machine system can be equivalently reduced to a single machine connected to infinite bus bar. The qualitative conclusions regarding system stability drawn from a two-machine or an equivalent one-machine infinite bus system can be easily extended to a multimachine system. In the last article we have studied the algorithm for determining the stability of a multimachine system.

It has been seen that transient stability is greatly affected by the type and location of a fault, so that a power system analyst must at the very outset of a stability study decide on these two factors. In our examples we have selected a 3–phase fault which is generally more severe from point of view of power transfer. Given the type of fault and its location let us now consider other factors which affect transient stability and therefrom draw the conclusions, regarding methods of improving the transient stability limit of a system and making it as close to the steady state limit as possible.

For the case of one machine connected to infinite bus, it is easily seen from Eq. (12.71) that an increase in the inertia constant M of the machine reduces the angle through which it swings in a given time interval offering thereby a method of improving stability but this cannot be employed in practice because of economic reasons and for the reason of slowing down the response of the speed governor loop (which can even become oscillatory) apart from an excessive rotor weight.

With reference to Fig. 12.30, it is easily seen that for a given clearing angle, the accelerating area decreases but the decelerating area increases as the maximum power limit of the various power angle curves is raised, thereby adding to the transient stability limit of the system. The maximum steady power of a system can be increased by raising the voltage profile of the system and by reducing the transfer reactance. These conclusions along with the various transient stability cases studied, suggest the following method of improving the transient stability limit of a power system.

1. Increase of system voltages, use of AVR.
2. Use of high speed excitation systems.
3. Reduction in system transfer reactance.
4. Use of high speed reclosing breakers (see Fig. 12.32). Modern tendency is to employ single-pole operation of reclosing circuit breakers.

When a fault takes place on a system, the voltages at all buses are reduced. At generator terminals, these are sensed by the automatic voltage regulators which help restore generator terminal voltages by acting within the excitation system. Modern exciter systems having solid state controls quickly respond to bus voltage reduction and can achieve from one-half to one and one-half cycles ($1/2$–$1\frac{1}{2}$) gain in critical clearing times for three-phase faults on the HT bus of the generator transformer.

Reducing transfer reactance is another important practical method of increasing stability limit. Incidentally this also raises system voltage profile. The reactance of a transmission line can be decreased (i) by reducing the conductor spacing, and (ii) by increasing conductor diameter (see Eq. (2.37)). Usually, however, the conductor spacing is controlled by other features such as lightning protection and minimum clearance to prevent the arc from one phase moving to another phase. The conductor diameter can be increased by using material of low conductivity or by hollow cores. However, normally, the conductor configuration is fixed by economic considerations quite apart from stability. The use of bundled conductors is, of course, an effective means of reducing series reactance.

Compensation for line reactance by series capacitors is an effective and economical method of increasing stability limit specially for transmission distances of more than 350 km. The degree of series compensation, however, accentuates the problems of protective relaying, normal voltage profiles, and overvoltages during line-to-ground faults. Series compensation becomes more effective and economical if part of it is switched on so as to increase the degree of compensation upon the occurrence of a disturbance likely to cause instability. Switched series capacitors simultaneously decrease fluctuation of load voltages and raise the transient stability limit to a value almost equal to the steady state limit. Switching shunt capacitors on or switching shunt reactors off also raises stability limits (see Example 12.2) but the MVA rating of shunt capacitors required is three to six times the rating of switched series capacitors for the same increase in stability limit. Thus series capacitors are preferred unless shunt elements are required for other purposes, say, control of voltage profile.

Increasing the number of parallel lines between transmission points is quite often used to reduce transfer reactance. It adds at the same time to reliability of the transmission system. Additional line circuits are not likely to prove economical unit l after all feasible improvements have been carried out in the first two circuits.

As the majority of faults are transient in nature, rapid switching and isolation of unhealthy lines followed by reclosing has been shown earlier to be a great help in improving the stability margins. The modern circuit breaker technology has now made it possible for line clearing to be done as fast as in two cycles. Further, a great majority of transient faults are line-to-ground in nature. It is natural that methods have been developed for selective single pole opening and reclosing which further aid the stability limits. With reference to Fig. 12.17, if a transient LG fault is assumed to occur on the generator bus, it is immediately seen that during the fault there will now be a definite amount of power transfer, as different from zero power transfer for the case of a three-phase fault. Also when the circuit breaker pole corresponding to the faulty line is opened, the other two lines (healthy ones) remain intact so that considerable power transfer continues to take place via these lines in comparison to the case of three-pole switching when the power transfer on fault clearing will be reduced to zero. It is, therefore, easy to see why the single pole switching and reclosing aids in stability problem and is widely adopted. These facts are illustrated by means of Example 12.12. Even when the stability margins are sufficient, single pole switching is adopted to prevent large swings and consequent voltage dips. Single pole switching and reclosing is, of course, expensive in terms of relaying and introduces the associated problems of overvoltages caused by single pole opening owing to line capacitances. Methods are available to nullify these capacitive coupling effects.

Recent Trends

Recent trends in design of large alternators tend towards lower short circuit ratio (SCR = $1/X_d$), which is achieved by reducing machine air gap with consequent savings in machine mmf, size, weight and cost. Reduction in the size of rotor reduces inertia constant, lowering thereby the stability margin. The loss in stability margin is made up by such features as lower reactance lines, faster circuit breakers and faster excitation systems as discussed already, and a faster system valving to be discussed later in this article.

A stage has now been reached in technology whereby the methods of improving stability, discussed above, have been pushed to their limits, e.g., clearing times of circuit breakers have been brought down to virtually irreducible values of the order of two cycles. With the trend to reduce machine inertias there is a constant need to determine availability, feasibility and applicability of new methods for maintaining and/or improving system stability. A brief account of some of the recent methods of maintaining stability is given below:

HVDC Links

Increased use of HVDC links employing thyristors would alleviate stability problems. A dc link is asynchronous, i.e., the two ac system at either end do not have to be controlled in phase or even be at exactly the same frequency as they do for an ac link, and the power transmitted can be readily controlled. There is no risk of a fault in one system causing loss of stability in the other system.

Breaking Resistors

For improving stability where clearing is delayed or a large load is suddenly lost, a resistive load called a breaking resistor is connected at or near the generator bus. This load compensates for at least some of the reduction of load on the generators and so reduces the acceleration. During a fault, the resistors are applied to the terminals of the generators through circuit breakers by means of an elaborate control scheme. The control scheme determines the amount of resistance to be applied and its duration. The breaking resistors remain on for a matter of cycles both during fault clearing and after system voltage is restored.

Short Circuit Current Limiters

These are generally used to limit the short circuit duty of distribution lines. These may also be used in long transmission lines to modify favourably the transfer impedance during fault conditions so that the voltage profile of the system is somewhat improved, thereby raising the system load level during the fault.

Turbine Fast Valving or Bypass Valving

The two methods just discussed above are an attempt at replacing the system load so as to increase the electrical output of the generator during fault conditions. Another recent method of improving the stability of a unit is to decrease the mechanical input power to the turbine. This can be accomplished by means of fast valving, where the difference between mechanical input and reduced electrical output of a generator under a fault, as sensed by a control scheme, initiates the closing of a turbine valve to reduce the power input. Briefly, during a fast valving operation, the interceptor valves are rapidly shut (in 0.1 to 0.2 sec) and immediately reopened. This procedure increases the critical switching time long enough so that in most cases, the unit will remain stable for faults with stuck-breaker clearing times. The scheme has been put to use in some stations in the USA.

Full Load Rejection Technique

Fast valving combined with high-speed clearing time will suffice to maintain stability in most of the cases. However, there are still situations where stability is difficult to maintain. In such cases, the normal procedure is to automatically trip the unit off the line. This, however, causes several hours of delay before the unit can be put back into operation. The loss of a major unit for this length of time can seriously jeopardize the remaining system.

To remedy these situations, a full load rejection scheme could be utilized after the unit is separated from the system. To do this, the unit has to be equipped with a large steam bypass system. After the system has recovered from the shock caused by the fault, the unit could be resynchronized and reloaded. The main disadvantage of this method is the extra cost of a large bypass system.

Example 12.12

The system shown in Fig. 12.44 is loaded to 1 pu. Calculate the swing curve and ascertain system stability for:

(i) LG fault three pole switching followed by reclosure, line found healthy.

(ii) LG fault single pole switching followed by reclosure, line found healthy.

Switching occurs at 3.75 cycles (0.075 sec) and reclosure occurs at 16.25 cycles (0.325 sec). All values shown in the figure are in pu.

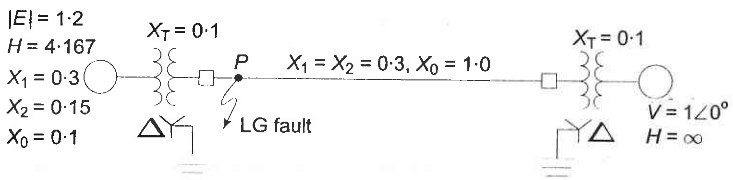


Fig. 12.44

Solution The sequence networks of the system are drawn and suitably reduced in Figs. 12.45a, b and c.

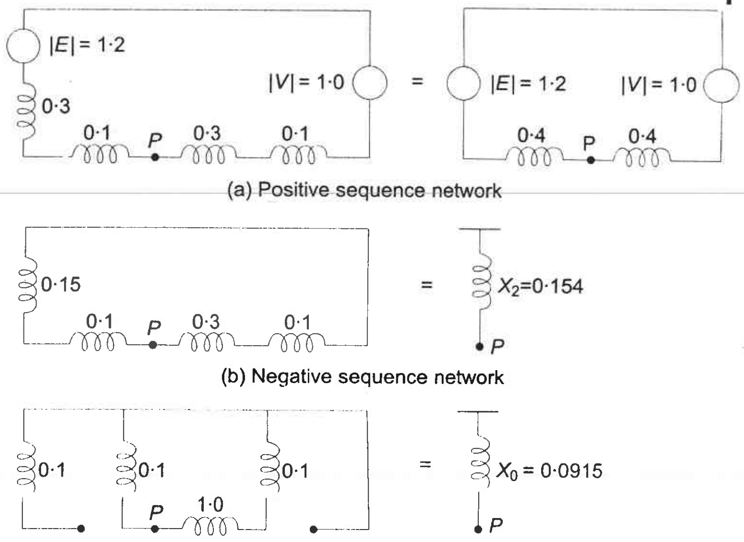


Fig. 12.45

For an LG fault at P the sequence networks will be connected in series as shown in Fig. 12.46. A star-delta transformation reduces Fig. 12.38 to that of Fig. 12.47 from which we have the transfer reactance

$$X_{12}(\text{LG fault}) = 0.4 + 0.4 + \frac{0.4 \times 0.4}{0.246} = 1.45$$

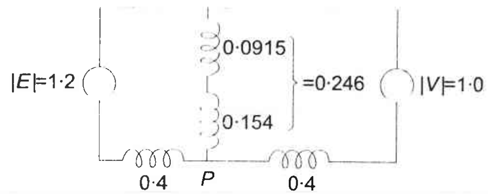


Fig. 12.46 Connection of sequence networks for an LG fault

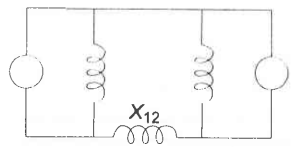


Fig. 12.47 Transfer impedance for an LG fault

When the circuit breaker poles corresponding to the faulted line are opened (it corresponds to a single-line open fault) the connection of sequence networks is shown in Fig. 12.48. From the reduced network of Fig. 12.49 the transfer reactance with faulted line switched off is

X_{12} (faulted line open) = 0.4 + 0.42 + 0.4 = 1.22

Under healthy conditions transfer reactance is easily obtained from the positive sequence network of Fig. 12.45 a as

X_{12}(line healthy) = 0.8

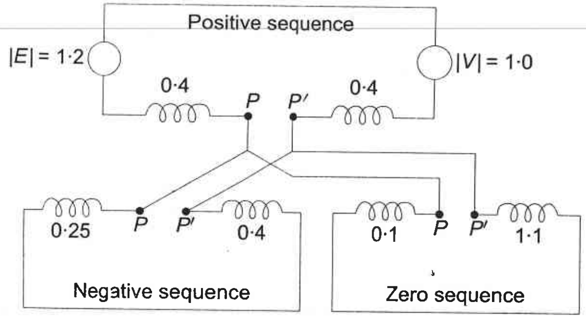


Fig. 12.48 Connection of sequence networks with faulted line switched off

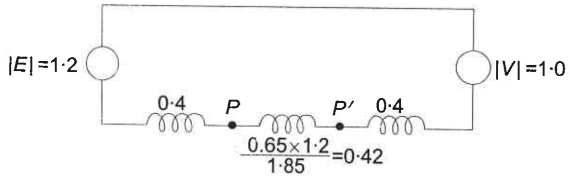


Fig. 12.49 Reduced network of Fig. 12.48 giving transfer reactance

Power angle equations

Prefault

$$P_{eI} = \frac{|E||V|}{X_{12}} \sin\ \delta = \frac{1.2\times 1}{0.8} \sin\ \delta = 1.5 \sin\ \delta$$

Initial load = 1.0 pu

Initial torque angle is given by

$$1 = 1.5 \sin\ \delta_o$$

or $$\delta_o = 41.8°$$

During fault

$$P_{eII} = \frac{1.2\times 1}{1.45} \sin\ \delta = 0.827 \sin\ \delta$$

During single pole switching

$$P_{eIII} = \frac{1.2\times 1}{1.22} \sin\ \delta = 0.985 \sin\ \delta$$

During three pole switching

$$P_{eIII} = 0$$

Postfault

$$P_{eIV} = P_{eI} = 1.5 \sin\ \delta$$

Now

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)}$$

$$H = 4.167 \text{ MJ/MVA}$$

$$M = \frac{4.167}{180\times 50} = 4.63 \times 10^{-4} \text{ sec}^2\text{/electrical degree}$$

Taking Δt = 0.05 sec

$$\frac{(\Delta t)^2}{M} = \frac{(0.05)^2}{4.63\times 10^{-4}} = 5.4$$

Time when single/three pole switching occurs

= 0.075 sec (during middle of Δt)

Time when reclosing occurs = 0.325 (during middle of Δt)

Table 12.9 Swing curve calculation—three pole switching

	t sec	P_{max} (pu)	$\sin\ \delta$	P_e (pu)	P_a (pu)	$5.4P_a$ elec deg	$\Delta\delta$ elec deg	δ elec deg
	0_-	1.5	0.667	1.0	0.0			41.8
	0_+	0.827	0.667	0.552	0.448			41.8
	0_{avg}				0.224	1.2	1.2	41.8
	0.05	0.827	0.682	0.564	0.436	2.4	3.6	43.0
0.075→								
	0.10	0.0	0.726	0.0	1.0	5.4	9.0	46.6
	0.15	0.0		0.0	1.0	5.4	14.4	55.6
	0.20	0.0		0.0	1.0	5.4	19.8	70.0
	0.25	0.0		0.0	1.0	5.4	25.2	89.8
	0.30	0.0		0.0	1.0	5.4	30.6	115.0
0.325→								
	0.35	1.5	0.565	0.85	0.15	0.8	31.4	145.6
	0.40	1.5	0.052	0.078	0.922	5.0	36.4	177.0
	0.45	1.5	− 0.55	− 0.827	1.827	9.9	46.3	213.4
	0.50	1.5	− 0.984	− 1.48	2.48	13.4	59.7	259.7
	0.55	1.5	− 0.651	− 0.98	1.98	10.7	70.4	319.4
	0.60	1.5	0.497	0.746	0.254	1.4	71.8	389.8
	0.65							461.6

The swing curve is plotted in Fig. 12.50 from which it is obvious that the

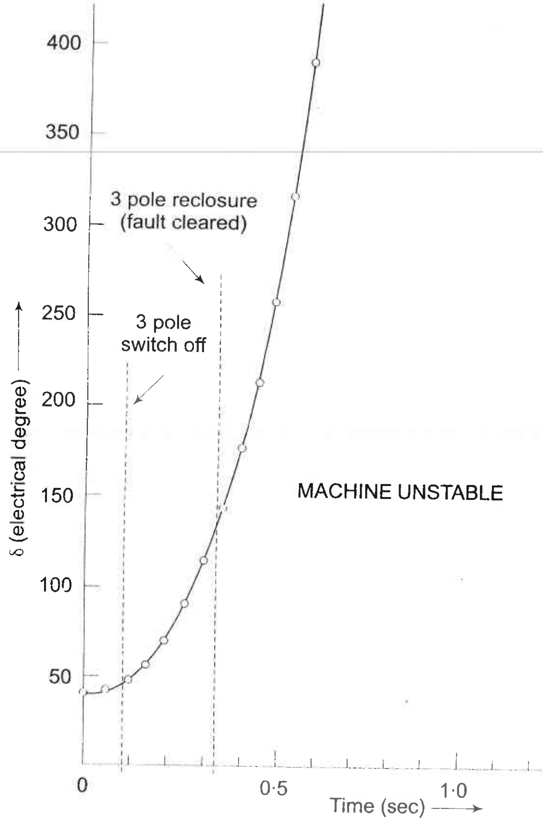


Fig. 12.50 Swing curve for three pole switching with reclosure

Table 12.10 Swing curve calculation—single pole switching

t sec	P_{max} (pu)	$\sin\delta$	P_e (pu)	P_a (pu)	$5.4P_a$ elec deg	$\Delta\delta$ elec deg	δ elec deg
0	1.5	0.667	1.0	0.0			41.80
0_+	0.827	0.667	0.552	0.448			41.8
0_{avg}				0.224	1.2	1.2	41.8
0.05	0.827	0.682	0.564	0.436	2.4	3.6	43.0
0.075→							
0.10	0.985	0.726	0.715	0.285	1.5	5.1	46.6
0.15	0.985	0.784	0.77	0.230	1.2	6.3	51.7
0.20	0.985	0.848	0.834	0.166	0.9	7.2	58.0
0.25	0.985	0.908	0.893	0.107	0.6	7.8	65.2
0.30	0.985	0.956	0.940	0.060	0.3	8.1	73.0
0.325→							
0.35	1.5	0.988	1.485	- 0.485	– 2.6	5.5	81.1

(Contd....)

0.40	1.5	0.998	1.5	– 0.5	– 2.7	2.8	86.6
0.45	1.5	1.0	1.5	– 0.5	– 2.7	0.1	89.4
0.50	1.5	1.0	1.5	– 0.5	– 2.7	– 2.6	89.5
0.55	1.5	0.9985	1.5	– 0.5	–2.7	–5.3	86.9
0.60	1.5	0.989	1.485	– 0.485	– 2.6	– 7.9	81.6
0.65	1.5	0.96	1.44	– 0.44	– 2.4	– 10.3	73.7
0.70	1.5	0.894	1.34	– 0.34	– 1.8	– 12.1	63.4
0.75	1.5	0.781	1.17	– 0.17	– 0.9	– 13.0	51.3
0.80	1.5	0.62	0.932	0.068	0.4	– 12.6	38.3
0.85	1.5	0.433	0.65	0.35	1.9	– 10.7	25.7
0.90	1.5	0.259	0.39	0.61	3.3	– 7.4	15.0
0.95	1.5	0.133	0.2	0.8	4.3	– 3.1	7.6
1.00	1.5	0.079	0.119	0.881	4.8	1.7	4.5
1.05	1.5	0.107	0.161	0.839	4.5	6.2	6.2
1.10	1.5	0.214	0.322	0.678	3.7	9.9	12.4
1.15	1.5	0.38	0.57	0.43	2.3	12.2	22.3
1.20	1.5	0.566	0.84	0.16	0.9	13.1	34.5
1.25	1.5	0.738	1.11	– 0.11	– 0.6	12.5	47.6
1.30	1.5	0.867	1.3	– 0.3	– 1.6	10.9	60.1
1.35	1.5	0.946	1.42	– 0.42	– 2.3	8.6	71.0
1.40	1.5	0.983	1.48	– 0.48	– 2.6	6.0	79.6
1.45	1.5	0.997	1.5	– 0.5	– 2.7	3.3	85.6
1.50	1.5						88.9

The swing curve is plotted in Fig. 12.51 from which it follows that the system is stable.

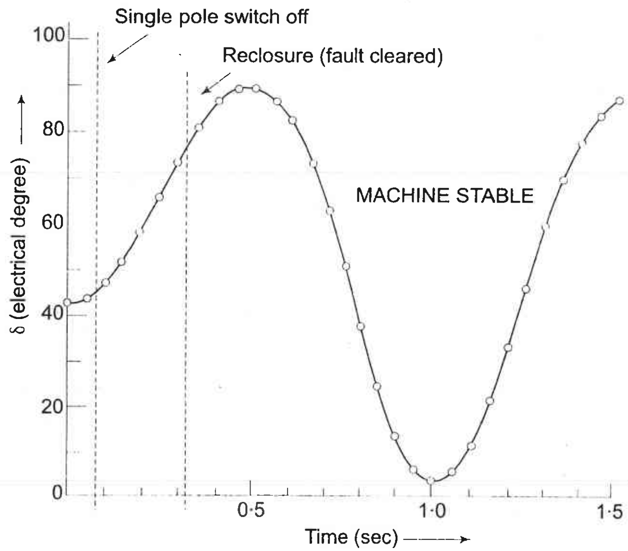


Fig. 12.51 Swing curve for single pole switching with reclosure

PROBLEMS

12.1 A two-pole, 50 Hz, 11 kV turboalternator has a rating of 100 MW, power factor 0.85 lagging. The rotor has a moment of inertia of a 10,000 kg-m^2. Calculate H and M.

12.2 Two turboalternators with ratings given below are interconnected via a short transmission line.

Machine 1: 4 pole, 50 Hz, 60 MW, power factor 0.80 lagging, moment of inertia 30,000 kg-m^2

Machine 2: 2 pole, 50 Hz, 80 MW, power factor 0.85 lagging, moment of inertia 10,000 kg-m^2

Calculate the inertia constant of the single equivalent machine on a base of 200 MVA.

12.3 Power station 1 has four identical generator sets each rated 80 MVA and each having an inertia constant 7 MJ/MVA; while power station 2 has three sets each rated 200 MVA, 3 MJ/MVA. The stations are located close together to be regarded as a single equivalent machine for stability studies. Calculate the inertia constant of the equivalent machine on 100 MVA base.

12.4 A 50 Hz transmission line 500 km long with constants given below ties up two large power areas

$R = 0.11\ \Omega$/km $\qquad L = 1.45$ mH/km

$C = 0.009\ \mu$F/km $\qquad G = 0$

Find the steady state stability limit if $|V_S| = |V_R| = 200$ kV (constant). What will the steady state stability limit be if line capacitance is also neglected? What will the steady state stability limit be if line resistance is also neglected? Comment on the results.

12.5 A power deficient area receives 50 MW over a tie line from another area. The maximum steady state capacity of the tie line is 100 MW. Find the allowable sudden load that can be switched on without loss of stability.

12.6 A synchronous motor is drawing 30% of the maximum steady state power from an infinite bus bar. If the load on motor is suddenly increased by 100 per cent, would the synchronism be lost? If not, what is the maximum excursion of torque angle about the new steady state rotor position.

12.7 The transfer reactances between a generator and an infinite bus bar operating at 200 kV under various conditions on the interconnector are:

Prefault	150 Ω per phase
During fault	400 Ω per phase
Postfault	200 Ω per phase

If the fault is cleared when the rotor has advanced 60 degrees electrical from its prefault position, determine the maximum load that could be transferred without loss of stability.

12.8 A synchronous generator is feeding 250 MW to a large 50 Hz network over a double circuit transmission line. The maximum steady state power that can be transmitted over the line with both circuits in operation is 500 MW and is 350 MW with any one of the circuits.

A solid three-phase fault occurring at the network-end of one of the lines causes it to trip. Estimate the critical clearing angle in which the circuit breakers must trip so that synchronism is not lost.

What further information is needed to estimate the critical clearing time?

12.9 A synchronous generator represented by a voltage source of 1.05 pu in series with a transient reactance of $j0.15$ pu and in inertia constant $H =$ 4.0 sec, is connected to an infinite inertia system through a transmission line. The line has a series reactance of $j0.30$ pu, while the infinite inertia system is represented by a voltage source of 1.0 pu in series with a transient reactance of $j0.20$ pu.

The generator is transmitting an active power of 1.0 pu when a three-phase fault occurs at its terminals. If the fault is cleared in 100 millisec, determine if the system will remain stable by calculating the swing curve.

12.10 For Problem 12.9 find the critical clearing time from the swing curve for a sustained fault.

12.11 A synchronous generator represented by a voltage of 1.15 pu in series with a transient reactance is connected to a large power system with voltage 1.0 pu through a power network. The equivalent transient transfer reactance X between voltage sources is $j0.50$ pu.

After the occurrence of a three-phase to ground fault on one of the lines of the power network, two of the line circuit breakers A and B operate sequentially as follows with corresponding transient transfer reactance given therein.

(i) Short circuit occurs at $\delta = 30°$, A opens instantaneously to make X = 3.0 pu.

(ii) At $\delta = 60°$, A recloses, $X = 6.0$ pu.

(iii) At $\delta = 75°$, A reopens.

(iv) At $\delta = 90°$, B also opens to clear the fault making $X = 0.60$ pu

Check if the system will operate stably.

12.12 A 50 Hz synchronous generator with inertia constant $H = 2.5$ sec and a transient reactance of 0.20 pu feeds 0.80 pu active power into an infinite bus (voltage 1 pu) at 0.8 lagging power factor via a network with an equivalent reactance of 0.25 pu.

A three-phase fault is sustained for 150 millisec across generator terminals. Determine through swing curve calculation the torque angle δ, 250 millisec, after fault initiation.

12.13 A 50 Hz, 500 MVA, 400 kV generator (with transformer) is connected to a 400 kV infinite bus bar through an interconnector. The generator has $H = 2.5$ MJ/MVA, voltage behind transient reactance of 450 kV and is loaded 460 MW. The transfer reactances between generator and bus bar under various conditions are:

Prefault	0.5 pu
During fault	1.0 pu
Postfault	0.75 pu

Calculate the swing curve using intervals of 0.05 sec and assuming that the fault is cleared at 0.15 sec.

12.14 Plot swing curves and check system stability for the fault shown on the system of Example 12.10 for fault clearing by simultaneous opening of breakers at the ends of the faulted line at three cycles and eight cycles after the fault occurs. Also plot the swing curve over a period of 0.6 sec if the fault is sustained. For the generator assume $H = 3.5$ pu, $G = 1$ pu and carry out the computations in per unit.

12.15 Solve Example 12.10 for a LLG fault.

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