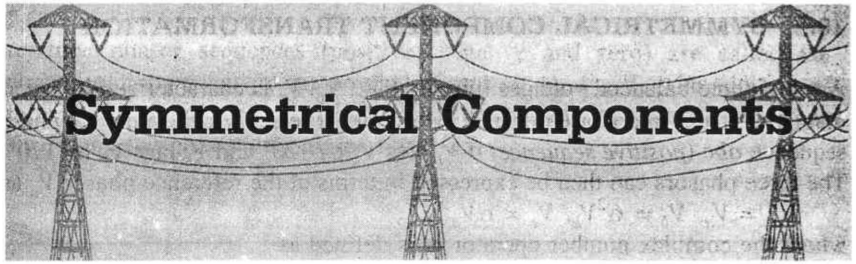
10

Symmetrical Components



10.1 INTRODUCTION

In our work so far, we have considered both normal and abnormal (short circuit) operations of power system under completely balanced (symmetrical) conditions. Under such operation the system impedances in each phase are identical and the three-phase voltages and currents throughout the system are completely balanced, i.e. they have equal magnitudes in each phase and are progressively displaced in time phase by 120° (phase a leads/lags phase b by 120° and phase b leads/lags phase c by 120°). In a balanced system, analysis can proceed on a single-phase basis. The knowledge of voltage and current in one phase is sufficient to completely determine voltages and currents in the other two phases. Real and reactive powers are simply three times the corresponding per phase values.

Unbalanced system operation can result in an otherwise balanced system due to unsymmetrical fault, e.g. line-to-ground fault or line-to-line fault. These faults are, in fact, of more common occurrence* than the symmetrical (three-phase) fault. System operation may also become unbalanced when loads are unbalanced as in the presence of large single-phase loads. Analysis under unbalanced conditions has to be carried out on a three-phase basis. Alternatively, a more convenient method of analyzing unbalanced operation is through symmetrical components where the three-phase voltages (and currents) which may be unbalanced are transformed into three sets of balanced voltages (and

* Typical relative frequencies of occurrence of different kinds of faults in a power system (in order of decreasing severity) are:

Three-phase (3L) faults	5%
Double line-to-ground (LLG) faults	10%
Double line (LL) faults	15%
Single line-to-ground (LG) faults	70%

currents) called symmetrical components. Fortunately, in such a transformation the impedances presented by various power system elements (synchronous generators, transformers, lines) to symmetrical components are decoupled from each other resulting in independent system networks for each component (balanced set). This is the basic reason for the simplicity of the symmetrical component method of analysis.

10.2 SYMMETRICAL COMPONENT TRANSFORMATION

A set of three balanced voltages (phasors) V_a, V_b, V_c is characterized by equal magnitudes and interphase differences of 120°. The set is said to have a phase sequence *abc* (*positive sequence*) if V_b lags V_a by 120° and V_c lags V_b by 120°. The three phasors can then be expressed in terms of the reference phasor V_a as

$$V_a = V_a,\ V_b = \alpha^2 V_a,\ V_c = \alpha V_a$$

where the complex number operator α is defined as

$$\alpha = e^{j120^\circ}$$

It has the following properties

$$\left.\begin{aligned} &\alpha^2 = e^{j240^\circ} = e^{-j120^\circ} = \alpha^* \\ &(\alpha^2)^* = \alpha \\ &\alpha^3 = 1 \\ &1+\alpha+\alpha^2 = 0 \end{aligned}\right] \qquad (10.1)$$

If the phase sequence is *acb* (*negative sequence*), then

$$V_a = V_a,\ V_b = \alpha V_a,\ V_c = \alpha^2 V_a$$

Thus a set of balanced phasors is fully characterized by its reference phasor (say V_a) and its phase sequence (positive or negative).

Suffix 1 is commonly used to indicate positive sequence. A set of (balanced) positive sequence phasors is written as

$$V_{a1},\ V_{b1} = \alpha^2 V_{a1},\ V_{c1} = \alpha V_{a1} \qquad (10.2)$$

Similarly, suffix 2 is used to indicate negative sequence. A set of (balanced) negative sequence phasors is written as

$$V_{a2},\ V_{b2} = \alpha V_{a2},\ V_{c2} = \alpha^2 V_{a2} \qquad (10.3)$$

A set of three voltages (phasors) equal in magnitude and having the same phase is said to have zero sequence. Thus a set of *zero sequence* phasors is written as

$$V_{a0},\ V_{b0} = V_{a0},\ V_{c0} = V_{a0} \qquad (10.4)$$

Consider now a set of three voltages (phasors) V_a, V_b, V_c which in general may be unbalanced. According to Fortesque's theorem* the three phasors can be

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* The theorem is a general one and applies to the case of n phasors [6].

expressed as the sum of positive, negative and zero sequence phasors defined above. Thus

$$V_a = V_{a1} + V_{a2} + V_{a0} \qquad (10.5)$$

$$V_b = V_{b1} + V_{b2} + V_{b0} \qquad (10.6)$$

$$V_c = V_{c1} + V_{c2} + V_{c0} \qquad (10.7)$$

The three phasor sequences (positive, negative and zero) are called the *symmetrical components* of the original phasor set V_a, V_b, V_c. The addition of symmetrical components as per Eqs. (10.5) to (10.7) to generate V_a, V_b, V_c is indicated by the phasor diagram of Fig. 10.1.

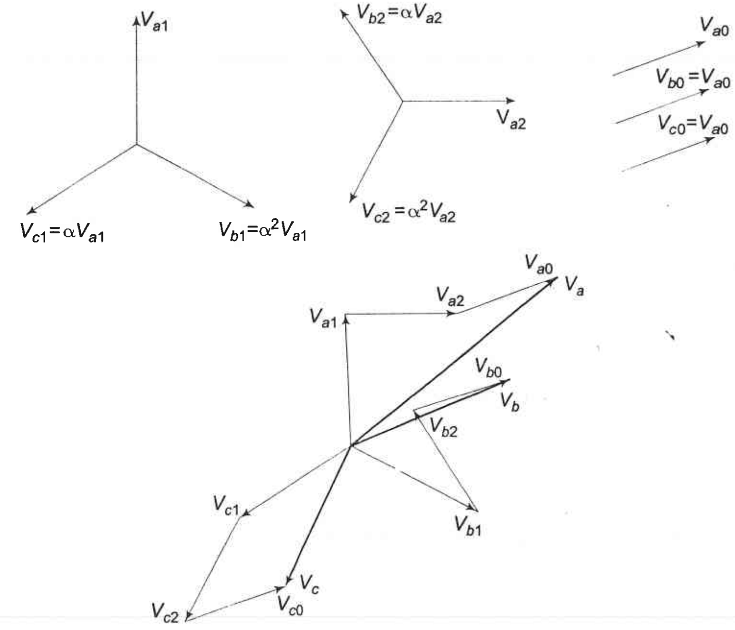


Fig. 10.1 Graphical addition of the symmetrical components to obtain the set of phasors V_a, V_b, V_c (unbalanced in general)

Let us now express Eqs. (10.5) to (10.7) in terms of reference phasors V_{a1}, V_{a2} and V_{a0}. Thus

$$V_a = V_{a1} + V_{a2} + V_{a0} \qquad (10.8)$$

$$V_b = \alpha^2 V_{a1} + \alpha V_{a2} + V_{a0} \qquad (10.9)$$

$$V_c = \alpha V_{a1} + \alpha^2 V_{a2} + V_{a0} \qquad (10.10)$$

These equations can be expressed in the matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} \tag{10.11}$$

or

$$\mathbf{V}_p = \mathbf{A}\mathbf{V}_s \tag{10.12}$$

where

$$V_p = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \text{vector of original phasors}$$

$$V_s = \begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \text{vector of symmetrical components}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \tag{10.13}$$

We can write Eq. (10.12) as

$$\mathbf{V}_s = A^{-1}V_p \tag{10.14}$$

Computing A^{-1} and utilizing relations (10.1), we get

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \tag{10.15}$$

In expanded form we can write Eq. (10.14) as

$$V_{a1} = \frac{1}{3}(V_a + \alpha V_b + \alpha^2 V_c) \tag{10.16}$$

$$V_{a2} = \frac{1}{3}(V_a + \alpha^2 V_b + \alpha V_c) \tag{10.17}$$

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c) \tag{10.18}$$

Equations (10.16) to (10.18) give the necessary relationships for obtaining symmetrical components of the original phasors, while Eqs. (10.5) to (10.7) give the relationships for obtaining original phasors from the symmetrical components.

The symmetrical component transformations though given above in terms of voltages hold for any set of phasors and therefore automatically apply for a set of currents. Thus

$$I_p = AI_s \tag{10.19}$$

and

$$I_s = A^{-1}I_p \tag{10.20}$$

where

$$I_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}; \text{ and } I_s = \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix}$$

Of course A and A^{-1} are the same as given earlier.

In expanded form the relations (10.19) and (10.20) can be expressed as follows:

(i) Construction of current phasors from their symmetrical components:

$$I_a = I_{a1} + I_{a2} + I_{a0} \tag{10.21}$$

$$I_b = \alpha^2 I_{a1} + \alpha I_{a2} + I_{a0} \tag{10.22}$$

$$I_c = \alpha I_{a1} + \alpha^2 I_{a2} + I_{a0} \tag{10.23}$$

(ii) Obtaining symmetrical components of current phasors:

$$I_{a1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c) \tag{10.24}$$

$$I_{a2} = \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c) \tag{10.25}$$

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) \tag{10.26}$$

Certain observations can now be made regarding a three-phase system with neutral return as shown in Fig. 10.2.

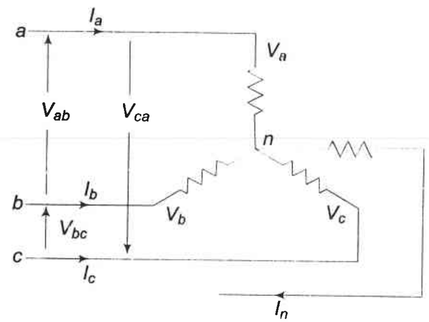


Fig. 10.2 Three-phase system with neutral return

The sum of the three line voltages will always be zero. Therefore, the *zero sequence component of line voltages is always zero*, i.e.

$$V_{ab0} = \frac{1}{3}(V_{ab} + V_{bc} + V_{ca}) = 0 \tag{10.27}$$

On the other hand, the sum of phase voltages (line to neutral) may not be zero so that their zero sequence component V_{a0} may exist.

Since the sum of the three line currents equals the current in the neutral wire, we have

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3}I_n \qquad (10.28)$$

i.e. the current in the neutral is three times the zero sequence line current. If the neutral connection is severed,

$$I_{a0} = \frac{1}{3}I_n = 0 \qquad (10.29)$$

i.e. *in the absence of a neutral connection the zero sequence line current is always zero.*

Power Invariance

We shall now show that the symmetrical component transformation is power invariant, which means that the sum of powers of the three symmetrical components equals the three-phase power.

Total complex power in a three-phase circuit is given by

$$S = V_p^T I_p^* = V_a I_a^* + V_b I_b^* + V_c I_c^* \qquad (10.30)$$

or

$$S = [AV_s]^T [AI_s]^*$$

$$= V_s^T A^T A^* I_s^* \qquad (10.31)$$

Now

$$A^T A^* = \begin{bmatrix} 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & \alpha & 1 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3U \qquad (10.32)$$

$$\therefore \quad S = 3V_s^T U I_s^* = 3V_s^T I_s^*$$

$$= 3V_{a1}I_{a1}^* + 3V_{a2}I_{a2}^* + 3V_{a0}I_{a0}^* \qquad (10.33)$$

$$= \text{sum of symmetrical component powers}$$

Example 10.1

A delta connected balanced resistive load is connected across an unbalanced three-phase supply as shown in Fig. 10.3. With currents in lines A and B specified, find the symmetrical components of line currents. Also find the symmetrical components of delta currents. Do you notice any relationship between symmetrical components of line and delta currents ? Comment.

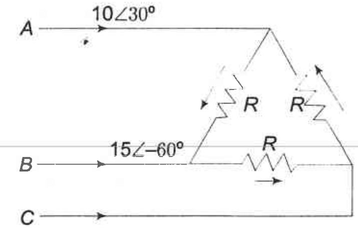


Fig. 10.3

Solution $I_A + I_B + I_C = 0$

or

$$10\angle 30° + 15\angle -60° + I_C = 0$$

$$\therefore \quad I_C = -16.2 + j8.0 = 18\angle 154° \text{ A}$$

From Eqs. (10.24) to (10.26)

$$I_{A1} = \frac{1}{3}(10\angle 30° + 15\angle(-60° + 120°) + 18\angle(154° + 240°))$$

$$= 10.35 + j9.3 = 14\angle 42° \text{ A} \qquad \text{(i)}$$

$$I_{A2} = \frac{1}{3}(10\angle 30° + 15\angle(-60° + 240°) + 18\angle(154° + 120°))$$

$$= -1.7 - j4.3 = 4.65\angle 248° \text{ A} \qquad \text{(ii)}$$

$$I_{A0} = \frac{1}{3}(I_A + I_B + I_C) = 0 \qquad \text{(iii)}$$

From Eq. (10.2)

$I_{B1} = 14\angle 282°$ A $\qquad I_{C1} = 14\angle 162°$ A

$I_{B2} = 4.65\angle 8°$ A $\qquad I_{C2} = 4.65\angle 128°$ A

$I_{B0} = 0$ A $\qquad I_{C0} = 0$ A

Check:

$$I_A = I_{A1} + I_{A2} + I_{A0} = 8.65 + j5 = 10\angle 30°$$

Converting delta load into equivalent star, we can redraw Fig. 10.3 as in Fig. 10.4.

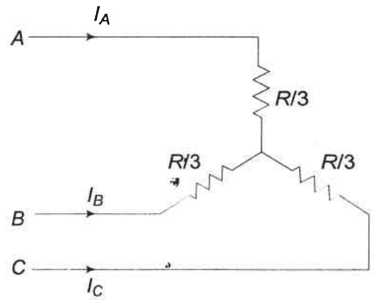


Fig. 10.4

Delta currents are obtained as follows

$$V_{AB} = \frac{1}{3} R\,(I_A - I_B)$$

Now

$$I_{AB} = V_{AB}/R = \frac{1}{3}(I_A - I_B)$$

Similarly,

$$I_{BC} = \frac{1}{3}(I_B - I_C)$$

$$I_{CA} = \frac{1}{3}(I_C - I_A)$$

Substituting the values of I_A, I_B and I_C, we have

$$I_{AB} = \frac{1}{3}(10\angle 30° - 15\angle -60°) = 6\angle 86° \text{ A}$$

$$I_{BC} = \frac{1}{3}(15\angle -60° - 18\angle 154°) = 10.5\angle -41.5° \text{ A}$$

$$I_{CA} = \frac{1}{3}(18\angle 154° - 10\angle 30°) = 8.3\angle 173° \text{ A}$$

The symmetrical components of delta currents are

$$I_{AB1} = \frac{1}{3}(6\angle 86° + 10.5\angle(-41.5° + 120°) + 8.3\angle(173° + 240°)) \quad \text{(iv)}$$

$$= 8\angle 72° \text{ A}$$

$$I_{AB2} = \frac{1}{3}(6\angle 86° + 10.5\angle(-41.5° + 240°) + 8.3\angle(173° + 120°)) \quad \text{(v)}$$

$$= 2.7\angle 218° \text{ A}$$

$$I_{AB0} = 0 \quad \text{(vi)}$$

I_{BC1}, I_{BC2}, I_{BC0}, I_{CA1}, I_{CA2} and I_{CA0} can be found by using Eq. (10.2).

Comparing Eqs. (i) and (iv), and (ii) and (v), the following relationship between symmetrical components of line and delta currents are immediately observed:

$$I_{AB1} = \frac{I_{A1}}{\sqrt{3}}\angle 30° \quad \text{(vii)}$$

$$I_{AB2} = \frac{I_{A2}}{\sqrt{3}}\angle -30° \quad \text{(viii)}$$

The reader should verify these by calculating I_{AB1} and I_{AB2} from Eqs. (vii) and (viii) and comparing the results with Eqs. (iv) and (v).

10.3 PHASE SHIFT IN STAR-DELTA TRANSFORMERS

Positive and negative sequence voltages and currents undergo a phase shift in passing through a star-delta transformer which depends upon the labelling of terminals. Before considering this phase shift, we need to discuss the standard polarity marking of a single-phase transformer as shown in Fig. 10.5. The transformer ends marked with a dot have the same polarity. Therefore, voltage $V_{HH'}$ is in phase with voltage $V_{LL'}$. Assuming that the small amount of magnetizing current can be neglected, the primary current I_1, entering the dotted end cancels the demagnetizing ampere-turns of the secondary current I_2 so that I_1 and I_2 with directions of flow as indicated in the diagram are in phase. If the direction of I_2 is reversed, I_1 and I_2 will be in phase opposition.

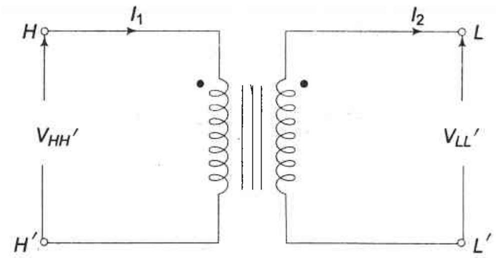


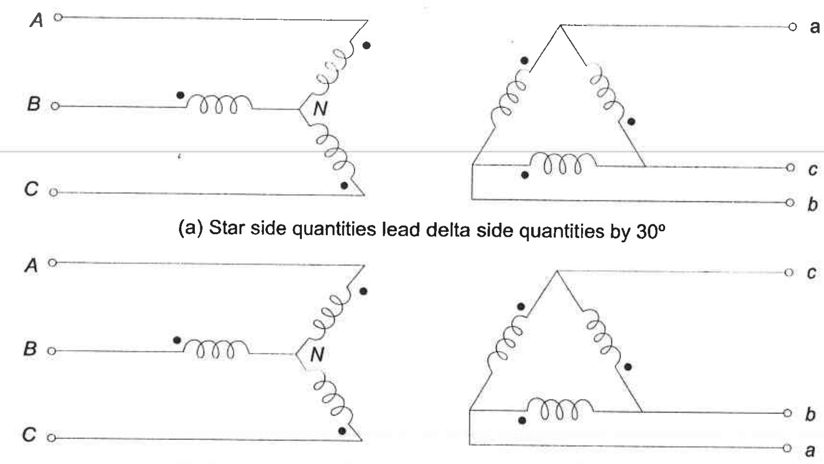
Fig. 10.5 Polarity marking of a single-phase transformer

Consider now a star/delta transformer with terminal labelling as indicated in Fig. 10.6 (a). Windings shown parallel to each other are magnetically coupled. Assume that the transformer is excited with positive sequence voltages and carries positive sequence currents. With the polarity marks shown, we can immediately draw the phasor diagram of Fig. 10.7. The following interrelationship between the voltages on the two sides of the transformer is immediately observed from the phasor diagram

$$V_{AB1} = x\, V_{ab1} \angle 30°, \; x = \text{phase transformation ratio} \quad (10.34)$$

As per Eq. (10.34), the positive sequence line voltages on star side lead the corresponding voltages on the delta side by 30° (The same result would apply to line-to-neutral voltages on the two sides). The same also applies for line currents.

If the delta side is connected as in Fig. 10.6(b) the phase shift reverses (the reader should draw the phasor diagram); the delta side quantities lead the star side quantities by 30°.



(a) Star side quantities lead delta side quantities by 30°

(b) Delta side quantities lead star side quantities by 30°

Fig. 10.6 Labelling of star/delta transformer

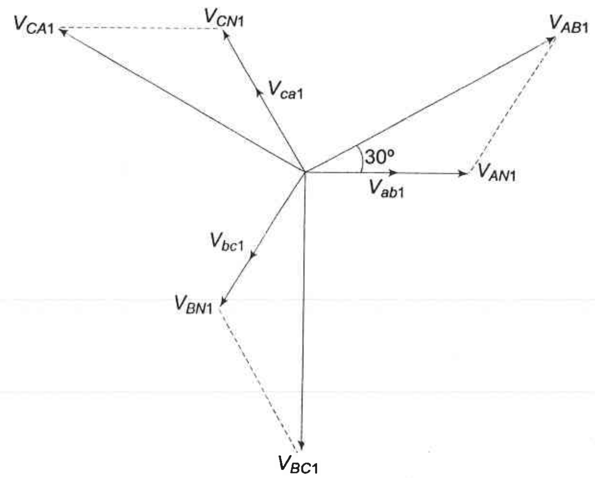


Fig. 10.7 Positive sequence voltages on a star/delta transformer

Instead, if the transformer of Fig. 10.6(a) is now excited by negative sequence voltages and currents, the voltage phasor diagram will be as in Fig. 10.8. The phase shift in comparison to the positive sequence case now reverses, i.e., the star side quantities lag the delta side quantities by 30°. The result for Fig. 10.6(b) also correspondingly reverses.

It shall from now onwards be assumed that *a star/delta transformer is so labelled that the positive sequence quantities on the HV side lead their corresponding positive sequence quantities on the LV side by 30°. The reverse is the case for negative sequence quantities wherein HV quantities lag the corresponding LV quantities by 30°.*

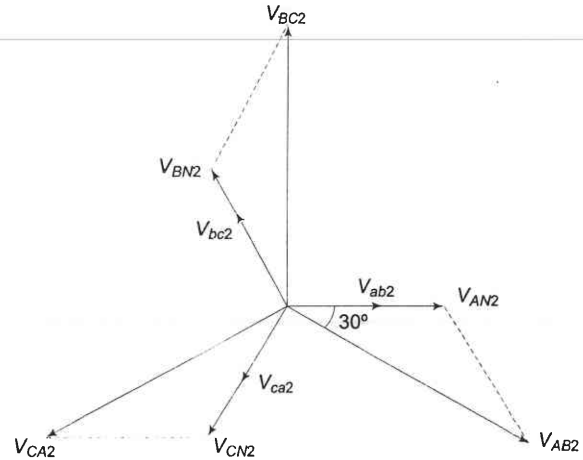


Fig. 10.8 Negative sequence voltages on a star/delta transformer

10.4 SEQUENCE IMPEDANCES OF TRANSMISSION LINES

Figure 10.9 shows the circuit of a fully transposed line carrying unbalanced currents. The return path for I_n is sufficiently away for the mutual effect to be ignored. Let

X_s = self reactance of each line

X_m = mutual reactance of any line pair

The following KVL equations can be written down from Fig. 10.9.

$$V_a - V'_a = jX_sI_a + jX_mI_b + jX_mI_c$$

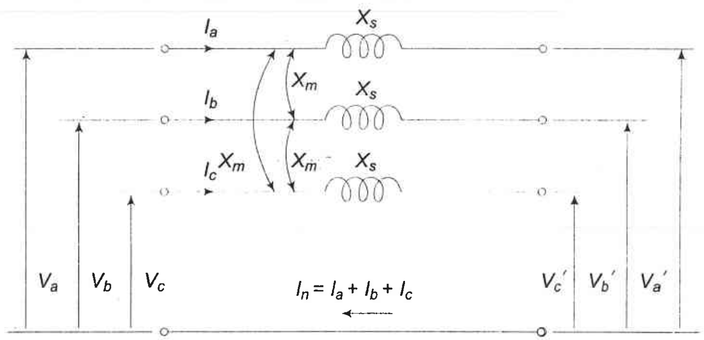


Fig. 10.9

$$V_b - V_b' = jX_m I_a + jX_s I_b + jX_m I_c \tag{10.35}$$

$$V_c - V_c' = jX_m I_a + jX_m I_b + jX_s I_c$$

or in matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \begin{bmatrix} V_a' \\ V_b' \\ V_c' \end{bmatrix} = j \begin{bmatrix} X_s X_m X_m \\ X_m X_s X_m \\ X_m X_m X_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{10.36}$$

or $$V_p - V_p' = ZI_p \tag{10.37}$$

or $$A\,(V_s - V_s') = ZAI_s \tag{10.38}$$

or $$V_s - V_s' = A^{-1}ZAI_s \tag{10.39}$$

Now

$$A^{-1}ZA = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} jX_s & jX_m & jX_m \\ jX_m & jX_s & jX_m \\ jX_m & jX_m & jX_s \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \tag{10.40}$$

$$= j\begin{bmatrix} X_s - X_m & 0 & 0 \\ 0 & X_s - X_m & 0 \\ 0 & 0 & X_s + 2X_m \end{bmatrix}$$

Thus Eq. (10.37) can be written as

$$\begin{bmatrix} V_1 \\ V_2 \\ V_0 \end{bmatrix} - \begin{bmatrix} V_1' \\ V_2' \\ V_0' \end{bmatrix} = j\begin{bmatrix} X_s - X_m & 0 & 0 \\ 0 & X_s - X_m & 0 \\ 0 & 0 & X_s + 2X_m \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_0 \end{bmatrix} \tag{10.41}$$

$$= \begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_2 & 0 \\ 0 & 0 & Z_0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_0 \end{bmatrix} \tag{10.42}$$

wherein

$$Z_1 = j(X_s - X_m) = \textit{positive sequence impedance} \tag{10.43}$$

$$Z_2 = j(X_s - X_m) = \textit{negative sequence impedance} \tag{10.44}$$

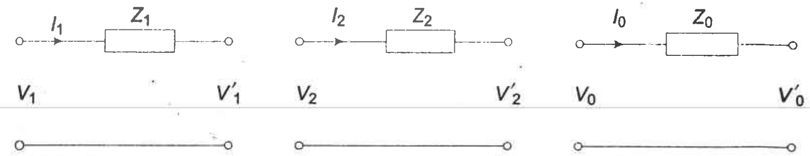
$$Z_0 = j(X_s + 2X_m) = \textit{zero sequence impedance} \tag{10.45}$$

We conclude that a fully transposed transmission has:

(i) equal positive and negative sequence impedances.

(ii) zero sequence impedance much larger than the positive (or negative) sequence impedance (it is approximately 2.5 times).

It is further observed that the sequence circuit equations (10.42) are in *decoupled* form, i.e. there are no mutual sequence inductances. Equation (10.42) can be represented in network form as in Fig. 10.10.



(a) Positive sequence network (b) Nagative sequence network (ç) Zero sequence network

Fig. 10.10

The decoupling between sequence networks of a fully transposed transmission holds also in 3-phase synchronous machines and 3-phase transformers. This fact leads to considerable simplications in the use of symmetrical components method in unsymmetrical fault analysis.

In case of three static unbalanced impedances, coupling appears between sequence networks and the method is no more helpful than a straight forward 3-phase analysis.

10.5 SEQUENCE IMPEDANCES AND SEQUENCE NETWORK OF POWER SYSTEM

Power system elements—transmission lines, transformers and synchronous machines—have a three-phase symmetry because of which when currents of a particular sequence are passed through these elements, voltage drops of the same sequence appear, i.e. the elements possess only self impedances to sequence currents. Each element can therefore be represented by three decoupled *sequence networks* (on single-phase basis) pertaining to positive, negative and zero sequences, respectively. EMFs are involved only in a positive sequence network of synchronous machines. For finding a particular sequence impedance, the element in question is subjected to currents and voltages of that sequence only. With the element operating under these conditions, the sequence impedance can be determined analytically or through experimental test results.

With the knowledge of sequence networks of elements, complete positive, negative and zero sequence networks of any power system can be assembled. As will be explained in the next chapter, these networks are suitably interconnected to simulate different unsymmetrical faults. The sequence currents and voltages during the fault are then calculated from which actual fault currents and voltages can be found.

10.6 SEQUENCE IMPEDANCES AND NETWORKS OF SYNCHRONOUS MACHINE

Figure 10.11 depicts an unloaded synchronous machine (generator or motor) grounded through a reactor (impedance Z_n). E_a, E_b and E_c are the induced emfs of the three phases. When a fault (not shown in the figure) takes place at machine terminals, currents I_a, I_b and I_c flow in the lines. Whenever the fault involves ground, current $I_n = I_a + I_b + I_c$ flows to neutral from ground via Z_n. Unbalanced line currents can be resolved into their symmetrical components I_{a1}, I_{a2} and I_{a0}. Before we can proceed with fault analysis (Chapter 11), we must know the equivalent circuits presented by the machine to the flow of positive, negative and zero sequence currents, respectively. Because of winding symmetry currents of a particular sequence produce voltage drops of that sequence only. Therefore, there is a no coupling between the equivalent circuits of various sequences*.

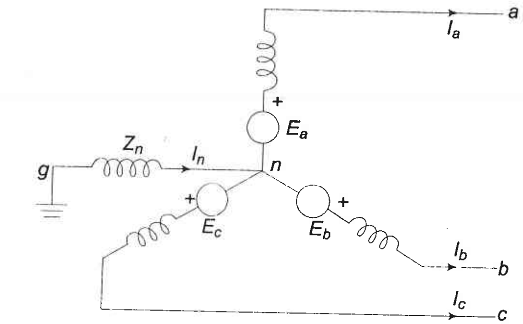


Fig. 10.11 Three-phase synchronous generator with grounded neutral

Positive Sequence Impedance and Network

Since a synchronous machine is designed with symmetrical windings, it induces emfs of positive sequence only, i.e. no negative or zero sequence voltages are induced in it. When the machine carries positive sequence currents only, this mode of operation is the balanced mode discussed at length in Chapter 9. The armature reaction field caused by positive sequence currents rotates at synchronous speed in the same direction as the rotor, i.e., it is stationary with respect to field excitation. The machine equivalently offers a direct axis reactance whose value reduces from subtransient reactance (X''_d) to transient reactance (X'_d) and finally to steady state (synchronous) reactance (X_d), as the short circuit transient progresses in time. If armature resistance is assumed negligible, the positive sequence impedance of the machine is

$$Z_1 = jX''_d \text{ (if 1 cycle transient is of interest)} \quad (10.46)$$

$$= jX'_d \text{ (if 3-4 cycle transient is of interest)} \quad (10.47)$$

$$= jX_d \text{ (if steady state value is of interest)} \quad (10.48)$$

If the machine short circuit takes place from unloaded conditions, the terminal voltage constitutes the positive sequence voltage; on the other hand, if

*This can be shown to be so by synchronous machine theory [5].

the short circuit occurs from loaded conditions, the voltage behind appropriate reactance (subtransient, transient or synchronous) constitutes the positive sequence voltage.

Figure 10.12a shows the three-phase positive sequence network model of a synchronous machine. Z_n does not appear in the model as $I_n = 0$ for positive sequence currents. Since it is a balanced network it can be represented by the single-phase network model of Fig. 10.12b for purposes of analysis. The reference bus for a positive sequence network is at neutral potential. Further, since no current flows from ground to neutral, the neutral is at ground potential.

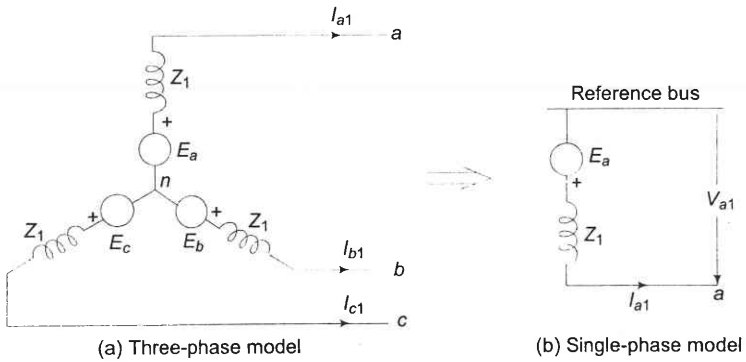


Fig. 10.12 Positive sequence network of synchronous machine

With reference to Fig. 10.12b, the positive sequence voltage of terminal a with respect to the reference bus is given by

$$V_{a1} = E_a - Z_1 I_{a1} \quad (10.49)$$

Negative Sequence Impedance and Network

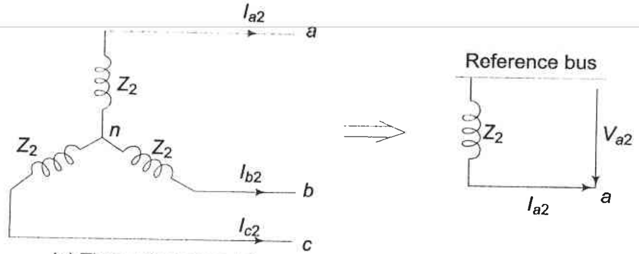
It has already been said that a synchronous machine has zero negative sequence induced voltages. With the flow of negative sequence currents in the stator a rotating field is created which rotates in the opposite direction to that of the positive sequence field and, therefore, at double synchronous speed with respect to rotor. Currents at double the stator frequency are therefore induced in rotor field and damper winding. In sweeping over the rotor surface, the negative sequence mmf is alternately presented with reluctances of direct and quadrature axes. The negative sequence impedance presented by the machine with consideration given to the damper windings, is often defined as

$$Z_2 = j\frac{X''_q + X''_d}{2}; \ |Z_2| < |Z_1| \quad (10.50)$$

Negative sequence network models of a synchronous machine, on a three-phase and single-phase basis are shown in Figs. 10.13a and b, respectively. The reference bus is of course at neutral potential which is the same as ground potential.

From Fig. 10.13b the negative sequence voltage of terminal a with respect to reference bus is

$$V_{a2} = -\ Z_2 I_{a2} \tag{10.51}$$

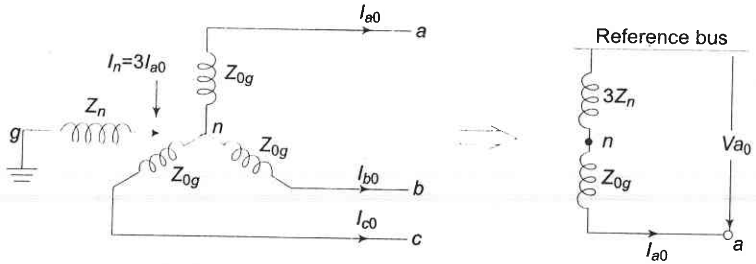


(a) Three-phase model (b) Single-phase model

Fig. 10.13 Negative sequence network of a synchronous machine

Zero Sequence Impedance and Network

We state once again that no zero sequence voltages are induced in a synchronous machine. The flow of zero sequence currents creates three mmfs which are in time phase but are distributed in space phase by 120°. The resultant air gap field caused by zero sequence currents is therefore zero. Hence, the rotor windings present leakage reactance only to the flow of zero sequence currents ($Z_{0g} < Z_2 < Z_1$).



(a) Three-phase model (b) Single-phase model

10.14 Zero sequence network of a synchronous machine

Zero sequence network models on a three- and single-phase basis are shown in Figs. 10.14a and b. In Fig. 10.14a, the current flowing in the impedance Z_n between neutral and ground is $I_n = 3I_{a0}$. The zero sequence voltage of terminal a with respect to ground, the reference bus, is therefore

$$V_{a0} = -\ 3Z_n I_{a0} - Z_{0g} I_{a0} = -\ (3Z_n + Z_{0g})\, I_{a0} \tag{10.52}$$

where Z_{0g} is the zero sequence impedance per phase of the machine.

Since the single-phase zero sequence network of Fig. 10.14b carries only per phase zero sequence current, its total zero sequence impedance must be

$$Z_0 = 3Z_n + Z_{0g} \tag{10.53}$$

in order for it to have the same voltage from a to reference bus. The reference bus here is, of course, at ground potential.

From Fig. 10.14b zero sequence voltage of point a with respect to the reference bus is

$$V_{a0} = -\ Z_0 I_{a0} \tag{10.54}$$

Order of Values of Sequence Impedances of a Synchronous Generator

Typical values of sequence impedances of a turbo-generator rated 5 MVA, 6.6 kV, 3,000 rpm are:

Z_1 = 12% (subtransient)

Z_1 = 20% (transient)

Z_1 = 110% (synchronous)

Z_2 = 12%

Z_0 = 5%

For typical values of positive, negative and zero sequence reactances of a synchronous machine refer to Table 9.1.

10.7 SEQUENCE IMPEDANCES OF TRANSMISSION LINES

A fully transposed three-phase line is completely symmetrical and therefore the per phase impedance offered by it is independent of the phase sequence of a balanced set of currents. In other words, the impedances offered by it to positive and negative sequence currents are identical. The expression for its per phase inductive reactance accounting for both self and mutual linkages has been derived in Chapter 2.

When only zero sequence currents flow in a transmission line, the currents in each phase are identical in both magnitude and phase angle. Part of these currents return via the ground, while the rest return through the overhead ground wires. The ground wires being grounded at several towers, the return currents in the ground wires are not necessarily uniform along the entire length. The flow of zero sequence currents through the transmission lines, ground wires and ground creates a magnetic field pattern which is very different from that caused by the flow of positive or negative sequence currents where the currents have a phase difference of 120° and the return current is zero. The zero sequence impedance of a transmission line also accounts for the ground impedance ($Z_0 = Z_{l0} + 3Z_{g0}$). Since the ground impedance heavily depends on soil conditions, it is essential to make some simplifying assumptions to obtain analytical results. The zero sequence impedance of transmission lines usually ranges from 2 to 3.5 times the positive sequence impedance*. This ratio is on the higher side for double circuit lines without ground wires.

10.8 SEQUENCE IMPEDANCES AND NETWORKS OF TRANSFORMERS

It is well known that almost all present day installations have three-phase transformers since they entail lower initial cost, have smaller space requirements and higher efficiency.

The positive sequence series impedance of a transformer equals its leakage impedance. Since a transformer is a static device, the leakage impedance does not change with alteration of phase sequence of balanced applied voltages. The transformer negative sequence impedance is also therefore equal to its leakage reactance. Thus, for a transformer

$$Z_1 = Z_2 = Z_{\text{leakage}} \tag{10.55}$$

Assuming such transformer connections that zero sequence currents can flow on both sides, a transformer offers a zero sequence impedance which may differ slightly from the corresponding positive and negative sequence values. It is, however, normal practice to assume that the series impedances of all sequences are equal regardless of the type of transformer.

The zero sequence magnetizing current is somewhat higher in a core type than in a shell type transformer. This difference does not matter as the magnetizing current of a transformer is always neglected in short circuit analysis.

Above a certain rating (1,000 kVA) the reactance and impedance of a transformer are almost equal and are therefore not distinguished.

*We can easily compare the forward path positive and zero sequence impedances of a transmission line with ground return path infinitely away. Assume that each line has a self inductance, L and mutual inductance M between any two lines (completely symmetrical case). The voltage drop in line a caused by positive sequence currents is

$$V_{Aa1} = \omega L I_{a1} + \omega M I_{b1} + \omega M I_{c1}$$

$$= [\omega L + (\alpha^2 + \alpha)\,\omega M] I_{a1} = \omega(L-M) I_{a1}$$

$\therefore$ Positive sequence reactance $= \omega(L - M)$

The voltage drop in line a caused by zero sequence currents is

$$V_{Aa0} = \omega L I_{a0} + \omega M I_{b0} + \omega M I_{c0}$$

$$= \omega(L + 2M) I_{a0}$$

$\therefore$ Zero sequence reactance $= \omega(L + 2M)$

Obviously, zero sequence reactance is much more than positive sequence reactance. This result has already been derived in Eq. (10.45).

Zero Sequence Networks of Transformers

Before considering the zero sequence networks of various types of transformer connections, three important observations are made:

(i) When magnetizing current is neglected, transformer primary would carry current only if there is current flow on the secondary side.

(ii) Zero sequence currents can flow in the legs of a star connection only if the star point is grounded which provides the necessary return path for zero sequence currents. This fact is illustrated by Figs. 10.15a and b.

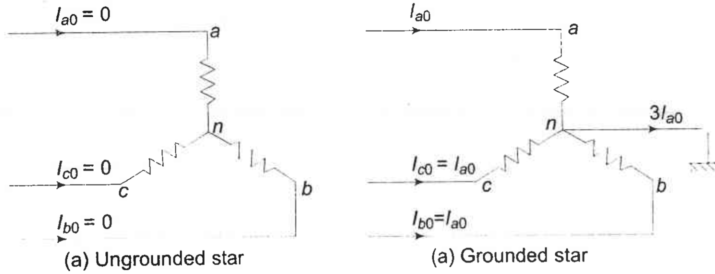


Fig. 10.15 Flow of zero sequence currents in a star connection

(iii) No zero sequence currents can flow in the lines connected to a delta connection as no return path is available for these currents. Zero sequence currents can, however, flow in the legs of a delta—such currents are caused by the presence of zero sequence voltages in the delta connection. This fact is illustrated by Fig. 10.16.

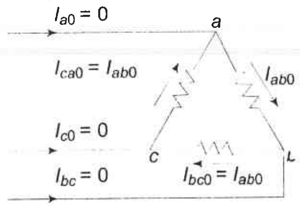


Fig. 10.16 Flow of zero sequence currents in a delta connection

Let us now consider various types of transformer connections.

Case 1: Y-Y transformer bank with any one neutral grounded.

If any one of the two neutrals of a Y-Y transformer is ungrounded, zero sequence currents cannot flow in the ungrounded star and consequently, these cannot flow in the grounded star. Hence, an open circuit exists in the zero sequence network between H and L, i.e. between the two parts of the system connected by the transformer as shown in Fig. 10.17.

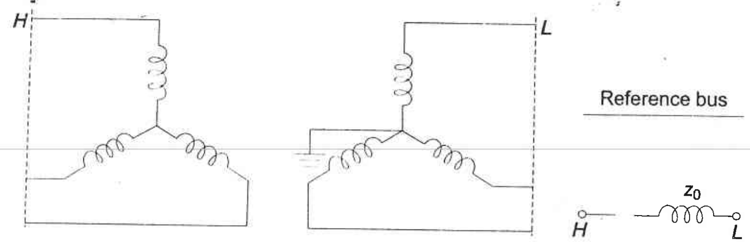


Fig. 10.17 *Y-Y* transformer bank with one neutral grounded and its zero sequence network

Case 2: Y-Y transformer bank both neutrals grounded

When both the neutrals of a *Y–Y* transformer are grounded, a path through the transformer exists for zero sequence currents in both windings via the two grounded neutrals. Hence, in the zero sequence network *H* and *L* are connected by the zero sequence impedance of the transformer as shown in Fig. 10.18.

Case 3: Y-Δ transformer bank with grounded Y neutral

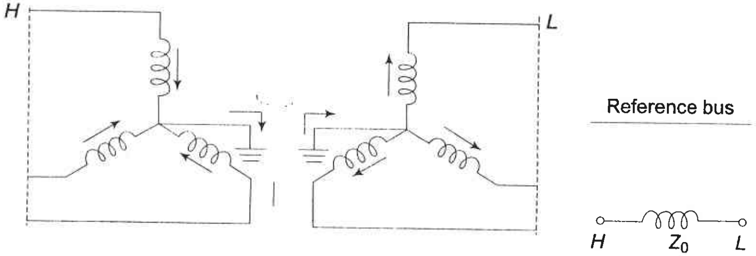


Fig. 10.18 *Y-Y* transformer bank with neutrals grounded and its zero sequence network

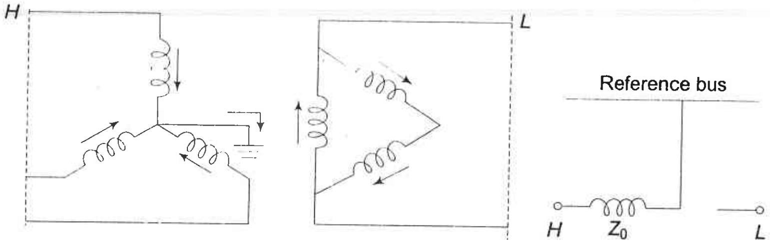


Fig. 10.19 *Y-Δ* transformer bank with grounded *Y* neutral and its zero sequence network

If the neutral of star side is grounded, zero sequence currents can flow in star because a path is available to ground and the balancing zero sequence currents can flow in delta. Of course no zero sequence currents can flow in the line on the delta side. The zero sequence network must therefore have a path from the line *H* on the star side through the zero sequence impedance of the transformer to the reference bus, while an open circuit must exist on the line *L* side of delta (*see* Fig. 10.19). If the star neutral is grounded through Z_n, an impedance $3Z_n$ appears in series with Z_0 in the sequence network.

Case 4: Y-Δ transformer bank with ungrounded star

This is the special case of Case 3 where the neutral is grounded through $Z_n = \infty$. Therefore no zero sequence current can flow in the transformer windings. The zero sequence network then modifies to that shown in Fig. 10.20.

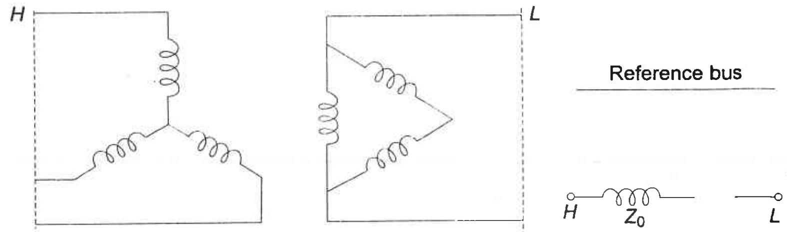


Fig. 10.20 *Y-Δ* transformer bank with ungrounded star and its zero sequence network

Case 5: Δ-Δ transformer bank

Since a delta circuit provides no return path, the zero sequence currents cannot flow in or out of *Δ-Δ* transformer; however, it can circulate in the delta windings*. Therefore, there is an open circuit between *H* and *L* and Z_0 is connected to the reference bus on both ends to account for any circulating zero sequence current in the two deltas (*see* Fig. 10.21).

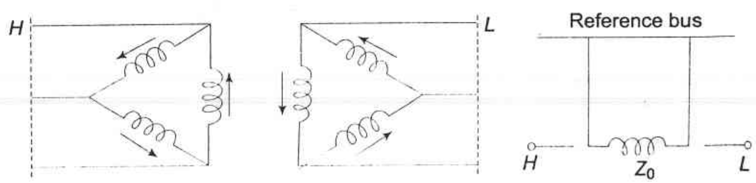


Fig. 10.21 *Δ-Δ* transformer bank and its zero sequence network

10.9 CONSTRUCTION OF SEQUENCE NETWORKS OF A POWER SYSTEM

In the previous sections the sequence networks for various power system elements—synchronous machines, transformers and lines—have been given. Using these, complete sequence networks of a power system can be easily constructed. To start with, the positive sequence network is constructed by

*Such circulating currents would exist only if zero sequence voltages are somehow induced in either delta winding.

examination of the one-line diagram of the system. It is to be noted that positive sequence voltages are present in synchronous machines (generators and motors) only. The transition from positive sequence network to negative sequence network is straightforward. Since the positive and negative sequence impedances are identical for static elements (lines and transformers), the only change necessary in positive sequence network to obtain negative sequence network is in respect of synchronous machines. Each machine is represented by its negative sequence impedance, the negative sequence voltage being zero.

The reference bus for positive and negative sequence networks is the system neutral. Any impedance connected between a neutral and ground is not included in these sequence networks as neither of these sequence currents can flow in such an impedance.

Zero sequence subnetworks for various parts of a system can be easily combined to form complete zero sequence network. No voltage sources are present in the zero sequence network. Any impedance included in generator or transformer neutral becomes three times its value in a zero sequence network. Special care needs to be taken of transformers in respect of zero sequence network. Zero sequence networks of all possible transformer connections have been dealt with in the preceding section.

The procedure for drawing sequence networks is illustrated through the following examples.

Example 10.2

A 25 MVA, 11 kV, three-phase generator has a subtransient reactance of 20%. The generator supplies two motors over a transmission line with transformers at both ends as shown in the one-line diagram of Fig. 10.22. The motors have rated inputs of 15 and 7.5 MVA, both 10 kV with 25% subtransient reactance. The three-phase transformers are both rated 30 MVA, 10.8/121 kV, connection Δ-Y with leakage reactance of 10% each. The series reactance of the line is 100 ohms. Draw the positive and negative sequence networks of the system with reactances marked in per unit.

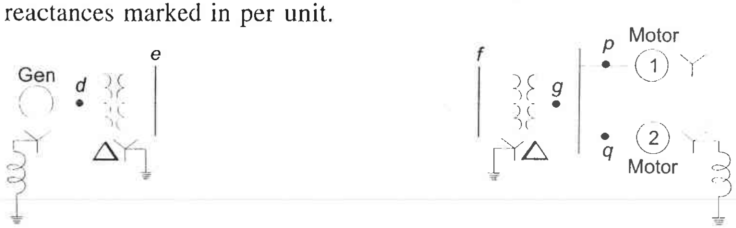


Fig. 10.22

Assume that the negative sequence reactance of each machine is equal to its subtransient reactance. Omit resistances. Select generator rating as base in the generator circuit.

Solution A base of 25 MVA, 11 kV in the generator circuit requires a 25 MVA base in all other circuits and the following voltage bases.

$$\text{Transmission line voltage base} = 11 \times \frac{121}{10.8} = 123.2 \text{ kV}$$

$$\text{Motor voltage base} = 123.2 \times \frac{10.8}{121} = 11 \text{ kV}$$

The reactances of transformers, line and motors are converted to pu values on appropriate bases as follows:

$$\text{Transformer reactance} = 0.1 \times \frac{25}{30} \times \left(\frac{10.8}{11}\right)^2 = 0.0805 \text{ pu}$$

$$\text{Line reactance} = \frac{100 \times 25}{(123.2)^2} = 0.164 \text{ pu}$$

$$\text{Reactance of motor 1} = 0.25 \times \frac{25}{15} \times \left(\frac{10}{11}\right)^2 = 0.345 \text{ pu}$$

$$\text{Reactance of motor 2} = 0.25 \times \frac{25}{7.5} \times \left(\frac{10}{11}\right)^2 = 0.69 \text{ pu}$$

The required positive sequence network is presented in Fig. 10.23.

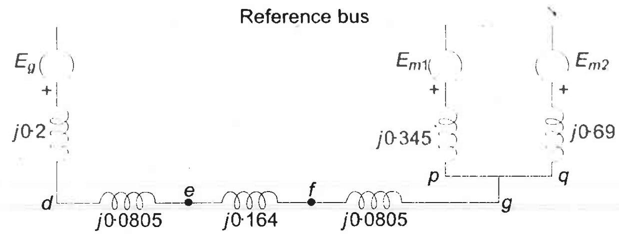


Fig. 10.23 Positive sequence network for Example 10.3

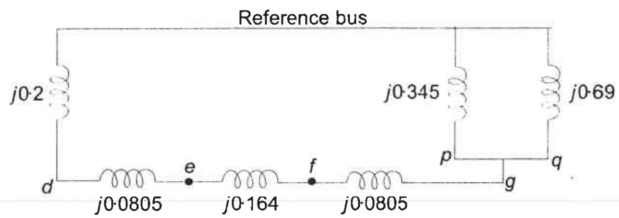


Fig. 10.24 Negative sequence network for Example 10.3

Since all the negative sequence reactances of the system are equal to the positive sequence reactances, the negative sequence network is identical to the positive sequence network but for the omission of voltage sources. The negative sequence network is drawn in Fig. 10.24.

Example 10.3

For the power system whose one-line diagram is shown in Fig. 10.25, sketch the zero sequence network.

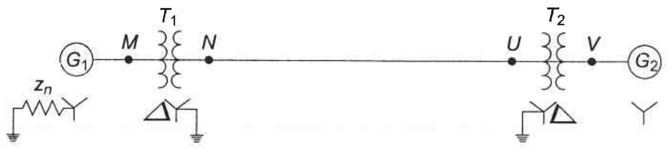


Fig. 10.25

Solution The zero sequence network is drawn in Fig. 10.26.

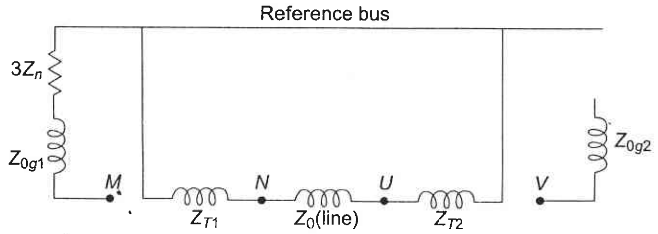


Fig. 10.26 Zero sequence network of the system presented in Fig. 10.25

Example 10.4

Draw the zero sequence network for the system described in Example 10.2. Assume zero sequence reactances for the generator and motors of 0.06 per unit. Current limiting reactors of 2.5 ohms each are connected in the neutral of the generator and motor No. 2. The zero sequence reactance of the transmission line is 300 ohms.

Solution The zero sequence reactance of the transformer is equal to its positive sequence reactance. Hence

Transformer zero sequence reactance = 0.0805 pu

Generator zero sequence reactances = 0.06 pu

$$\text{Zero sequence reactance of motor 1} = 0.06 \times \frac{25}{15} \times \left(\frac{10}{11}\right)^2$$
$$= 0.082 \text{ pu}$$

$$\text{Zero sequence reactance of motor 2} = 0.06 \times \frac{25}{7.5} \times \left(\frac{10}{11}\right)^2$$
$$= 0.164 \text{ pu}$$

$$\text{Reactance of current limiting reactors} = \frac{2.5 \times 25}{(11)^2} = 0.516 \text{ pu}$$

Reactance of current limiting reactor included in zero sequence network

$$= 3 \times 0.516 = 1.548 \text{ pu}$$

$$\text{Zero sequence reactance of transmission line} = \frac{300 \times 25}{(123.2)^2}$$
$$= 0.494 \text{ pu}$$

The zero sequence network is shown in Fig. 10.27.

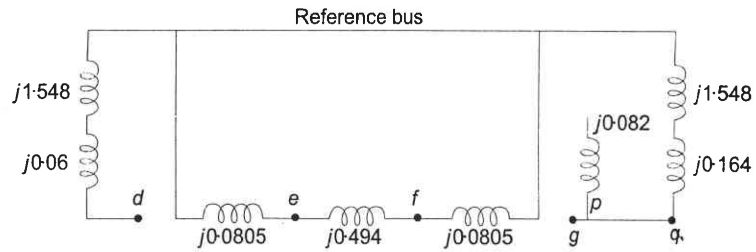


Fig. 10.27 Zero sequence network of Example 10.5

PROBLEMS

10.1 Compute the following in polar form

(i) $\alpha^2 - 1$ (ii) $1 - \alpha - \alpha^2$ (iii) $3\alpha^2 + 4\alpha + 2$ (iv) $j\alpha$

10.2 Three identical resistors are star connected and rated 2,500 V, 750 kVA. This three-phase unit of resistors is connected to the Y side of a Δ-Y transformer. The following are the voltages at the resistor load

$$|V_{ab}| = 2{,}000 \text{ V}; \ |V_{bc}| = 2{,}900 \text{ V}; \ |V_{ca}| = 2{,}500 \text{ V}$$

Choose base as 2,500 V, 750 kVA and determine the line voltages and currents in per unit on the delta side of the transformer. It may be assumed that the load neutral is not connected to the neutral of the transformer secondary.

10.3 Determine the symmetrical components of three voltages

$$V_a = 200\angle 0°, \ V_b = 200\angle 245° \text{ and } V_c = 200\angle 105° \text{ V}$$

10.4 A single-phase resistive load of 100 kVA is connected across lines *bc* of a balanced supply of 3 kV. Compute the symmetrical components of the line currents.

10.5 A delta connected resistive load is connected across a balanced three-phase supply

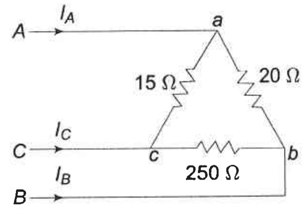


Fig. P-10.5 Phase sequence *ABC*

of 400 V as shown in Fig. P-10.5. Find the symmetrical components of line currents and delta currents.

10.6 Three resistances of 10, 15 and 20 ohms are connected in star across a three-phase supply of 200 V per phase as shown in Fig. P-10.6. The supply neutral is earthed while the load neutral is isolated. Find the currents in each load branch and the voltage of load neutral above earth. Use the method of symmetrical components.

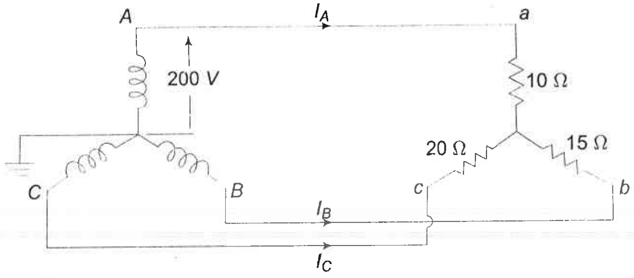


Fig. P-10.6

10.7 The voltages at the terminals of a balanced load consisting of three 20 ohm *Y*-connected resistors are $200\angle 0°$, $100\ \angle 255.5°$ and $200\ \angle 151°$ V. Find the line currents from the symmetrical components of the line voltages if the neutral of the load is isolated. What relation exists between the symmetrical components of the line and phase voltages? Find the power expanded in three 20 ohm resistors from the symmetrical components of currents and voltages.

10.8. Draw the positive, negative and zero sequence impedance networks for the power system of Fig. P-10.8.

Choose a base of 50 MVA, 220 kV in the 50 Ω transmission lines, and mark all reactances in pu. The ratings of the generators and transformers are:

Generator 1: 25 MVA, 11 kV, $X'' = 20\%$

Generator 2: 25 MVA, 11 kV, $X'' = 20\%$

Three-phase transformer (each): 20 MVA, 11 Y/220 Y kV, $X = 15\%$

The negative sequence reactance of each synchronous machine is equal to its subtransient reactance. The zero sequence reactance of each machine is 8%. Assume that the zero sequence reactances of lines are 250% of their positive sequence reactances.

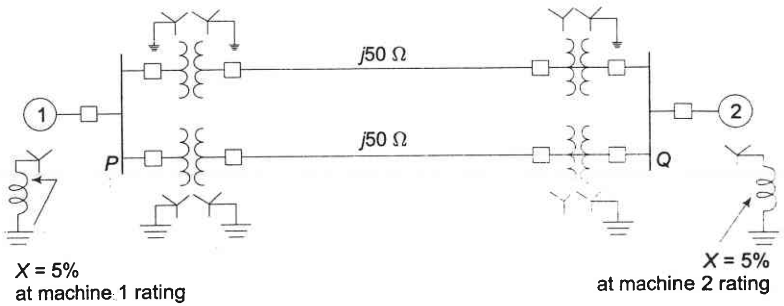


Fig. P-10.8

10.9 For the power system of Fig. P-10.9 draw the positive, negative and zero sequence networks. The generators and transformers are rated as follows:

Generator 1: 25 MVA, 11 kV, $X'' = 0.2$, $X_2 = 0.15$, $X_0 = 0.03$ pu

Generator 2: 15 MVA, 11 kV, $X'' = 0.2$, $X_2 = 0.15$, $X_0 = 0.05$ pu

Synchronous Motor 3: 25 MVA, 11 kV, $X'' = 0.2$, $X_2 = 0.2$, $X_0 = 0.1$ pu

Transformer 1: 25 MVA, 11 Δ/120 Y kV, $X = 10\%$

2: 12.5 MVA, 11 Δ/120 Y kV, $X = 10\%$

3: 10 MVA, 120 *Y*/11 Y kV, $X = 10\%$

Choose a base of 50 MVA, 11 kV in the circuit of generator 1.

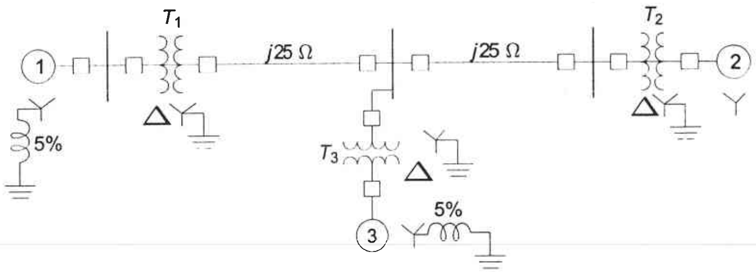


Fig. P-10.9

Note: Zero sequence reactance of each line is 250% of its positive sequence reactance.

10.10 Consider the circuit shown in Fig. P-10.10. Suppose

$$V_{an} = 100\ \angle 0 \qquad X_s = 12\ \Omega$$
$$V_{bn} = 60\ \angle 60^\circ \qquad X_{ab} = X_{bc} = X_{ca} = 5\ \Omega$$
$$V_{cn} = 60\ \angle 120^\circ$$

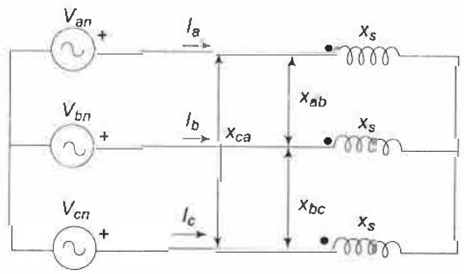


Fig. P-10.10

(a) Calculate I_a, I_b, and I_c without using symmetrical component.
(b) Calculate I_a, I_b, and I_c using symmetrical component.

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