9

9.1 INTRODUCTION

So far we have dealt with the steady state behaviour of power system under normal operating conditions and its dynamic behaviour under small scale perturbations. This chapter is devoted to abnormal system behaviour under conditions of symmetrical short circuit (symmetrical three-phase fault*). Such conditions are caused in the system accidentally through insulation failure of equipment or flashover of lines initiated by a lightning stroke or through accidental faulty operation. The system must be protected against flow of heavy short circuit currents (which can cause permanent damage to major equipment) by disconnecting the faulty part of the system by means of circuit breakers operated by protective relaying. For proper choice of circuit breakers and protective relaying, we must estimate the magnitude of currents that would flow under short circuit conditions—this is the scope of fault analysis (study).

The majority of system faults are not three-phase faults but faults involving one line to ground or occasionally two lines to ground. These are unsymmetrical faults requiring special tools like symmetrical components and form the subject of study of the next two chapters. Though the symmetrical faults are rare, the symmetrical fault analysis must be carried out, as this type of fault generally leads to most severe fault current flow against which the system must be protected. Symmetrical fault analysis is, of course, simpler to carry out.

A power network comprises synchronous generators, transformers, lines and loads. Though the operating conditions at the time of fault are important, the loads can be neglected during fault, as voltages dip very low so that currents drawn by loads can be neglected in comparison to fault currents.

*Symmetrical fault may be a solid three-phase short circuit or may involve are impedance.

The synchronous generator during short circuit has a characteristic time-varying behaviour. In the event of a short circuit, the flux per pole undergoes dynamic change with associated transients in damper and field windings. The reactance of the circuit model of the machine changes in the first few cycles from a low subtransient reactance to a higher transient value, finally settling at a still higher synchronous (steady state) value. Depending upon the arc interruption time of circuit breakers, a suitable reactance value is used for the circuit model of synchronous generators for short circuit analysis.

In a modern large interconnected power system, heavy currents flowing during a fault must be interrupted much before the steady state conditions are established. Furthermore, from the considerations of mechanical forces that act on circuit breaker components, the maximum current that a breaker has to carry momentarily must also be determined. For selecting a circuit breaker we must, therefore, determine the initial current that flows on occurrence of a short circuit and also the current in the transient that flows at the time of circuit interruption.

9.2 TRANSIENT ON A TRANSMISSION LINE

Let us consider the short circuit transient on a transmission line. Certain simplifying assumptions are made at this stage.

(i) The line is fed from a constant voltage source (the case when the line is fed from a realistic synchronous machine will be treated in Sec. 9.3).

(ii) Short circuit takes place when the line is unloaded (the case of short circuit on a loaded line will be treated later in this chapter).

(iii) Line capacitance is negligible and the line can be represented by a lumped *RL* series circuit.

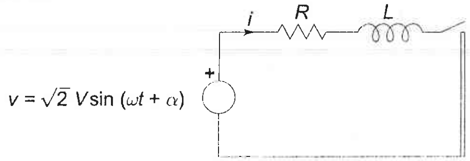


Fig. 9.1

With the above assumptions the line can be represented by the circuit model of Fig. 9.1. The short circuit is assumed to take place at $t = 0$. The parameter α controls the instant on the voltage wave when short circuit occurs. It is known from circuit theory that the current after short circuit is composed of two parts, i.e.

$$i = i_s + i_t$$

where

i_s = steady state current

$$= \frac{\sqrt{2}V}{|Z|}\sin(\omega t + \alpha - \theta)$$

$$Z = (R_2 + \omega^2L^2)^{1/2}\angle\left(\theta = \tan^{-1}\frac{\omega L}{R}\right)$$

i_t = transient current [it is such that $i(0) = i_s(0) + i_t(0) = 0$ being an inductive circuit; it decays corresponding to the time constant L/R].

$$= -\,i_s(0)e^{-(R/L)t}$$

$$= \frac{\sqrt{2}V}{|Z|}\sin(\theta - \alpha)e^{-(R/L)t}$$

Thus short circuit current is given by

$$i = \underbrace{\frac{\sqrt{2}V}{|Z|}\sin(\omega t + \alpha - \theta)}_{\text{Symmetrical short circuit current}} + \underbrace{\frac{\sqrt{2}V}{|Z|}\sin(\theta - \alpha)e^{-(R/L)t}}_{\text{DC off-set current}} \tag{9.1}$$

A plot of i_s, i_t and $i = i_s + i_t$ is shown in Fig. 9.2. In power system terminology, the sinusoidal steady state current is called the *symmetrical short circuit current* and the unidirectional transient component is called the *DC off-set current*, which causes the total short circuit current to be unsymmetrical till the transient decays.

It easily follows from Fig. 9.2 that the *maximum momentary short circuit current* i_{mm} corresponds to the first peak. If the decay of transient current in this short time is neglected,

$$i_{mm} = \frac{\sqrt{2}V}{|Z|}\sin(\theta - \alpha) + \frac{\sqrt{2}V}{|Z|} \tag{9.2}$$

Since transmission line resistance is small, $\theta \simeq 90°$.

$$\therefore \quad i_{mm} = \frac{\sqrt{2}V}{|Z|}\cos\alpha + \frac{\sqrt{2}V}{|Z|} \tag{9.3}$$

This has the maximum possible value for $\alpha = 0$, i.e. short circuit occurring when the voltage wave is going through zero. Thus

$$i_{mm\ (\text{mass possible})} = 2\frac{\sqrt{2}V}{|Z|} \tag{9.4}$$

= twice the maximum of symmetrical short circuit current

(*doubling effect*)

For the selection of circuit breakers, momentary short circuit current is taken corresponding to its maximum possible value (a safe choice).

The next question is 'what is the current to be interrupted?' As has been pointed out earlier, modern day circuit breakers are designed to interrupt the current in the first few cycles (five cycles or less). With reference to Fig. 9.2 it means that when the current is interrupted, the DC off-set (i_t) has not yet died out and so contributes to the current to be interrupted. Rather than computing the value of the DC off-set at the time of interruption (this would be highly complex in a network of even moderately large size), the symmetrical short circuit current alone is calculated. This figure is then increased by an empirical multiplying factor to account for the DC off-set current. Details are given in Sec. 9.5.

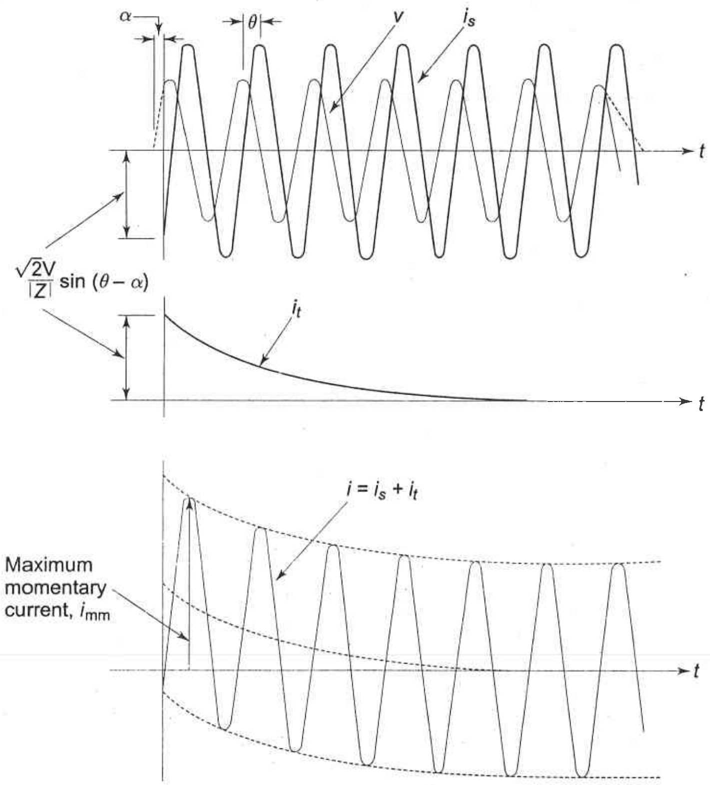
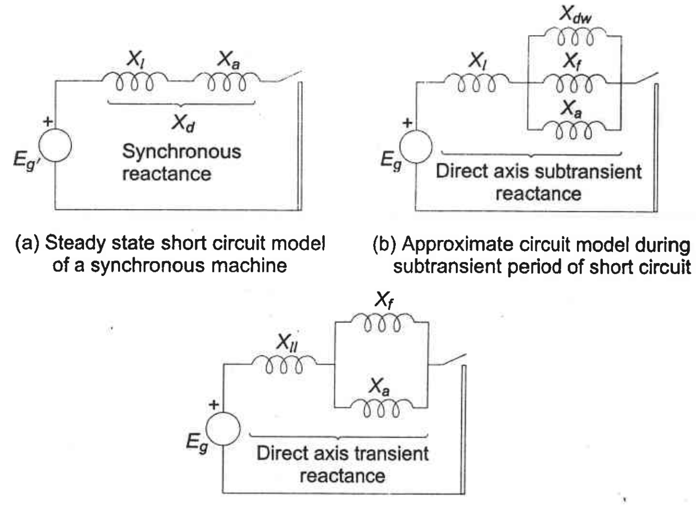


Fig. 9.2 Waveform of a short circuit current on a transmission line

9.3 SHORT CIRCUIT OF A SYNCHRONOUS MACHINE (ON NO LOAD)

Under steady state short circuit conditions, the armature reaction of a synchronous generator produces a demagnetizing flux. In terms of a circuit this

effect is modelled as a reactance X_a in series with the induced emf. This reactance when combined with the leakage reactance X_l of the machine is called *synchronous reactance* X_d (direct axis synchronous reactance in the case of salient pole machines). Armature resistance being small can be neglected. The steady state short circuit model of a synchronous machine is shown in Fig. 9.3a on per phase basis.



(a) Steady state short circuit model of a synchronous machine

(b) Approximate circuit model during subtransient period of short circuit

(c) Approximate circuit model during transient period of short circuit

Fig. 9.3

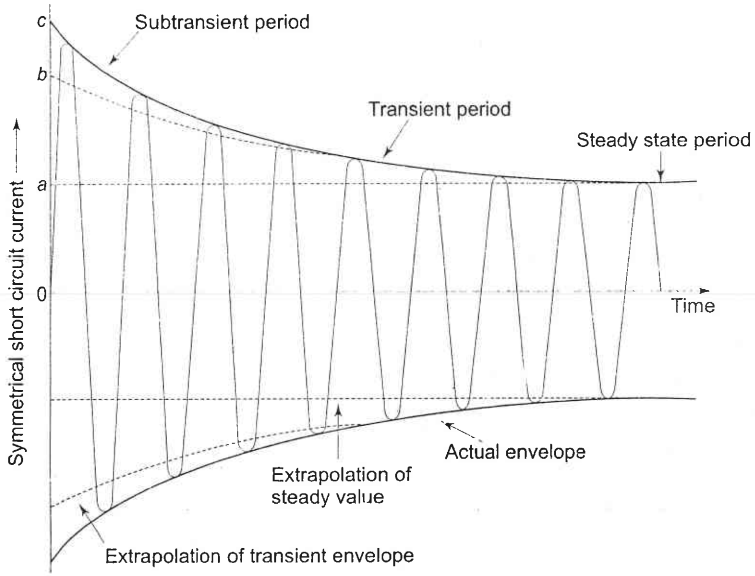
Consider now the sudden short circuit (three-phase) of a synchronous generator initially operating under open circuit conditions. The machine undergoes a transient in all the three phase finally ending up in steady state conditions described above. The circuit breaker must, of course, interrupt the current much before steady conditions are reached. Immediately upon short circuit, the DC off-set currents appear in all the three phases, each with a different magnitude since the point on the voltage wave at which short circuit occurs is different for each phase. These DC off-set currents are accounted for separately on an empirical basis and, therefore, for short circuit studies, we need to concentrate our attention on *symmetrical (sinusoidal) short circuit current only*. Immediately in the event of a short circuit, the symmetrical short circuit current is limited only by the leakage reactance of the machine. Since the air gap flux cannot change instantaneously (*theorem of constant flux linkages*), to counter the demagnetization of the armature short circuit current, currents appear in the field winding as well as in the damper winding in a direction to help the main flux. These currents decay in accordance with the winding time constants. The time constant of the damper winding which has low leakage inductance is much less than that of the field winding, which has high leakage inductance. Thus during the initial part of the short circuit, the damper and field windings have transformer currents induced in them so that in the circuit model their reactances—X_f of field winding and X_{dw} of damper winding—appear in parallel* with X_a as shown in Fig. 9.3b. As the damper winding currents are first to die out, X_{dw} effectively becomes open circuited and at a later stage X_f becomes open circuited. The machine reactance thus changes from the parallel combination of X_a, X_f and X_{dw} during the initial period of the short circuit to X_a and X_f in parallel (Fig. 9.3c) in the middle period of the short circuit, and finally to X_a in steady state (Fig. 9.3a). The reactance presented by the machine in the initial period of the short circuit, i.e.

$$X_l + \frac{1}{(1/X_a + 1/X_f + 1/X_{dw})} = X''_d \tag{9.5}$$

is called the *subtransient reactance* of the machine. While the reactance effective after the damper winding currents have died out, i.e.

$$X'_d = X_l + (X_a \parallel X_f) \tag{9.6}$$

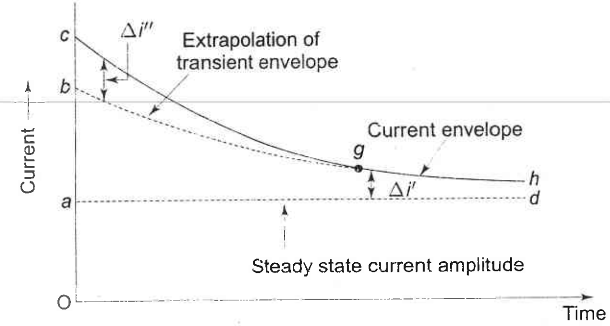
is called the transient reactance of the machine. Of course, the reactance under steady conditions is the synchronous reactance of the machine. Obviously $X''_d < X'_d < X_d$. The machine thus offers a time-varying reactance which changes from X''_d to X'_d and finally to X_d.



(a) Symmetrical short circuit armature current in synchronous machine

Fig. 9.4 (Contd.)

*Unity turn ratio is assumed here.



(b) Envelope of synchronous machine symmetrical short circuit current

Fig. 9.4

If we examine the oscillogram of the short circuit current of a synchronous machine after the DC off-set currents have been removed from it, we will find the current wave shape as given in Fig. 9.4a. The envelope of the current wave shape is plotted in Fig. 9.4b. The short circuit current can be divided into three periods—initial subtransient period when the current is large as the machine offers subtransient reactance, the middle transient period where the machine offers transient reactance, and finally the steady state period when the machine offers synchronous reactance.

If the transient envelope is extrapolated backwards in time, the difference between the transient and subtransient envelopes is the current $\Delta i''$ (corresponding to the damper winding current) which decays fast according to the damper winding time constant. Similarly, the difference $\Delta i'$ between the steady state and transient envelopes decays in accordance with the field time constant.

In terms of the oscillogram, the currents and reactances discussed above, we can write

$$|I| = \frac{oa}{\sqrt{2}} = \frac{|E_g|}{X_d} \tag{9.7a}$$

$$|I'| = \frac{ob}{\sqrt{2}} = \frac{|E_g|}{X'_d} \tag{9.7b}$$

$$|I''| = \frac{oc}{\sqrt{2}} = \frac{|E_g|}{X''_d} \tag{9.7c}$$

where

$|I|$ = steady state current (rms)

$|I'|$ = transient current (rms) excluding DC component

$|I''|$ = subtransient current (rms) excluding DC component

X_d = direct axis synchronous reactance

X'_d = direct axis transient reactance

X''_d = direct axis subtransient reactance

$|E_g|$ = per phase no load voltage (rms)

Oa, Ob, Oc = intercepts shown in Figs. 9.4a and b.

The intercept Ob for finding transient reactance can be determined accurately by means of a logarithmic plot. Both $\Delta i''$ and $\Delta i'$ decay exponentially as

$$\Delta i'' = \Delta i''_0 \exp(-t/\tau_{dw})$$

$$\Delta i' = \Delta i'_0 \exp(-t/\tau_f)$$

where τ_{dw} and τ_f are respectively damper, and field winding time constants with $\tau_{dw} \ll \tau_f$. At time $t \gg \tau_{dw}$, $\Delta i''$ practically dies out and we can write

$$\log(\Delta i'' + \Delta i')|_{t \gg \tau_{dw}}, \simeq \log \Delta i' = -\Delta i'_0 t/\tau_f$$

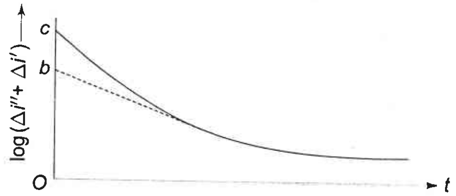


Fig. 9.5

The plot of log $(\Delta i'' + \Delta i')$ versus time for $t \gg \tau_{dw}$ therefore, becomes a straight line with a slope of $(-\Delta i'_0/\tau_f)$ as shown in Fig. 9.5. As the straight line portion of the plot is extrapolated (straight line extrapolation is much more accurate than, the exponential extrapolation of Fig. 9.4), the intercept corresponding to $t = 0$ is

$$\Delta i'|_{t=0} = \Delta i'_0 \exp(-t/\tau_f)|_{t=0} = \Delta i'_0 = ob$$

Table 9.1 Typical values of synchronous machine reactances (All values expressed in pu of rated MVA)

Type of machine	*Turbo-alternator (Turbine generator)*	*Salient pole (Hydroelectric)*	*Synchronous compensator (Condenser/ capacitor)*	*Synchronous motors**
X_s (or X_d)	1.00–2.0	0.6–1.5	1.5–12.5	0.8–1.10
X_q	0.9–1.5	0.4–1.0	0.95–1.5	0.65–0.8
X'_d	0.12–0.35	0.2–0.5	0.3–0.6	0.3–0.35
X''_d	0.1–0.25	0.13–0.35	0.18–0.38	0.18–0.2
X_2	$= X''_d$	$= X''_d$	0.17–0.37	0.19–0.35
X_0	0.04–0.14	0.02–0.2	0.025–0.16	0.05–0.07
r_a	0.003–0.008	0.003–0.015	0.004–0.01	0.003–0.012

r_a = AC resistance of the armature winding per phase.

* High-speed units tend to have low reactance and low speed units high reactance.

Though the machine reactances are dependent upon magnetic saturation (corresponding to excitation), the values of reactances normally lie within certain predictable limits for different types of machines. Table 9.1 gives typical values of machine reactances which can be used in fault calculations and in stability studies.

Normally both generator and motor subtransient reactances are used to determine the momentary current flowing on occurrence of a short circuit. To decide the interrupting capacity of circuit breakers, except those which open instantaneously, subtransient reactance is used for generators and transient reactance for synchronous motors. As we shall see later the transient reactances are used for stability studies.

The machine model to be employed when the short circuit takes place from loaded conditions will be explained in Sec. 9.4.

The method of computing short circuit currents is illustrated through examples given below.

Example 9.1

For the radial network shown in Fig. 9.6, a three-phase fault occurs at F. Determine the fault current and the line voltage at 11 kV bus under fault conditions.

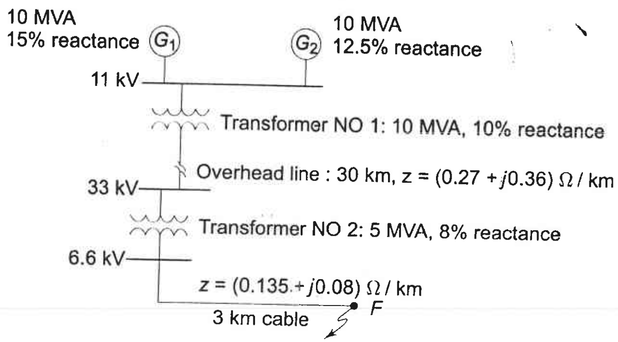


Fig. 9.6 Radial network for Example 9.1

Solution Select a system base of 100 MVA.

Voltage bases are: 11 kV in generators, 33 kV for overhead line and 6.6 kV for cable.

$$\text{Reactance of } G_1 = j\frac{0.15 \times 100}{10} = j1.5 \text{ pu}$$

$$\text{Reactance of } G_2 = j\frac{0.125 \times 100}{10} = j1.25 \text{ pu}$$

$$\text{Reactance of } T_1 = j\frac{0.1 \times 100}{10} = j1.0 \text{ pu}$$

$$\text{Reactance of } T_2 = j\frac{0.08 \times 100}{5} = j1.6 \text{ pu}$$

$$\text{Overhead line impedance} = \frac{Z\,(\text{in ohms}) \times \text{MVA}_{\text{Base}}}{(\text{kV}_{\text{Base}})^2}$$

$$= \frac{30 \times (0.27 + j0.36) \times 100}{(33)^2}$$

$$= (0.744 + j0.99) \text{ pu}$$

$$\text{Cable impedance} = \frac{3(0.135 + j0.08) \times 100}{(6.6)^2} = (0.93 + j0.55) \text{ pu}$$

Circuit model of the system for fault calculations is shown in Fig. 9.7. Since the system is on no load prior to occurrence of the fault, the voltages of the two generators are identical (in phase and magnitude) and are equal to 1 pu. The generator circuit can thus be replaced by a single voltage source in series with the parallel combination of generator reactances as shown.

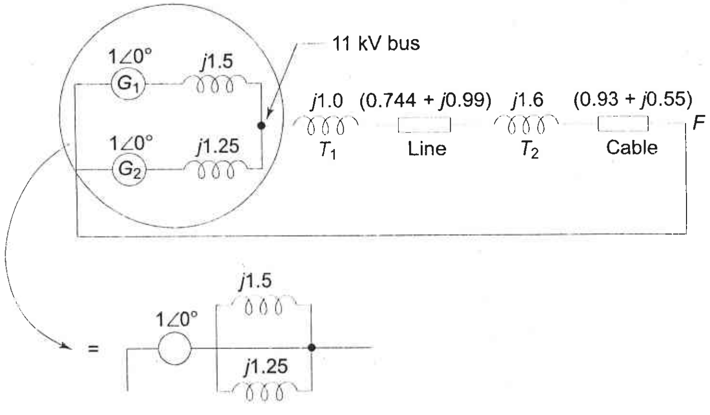


Fig. 9.7

$$\text{Total impedance} = (j1.5 \parallel j1.25) + (j1.0) + (0.744 + j0.99) + (j1.6) + (0.93 + j0.55)$$

$$= 1.674 + j4.82 = 5.1 \angle 70.8° \text{ pu}$$

$$I_{SC} = \frac{1\angle 0}{5.1\angle 70.8°} = 0.196 \angle -70.8° \text{ pu}$$

$$I_{\text{Base}} = \frac{100 \times 10^3}{\sqrt{3} \times 6.6} = 8{,}750 \text{ A}$$

$$\therefore \quad I_{SC} = 0.196 \times 8{,}750 = 1{,}715 \text{ A}$$

Total impedance between F and 11 kV bus

$$= (0.93 + j055) + (j1.6) + (0.744 + j0.99) + (j1.0)$$

$$= 1.674 + j4.14 = 4.43 \angle 76.8° \text{ pu}$$

$$\text{Voltage at 11 kV bus} = 4.43 \angle 67.8° \times 0.196 \angle -70.8°$$

$$= 0.88 \angle -3° \text{ pu} = 0.88 \times 11 = 9.68 \text{ kV}$$

Example 9.2

A 25 MVA, 11 kV generator with X''_d = 20% is connected through a transformer, line and a transformer to a bus that supplies three identical motors as shown in Fig. 9.8. Each motor has X''_d = 25% and X'_d = 30% on a base of 5 MVA, 6.6 kV. The three-phase rating of the step-up transformer is 25 MVA, 11/66 kV with a leakage reactance of 10% and that of the step-down transformer is 25 MVA, 66/6.6 kV with a leakage reactance of 10%. The bus voltage at the motors is 6.6 kV when a three-phase fault occurs at the point F. For the specified fault, calculate

(a) the subtransient current in the fault,

(b) the subtransient current in the breaker B,

(c) the momentary current in breaker B, and

(d) the current to be interrupted by breaker B in five cycles.

Given: Reactance of the transmission line = 15% on a base of 25 MVA, 66 kV. Assume that the system is operating on no load when the faul: occurs.

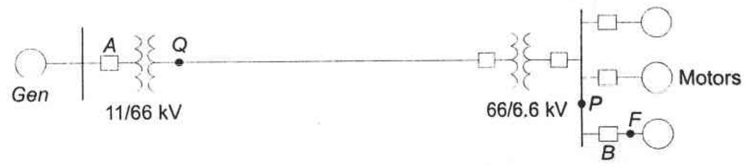


Fig. 9.8

Solution Choose a system base of 25 MVA.

For a generator voltage base of 11 kV, line voltage base is 66 kV and motor voltage base is 6.6 kV.

(a) For each motor

$$X''_{dm} = j0.25 \times \frac{25}{5} = j1.25 \text{ pu}$$

Line, transformers and generator reactances are already given on proper base values.

The circuit model of the system for fault calculations is given in Fig. 9.9a. The system being initially on no load, the generator and motor induced emfs are identical. The circuit can therefore be reduced to that of Fig. 9.9b and then to Fig. 9.9c. Now

$$I_{SC} = 3 \times \frac{1}{j1.25} + \frac{1}{j0.55} = -\,j4.22 \text{ pu}$$

$$\text{Base current in 6.6 kV circuit} = \frac{25 \times 1{,}000}{\sqrt{3} \times 6.6} = 2{,}187 \text{ A}$$

$$\therefore \qquad I_{SC} = 4.22 \times 2{,}187 = 9{,}229 \text{ A}$$

(b) From Fig. 9.9c, current through circuit breaker B is

$$I_{SC}(\text{B}) = 2 \times \frac{1}{j1.25} + \frac{1}{j0.55} = -j3.42$$

$$= 3.42 \times 2{,}187 = 7{,}479.5 \text{ A}$$

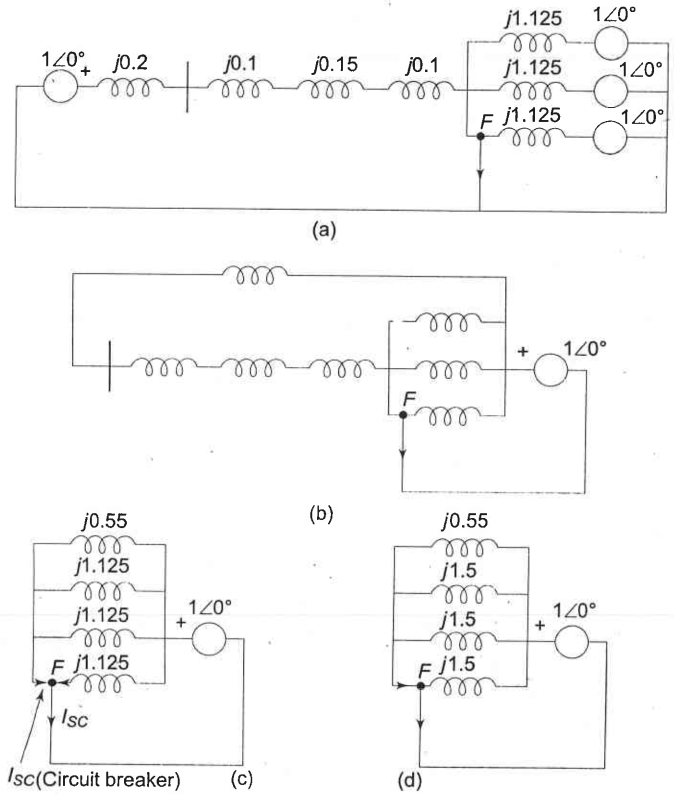


Fig. 9.9

(c) For finding momentary current through the breaker, we must add the DC off-set current to the symmetrical subtransient current obtained in part (b). Rather than calculating the DC off-set current, allowance is made for it on an empirical basis. As explained in Sec. 9.5,

momentary current through breaker $B = 1.6 \times 7{,}479.5$

$$= 11{,}967 \text{ A}$$

(d) To compute the current to be interrupted by the breaker, motor subtransient reactance ($X_d'' = j0.25$) is now replaced by transient reactance ($X'_d = j0.30$).

$$X_d' \text{ (motor)} = j0.3 \times \frac{25}{5} = j1.5 \text{ pu}$$

The reactances of the circuit of Fig. 9.9c now modify to that of Fig. **9.9d.** Current (symmetrical) to be interrupted by the breaker (as shown by arrow)

$$= 2 \times \frac{1}{j1.5} + \frac{1}{j0.55} = 3.1515 \text{ pu}$$

Allowance is made for the DC off-set value by multiplying with a factor of 1.1 (Sec. 9.5). Therefore, the current to be interrupted is

$$1.1 \times 3.1515 \times 2{,}187 = 7{,}581 \text{ A}$$

9.4 SHORT CIRCUIT OF A LOADED SYNCHRONOUS MACHINE

In the previous article on the short circuit of a synchronous machine, it was assumed that the machine was operating at no load prior to the occurrence of short circuit. The analysis of short circuit on a loaded synchronous machine is complicated and is beyond the scope of this book. We shall, however, present here the methods of computing short circuit current when short circuit occurs under loaded conditions.

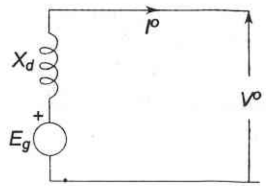


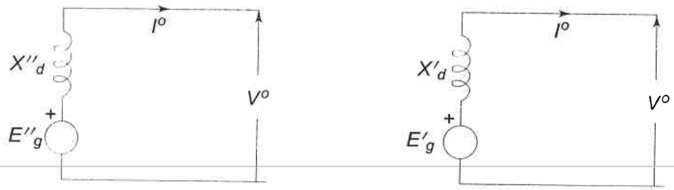
Fig. 9.10 Circuit model of a loaded machine

Figure 9.10 shows the circuit model of a synchronous generator operating under steady conditions supplying a load current I^o to the bus at a terminal voltage of V^o. E_g is the induced emf under loaded condition and X_d is the direct axis synchronous reactance of the machine. When short circuit occurs at the terminals of this machine, the circuit model to be used for computing short circuit current is given in Fig. 9.11a for subtransient current, and in Fig. 9.11b for transient current. The induced emfs to be used in these models are given by

$$E_g'' = V^o + jI^oX_d'' \tag{9.8}$$

$$E_g' = V^o + jI^oX'_d \tag{9.9}$$

The voltage E_g'' is known as the *voltage behind the subtransient reactance* and the voltage E_g' is known as the *voltage behind the transient reactance*. In fact, if I^o is zero (no load case), $E_g'' = E_g' = E_g$, the no load voltage, in which case the circuit model reduces to that discussed in Sec. 9.3.



(a) Circuit model for computing subtransient current

(b) Circuit model for computing transient current

Fig. 9.11

Synchronous motors have internal emfs and reactances similar to that of a generator except that the current direction is reversed. During short circuit conditions these can be replaced by similar circuit models except that the voltage behind subtransient/transient reactance is given by

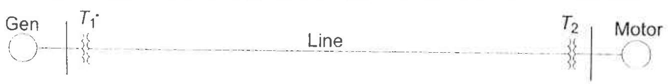
$$E''_m = V^o - jI^o X''_d \tag{9.10}$$

$$E'_m = V^o - jI^o X'_d \tag{9.11}$$

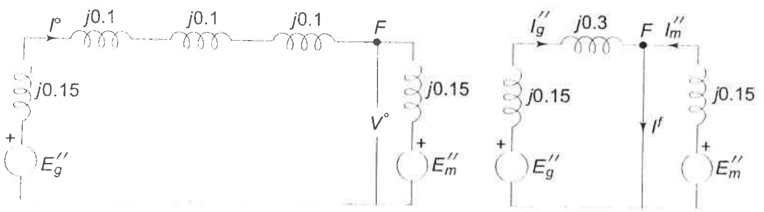
Whenever we are dealing with short circuit of an interconnected system, the synchronous machines (generators and motors) are replaced by their corresponding circuit models having voltage behind subtransient (transient) reactance in series with subtransient (transient) reactance. The rest of the network being passive remains unchanged.

Example 9.3

A synchronous generator and a synchronous motor each rated 25 MVA, 11 kV having 15% subtransient reactance are connected through transformers and a line as shown in Fig. 9.12a. The transformers are rated 25 MVA, 11/66 kV and 66/11 kV with leakage reactance of 10% each. The line has a reactance of 10% on a base of 25 MVA, 66 kV. The motor is drawing 15 MW at 0.8 power factor leading and a terminal voltage of 10.6 kV when a symmetrical three-phase fault occurs at the motor terminals. Find the subtransient current in the generator, motor and fault.



(a) One-line diagram for the system of Example 9.3



(b) Prefault equivalent circuit

(c) Equivalent circuit during fault

Fig. 9.12

Solution All reactances are given on a base of 25 MVA and appropriate voltages.

$$\text{Prefault voltage } V^o = \frac{10.6}{11} = 0.9636 \angle 0^\circ \text{ pu}$$

$$\text{Load} = 15 \text{ MW, } 0.8 \text{ pf leading}$$

$$= \frac{15}{25} = 0.6 \text{ pu, } 0.8 \text{ pf leading}$$

$$\text{Prefault current } I^o = \frac{0.6}{0.9636 \times 0.8} \angle 36.9^\circ = 0.7783 \angle 36.9^\circ \text{ pu}$$

Voltage behind subtransient reactance (generator)

$$E''_g = 0.9636 \angle 0^\circ + j0.45 \times 0.7783 \angle 36.9^\circ$$

$$= 0.7536 + j0.28 \text{ pu}$$

Voltage behind subtransient reactance (motor)

$$E''_m = 0.9636 \angle 0^\circ - j0.15 \times 0.7783 \angle 36.9^\circ$$

$$= 1.0336 - j0.0933 \text{ pu}$$

The prefault equivalent circuit is shown in Fig. 9.12b. Under faulted condition (Fig. 9.12c)

$$I''_g = \frac{0.7536 + j0.2800}{j0.45} = 0.6226 - j1.6746 \text{ pu}$$

$$I''_m = \frac{1.0336 - j0.0933}{j0.15} = -0.6226 - j6.8906 \text{ pu}$$

Current in fault

$$I^f = I''_g + I''_m = -j8.5653 \text{ pu}$$

$$\text{Base current (gen/motor)} = \frac{25 \times 10^3}{\sqrt{3} \times 11} = 1{,}312.2 \text{ A}$$

Now

$$I''_g = 1{,}312.0\ (0.6226 - j1.6746) = (816.4 - j2{,}197.4) \text{ A}$$

$$I''_m = 1{,}312.2\ (-0.6226 - j6.8906) = (-816.2 - j9{,}041.8) \text{ A}$$

$$I^f = -j11{,}239 \text{ A}$$

Short Circuit (SC) Current Computation through the Thevenin Theorem

An alternate method of computing short circuit currents is through the application of the Thevenin theorem. This method is faster and easily adopted to systematic computation for large networks. While the method is perfectly general, it is illustrated here through a simple example.

Consider a synchronous generator feeding a synchronous motor over a line. Figure 9.13a shows the circuit model of the system under conditions of steady load. Fault computations are to be made for a fault at F, at the motor terminals. As a first step the circuit model is replaced by the one shown in Fig. 9.13b, wherein the synchronous machines are represented by their transient reactances (or subtransient reactances if subtransient currents are of interest) in series with voltages behind transient reactances. This change does not disturb the prefault current I^o and prefault voltage V^o (at F).

As seen from FG the Thevenin equivalent circuit of Fig. 9.13b is drawn in Fig. 9.13c. It comprises prefault voltage V^o in series with the passive Thevenin impedance network. It is noticed that the prefault current I^o does not appear in the passive Thevenin impedance network. It is therefore to be remembered that this current must be accounted for by superposition after the SC solution is obtained through use of the Thevenin equivalent.

Consider now a fault at F through an impedance Z^f. Figure 9.13d shows the Thevenin equivalent of the system feeding the fault impedance. We can immediately write

$$I^f = \frac{V^o}{jX_{\text{Th}} + Z^f} \tag{9.12}$$

Current caused by fault in generator circuit

$$\Delta I_g = \frac{X'_{dm}}{(X'_{dg} + X + X'_{dm})} I^f \tag{9.13}$$

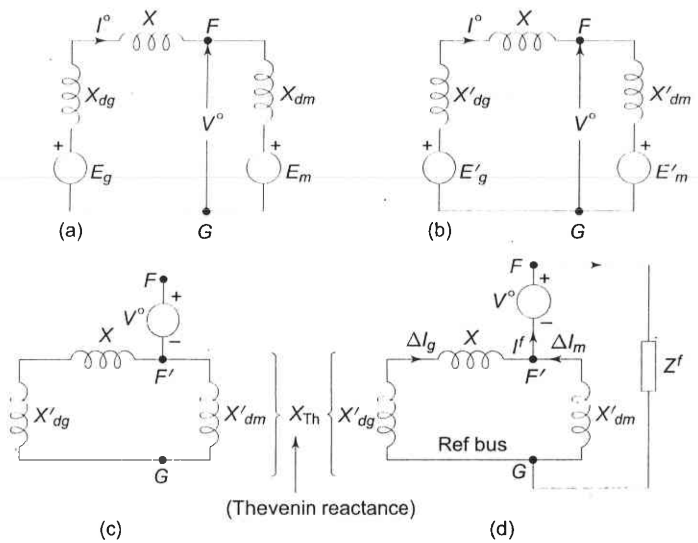


Fig. 9.13 Computation of SC current by the Thevenin equivalent

Current caused by fault in motor circuit

$$\Delta I_m = \frac{X'_{dg} + X}{(X'_{dm} + X + X'_{dg})} I^f \tag{9.14}$$

Postfault currents and voltages are obtained as follows by superposition:

$$I_g^f = I^o + \Delta I_g$$

$$I_m^f = -\, I^o + \Delta I_m \text{ (in the direction of } \Delta I_m) \tag{9.15}$$

Postfault voltage

$$V^f = V^o + (-\, jX_{\text{Th}} I^f) = V^o + \Delta V \tag{9.16}$$

where $\Delta V = -jX_{\text{Th}} I^f$ is the voltage of the fault point F' on the Thevenin passive network (with respect to the reference bus G) caused by the flow of fault current I^f.

An observation can be made here. Since the prefault current flowing out of fault point F is always zero, the postfault current out of F is independent of load for a given prefault voltage at F.

The above approach to SC computation is summarized in the following four steps:

Step 1: Obtain steady state solution of loaded system (load flow study).

Step 2: Replace reactances of synchronous machines by their subtransient/transient values. Short circuit all emf sources. The result is the passive Thevenin network.

Step 3: Excite the passive network of Step 2 at the fault point by negative of prefault voltage (see Fig. 9.13d) in series with the fault impedance. Compute voltages and currents at all points of interest.

Step 4: Postfault currents and voltages are obtained by adding results of Steps 1 and 3.

The following assumptions can be safely made in SC computations leading to considerable computational simplification:

Assumption 1: All prefault voltage magnitudes are 1 pu.

Assumption 2: All prefault currents are zero.

The first assumption is quite close to actual conditions as under normal operation all voltages (pu) are nearly unity.

The changes in current caused by short circuit are quite large, of the order of 10–20 pu and are purely reactive; whereas the prefault load currents are almost purely real. Hence the total postfault current which is the result of the two currents can be taken in magnitude equal to the larger component (caused by the fault). This justifies assumption 2.

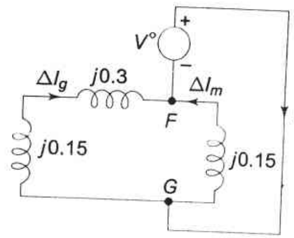


Fig. 9.14 F is the fault point on the passive Thevenin network

Let us illustrate the above method by recalculating the results of Example 9.3.

The circuit model for the system of Example 9.3 for computation of postfault condition is shown in Fig. 9.14.

$$I^f = \frac{V^o}{(j0.15 \| j0.45)} = \frac{0.9636 \times j0.60}{j0.15 \times j0.45} = -j8.565 \text{ pu}$$

Change in generator current due to fault,

$$\Delta I_g = -j8.565 \times \frac{j0.15}{j0.60} = -j2.141 \text{ pu}$$

Change in motor current due to fault,

$$\Delta I_m = -j8.565 \times \frac{j0.45}{j0.60} = -j6.424 \text{ pu}$$

To these changes we add the prefault current to obtain the subtransient current in machines. Thus

$$I''_g = I^o + \Delta I_g = (0.623 - j1.674) \text{ pu}$$

$$I''_m = -I^o + \Delta I_m = (-0.623 - j6.891) \text{ pu}$$

which are the same (and should be) as calculated already.

We have thus solved Example 9.3 alternatively through the Thevenin theorem and superposition. This, indeed, is a powerful method for large networks.

9.5 SELECTION OF CIRCUIT BREAKERS

Two of the circuit breaker ratings which require the computation of SC current are: *rated momentary current* and *rated symmetrical interrupting current*. Symmetrical SC current is obtained by using subtransient reactances for synchronous machines. Momentary current (rms) is then calculated by multiplying the symmetrical momentary current by a factor of 1.6 to account for the presence of DC off-set current.

Symmetrical current to be interrupted is computed by using subtransient reactances for synchronous generators and transient reactances for synchronous motors—induction motors are neglected*. The DC off-set value to be added to obtain the current to be interrupted is accounted for by multiplying the symmetrical SC current by a factor as tabulated below:

Circuit Breaker Speed	*Multiplying Factor*
8 cycles or slower	1.0
5 cycles	1.1
3 cycles	1.2
2 cycles	1.4

*In some recent attempts, currents contributed by induction motors during a short circuit have been accounted for.

If SC MVA (explained below) is more than 500, the above multiplying factors are increased by 0.1 each. The multiplying factor for air breakers rated 600 V or lower is 1.25.

The current that a circuit breaker can interrupt is inversely proportional to the operating voltage over a certain range, i.e.

Amperes at operating voltage

= amperes at rated voltage × rated voltage/operating voltage

Of course, operating voltage cannot exceed the maximum design value. Also, no matter how low the voltage is, the rated interrupting current cannot exceed the *rated maximum interrupting current*. Over this range of voltages, the product of operating voltage and interrupting current is constant. It is therefore logical as well as convenient to express the circuit breaker rating in terms of SC MVA that can be interrupted, defined as

Rated interrupting MVA (three-phase) capacity

$$= \sqrt{3}\, |V(\text{line})|_{\text{rated}} \times |I(\text{line})|_{\text{rated interrupting current}}$$

where V(line) is in kV and I (line) is kA.

Thus, instead of computing the SC current to be interrupted, we compute three-phase SC MVA to be interrupted, where

$$\text{SC MVA (3-phase)} = \sqrt{3} \times \text{prefault line voltage in kV} \times \text{SC current in kA.}$$

If voltage and current are in per unit values on a three-phase basis

$$\text{SC MVA (3-phase)} = |V|_{\text{prefault}} \times |I|_{\text{SC}} \times (\text{MVA})_{\text{Base}} \tag{9.17}$$

Obviously, rated MVA interrupting capacity of a circuit breaker is to be more than (or equal to) the SC MVA required to be interrupted.

For the selection of a circuit breaker for a particular location, we must find the maximum possible SC MVA to be interrupted with respect to type and location of fault and generating capacity (also synchronous motor load) connected to the system. A three-phase fault though rare is generally the one which gives the highest SC MVA and a circuit breaker must be capable of interrupting it. An exception is an LG (line-to-ground) fault close to a synchronous generator*. In a simple system the fault location which gives the highest SC MVA may be obvious but in a large system various possible locations must be tried out to obtain the highest SC MVA requiring repeated SC computations. This is illustrated by the examples that follow.

Example 9.4

Three 6.6 kV generators A, B and C, each of 10% leakage reactance and MVA ratings 40, 50 and 25, respectively are interconnected electrically, as shown in

*This will be explained in Chapter 11.

Fig. 9.15, by a tie bar through *current limiting reactors*, each of 12% reactance based upon the rating of the machine to which it is connected. A three-phase feeder is supplied from the bus bar of generator A at a line voltage of 6.6 kV. The feeder has a resistance of 0.06 Ω/phase and an inductive reactance of 0.12 Ω/phase. Estimate the maximum MVA that can be fed into a symmetrical short circuit at the far end of the feeder.

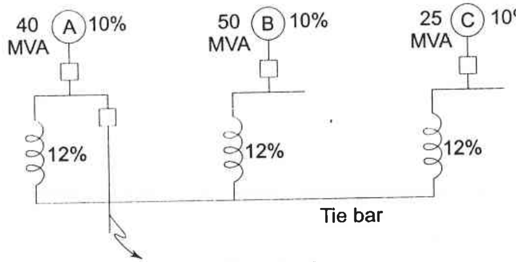


Fig. 9.15

Solution Choose as base 50 MVA, 6.6 kV.

Feeder impedance

$$= \frac{(0.06 + j0.12) \times 50}{(6.6)^2} = (0.069 + j0.138) \text{ pu}$$

$$\text{Gen A reactance} = \frac{0.1 \times 50}{40} = 0.125 \text{ pu}$$

$$\text{Gen B reactance} = 0.1 \text{ pu}$$

$$\text{Gen C reactance} = 0.1 \times \frac{50}{25} = 0.2 \text{ pu}$$

$$\text{Reactor A reactance} = \frac{0.12 \times 50}{40} = 0.15 \text{ pu}$$

$$\text{Reactor B reactance} = 0.12 \text{ pu}$$

$$\text{Reactor C reactance} = \frac{0.12 \times 50}{25} = 0.24 \text{ pu}$$

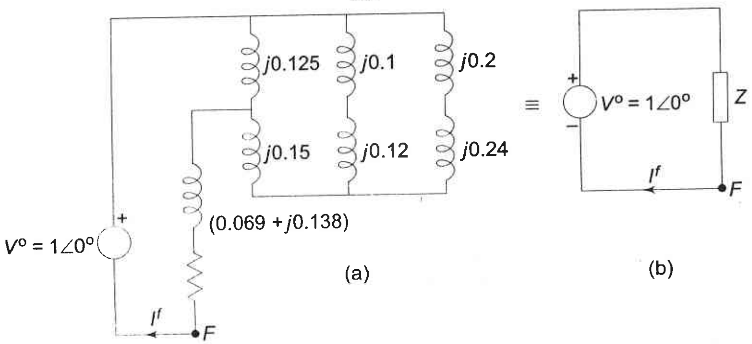


Fig. 9.16

Assume no load prefault conditions, i.e. prefault currents are zero. Postfault currents can then be calculated by the circuit model of Fig. 9.16a corresponding to Fig. 9.13d. The circuit is easily reduced to that of Fig. 9.16b, where

$$Z = (0.069 + j0.138) + j0.125 \parallel (j0.15 + j0.22 \parallel j0.44)$$

$$= 0.069 + j0.226 = 0.236\angle 73^\circ$$

$$\text{SC MVA} = V^o I^f = V^o\left(\frac{V^o}{Z}\right) = \frac{1}{Z} \text{ pu (since } V^o = 1 \text{ pu)}$$

$$= \frac{1}{Z} \times (\text{MVA})_{\text{Base}}$$

$$= \frac{50}{0.236} = 212 \text{ MVA}$$

Example 9.5

Consider the 4-bus system of Fig. 9.17. Buses 1 and 2 are generator buses and 3 and 4 are load buses. The generators are rated 11 kV, 100 MVA, with transient reactance of 10% each. Both the transformers are 11/110 kV, 100 MVA with a leakage reactance of 5%. The reactances of the lines to a base of 100 MVA, 110 kV are indicated on the figure. Obtain the short circuit solution for a three-phase solid fault on bus 4 (load bus).

Assume prefault voltages to be 1 pu and prefault currents to be zero.

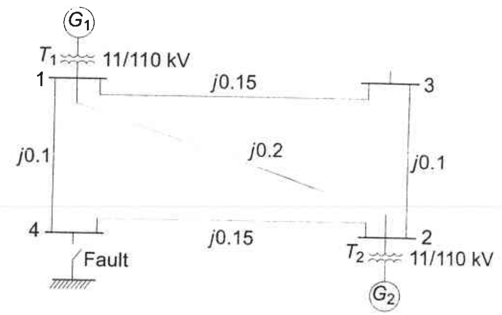


Fig. 9.17 Four-bus system of Example 9.5

Solution Changes in voltages and currents caused by a short circuit can be calculated from the circuit model of Fig. 9.18. Fault current I^f is calculated by systematic network reduction as in Fig. 9.19.

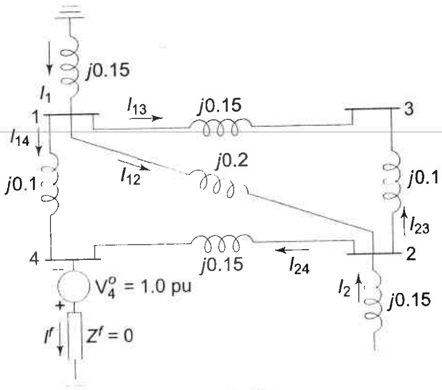


Fig. 9.18

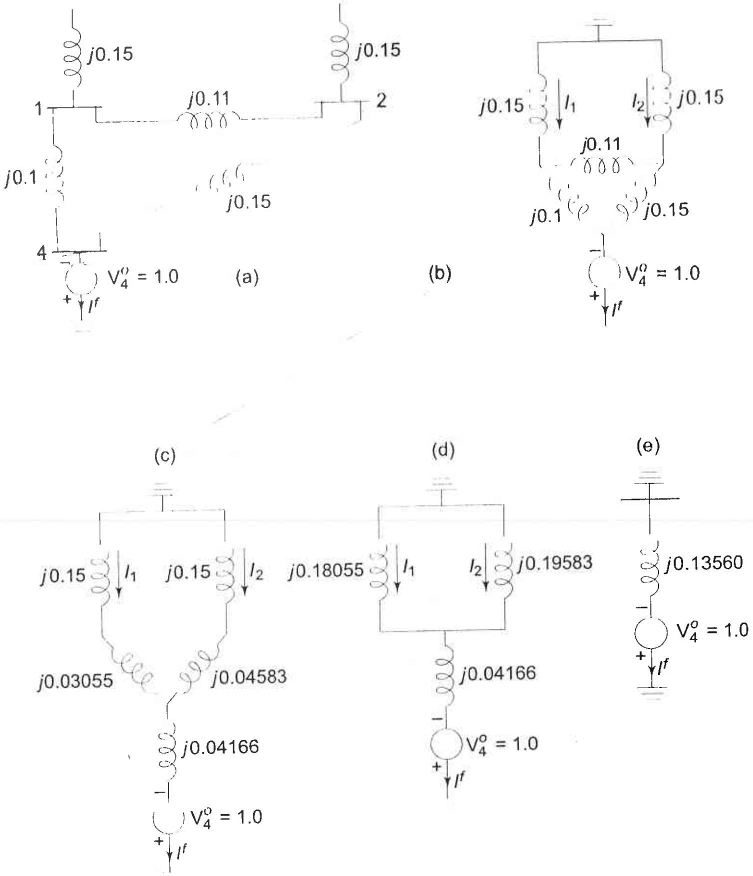


Fig. 9.19 Systematic reduction of the network of Fig. 9.18

From Fig. 9.19e, we get directly the fault current as

$$I^f = \frac{1.0}{j0.13560} = -j7.37463 \text{ pu}$$

From Fig. 9.19d, it is easy to see that

$$I_1 = I_f \times \frac{j0.19583}{j0.37638} = -j3.83701 \text{ pu}$$

$$I_2 = I_f \times \frac{j0.18055}{j0.37638} = -j3.53762 \text{ pu}$$

Let us now compute the voltage changes for buses 1, 2 and 3. From Fig. 9.19b, we give

$$\Delta V_1 = 0 - (j0.15)(-j3.83701) = -0.57555 \text{ pu}$$

$$\Delta V_2 = 0 - (j0.15)(-j3.53762) = -0.53064 \text{ pu}$$

Now

$$V_1^f = 1 + \Delta V_1 = 0.42445 \text{ pu}$$

$$V_2^f = 1 + \Delta V_2 = 0.46936 \text{ pu}$$

$$\therefore \quad I_{13} = \frac{V_1^f - V_2^f}{j0.15 + j0.1} = j0.17964 \text{ pu}$$

Now

$$\Delta V_3 = 0 - [(j0.15)(-j3.83701) + (j0.15)(j0.17964)]$$

$$= -0.54860 \text{ pu}$$

$$\therefore \quad V^f_3 = 1 - 0.54860 = 0.4514 \text{ pu}$$

$$V^f_4 = 0$$

The determination of currents in the remaining lines is left as an exercise to the reader.

Short circuit study is complete with the computation of SC MVA at bus 4.

$$(\text{SC MVA})_4 = 7.37463 \times 100 = 737.463 \text{ MVA}$$

It is obvious that the heuristic network reduction procedure adopted above is not practical for a real power network of even moderate size. It is, therefore, essential to adopt a suitable algorithm for carrying out short circuit study on a digital computer. This is discussed in Sec. 9.6.

9.6 ALGORITHM FOR SHORT CIRCUIT STUDIES

So far we have carried out short circuit calculations for simple systems whose passive networks can be easily reduced. In this section we extend our study to large systems. In order to apply the four steps of short circuit computation developed earlier to large systems, it is necessary to evolve a systematic general algorithm so that a digital computer can be used.

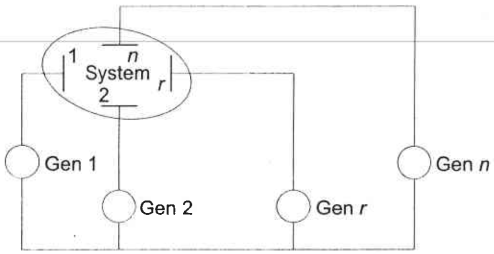


Fig. 9.20 *n*-bus system under steady load

Consider an n-bus system shown schematically in Fig. 9.20 operating at steady load. The first step towards short circuit computation is to obtain prefault voltages at all buses and currents in all lines through a load flow study. Let us indicate the prefault bus voltage vector as

$$V^0_{\text{BUS}} = \begin{bmatrix} V_1^0 \\ V_2^0 \\ \vdots \\ V_n^0 \end{bmatrix} \tag{9.18}$$

Let us assume that the rth bus is faulted through a fault impedance Z^f. The postfault bus voltage vector will be given by

$$V^f_{\text{BUS}} = V^0_{\text{BUS}} + \varDelta V \tag{9.19}$$

where $\varDelta V$ is the vector of changes in bus voltages caused by the fault.

As step 2, we drawn the passive Thevenin network of the system with generators replaced by transient/subtransient reactances with their emfs shorted (Fig. 9.21).

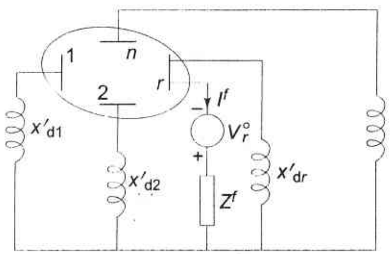


Fig. 9.21 Network of the system of Fig. 9.20 for computing changes in bus voltages caused by the fault

As per step 3 we now excite the passive Thevenin network with $-V_r^o$ in series with Z^f as in Fig. 9.21. The vector $\varDelta \mathbf{V}$ comprises the bus voltages of this network.

Now

$$\varDelta V = Z_{\text{BUS}} J^f \tag{9.20}$$

where

$$Z_{\text{BUS}} = \begin{bmatrix} Z_{11} & \dots & Z_{1n} \\ \vdots & & \vdots \\ Z_{n1} & \dots & Z_{nn} \end{bmatrix} = \text{bus impedance matrix of the passive Thevenin network} \tag{9.21}$$

J^f = bus current injection vector

Since the network is injected with current $-I^f$ only at the rth bus, we have

$$J^f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_r^f = -I^f \\ \vdots \\ 0 \end{bmatrix} \tag{9.22}$$

Substituting Eq. (9.22) in Eq. (9.20), we have for the rth bus

$$\varDelta V_r = -Z_{rr} I^f$$

By step 4, the voltage at the rth bus under fault is

$$V_r^f = V^0_r + \varDelta V^0_r = V^0_r - Z_{rr} I^f \tag{9.23}$$

However, this voltage must equal

$$V_r^f = Z^f I^f \tag{9.24}$$

We have from Eqs. (9.23) and (9.24)

$$Z^f I^f = V_r^0 - Z_{rr} I^f$$

or

$$I^f = \frac{V_r^0}{Z_{rr} + Z^f} \tag{9.25}$$

At the ith bus (from Eqs (9.20) and (9.22))

$$\varDelta V_i = -Z_{ir} I^f$$

$\therefore$

$$V_i^f = V_i^0 - Z_{ir} I^f,\ i = 1, 2, \dots, n \tag{9.26}$$

substituting for I^f from Eq. (9.25), we have

$$V_i^f = V_i^0 - \frac{Z_{ir}}{Z_{rr} + Z^f} V_r^0 \tag{9.27}$$

For $i = r$ in Eq. (9.27)

$$V_r^f = \frac{Z^f}{Z_{rr} + Z^f} V_r^0 \tag{9.28}$$

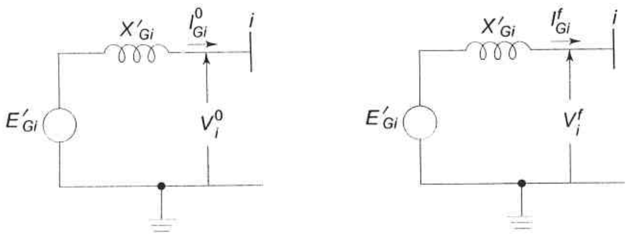
In the above relationship V_i^0's, the prefault bus voltages are assumed to be known from a load flow study. Z_{BUS} matrix of the short-circuit study network of Fig. 9.21 can be obtained by the inversion of its Y_{BUS} matrix as in Example 9.6 or the Z_{BUS} building algorithm presented in Section 9.7. It should be observed here that the SC study network of Fig. 9.21 is different from the corresponding load flow study network by the fact that the shunt branches corresponding to the generator reactances do not appear in the load flow study network. Further, in formulating the SC study network, the load impedances are ignored, these being very much larger than the impedances of lines and generators. Of course synchronous motors must be included in Z_{BUS} formulation for the SC study.

Postfault currents in lines are given by

$$I^f_{ij} = Y_{ij} (V^f_i - V^f_j) \tag{9.29}$$

For calculation of postfault generator current, examine Figs. 9.22(a) and (b). From the load flow study (Fig. 9.22(a))

Prefault generator output = $P_{Gi} + jQ_{Gi}$



(a) (b)

Fig. 9.22

$$\therefore \quad I^0_{Gi} = \frac{P_{Gi} - jQ_{Gi}}{V_i^0}; \text{ (prefault generator output} = P_{Gi} + jQ_{Gi}) \tag{9.30}$$

$$E'_{Gi} = V_i + jX'_{Gi} I^0_{Gi} \tag{9.31}$$

From the SC study, V^f_i is obtained. It then follows from Fig. 9.22(b) that

$$I^f_{Gi} = \frac{E'_{Gi} - V_i^f}{jX'_{Gi}} \tag{9.32}$$

Example 9.6

To illustrate the algorithm discussed above, we shall recompute the short circuit solution for Example 9.5 which was solved earlier using the network reduction technique.

First of all the bus admittance matrix for the network of Fig. 9.18 is formed as follows:

$$Y_{11} = \frac{1}{j0.15} + \frac{1}{j0.15} + \frac{1}{j0.1} + \frac{1}{j0.2} = -j28.333$$

$$Y_{12} = Y_{21} = \frac{-1}{j0.2} = j5.000$$

$$Y_{13} = Y_{31} = \frac{-1}{j0.15} = j6.667$$

$$Y_{14} = Y_{41} = \frac{-1}{j0.1} = j10.000$$

$$Y_{22} = \frac{1}{j0.15} + \frac{1}{j0.15} + \frac{1}{j0.1} + \frac{1}{j0.2} = -j28.333$$

$$Y_{23} = Y_{32} = \frac{-1}{j0.1} = j10.000$$

$$Y_{24} = Y_{42} = \frac{-1}{j0.15} = j6.667$$

$$Y_{33} = \frac{1}{j0.15} + \frac{1}{j0.1} = -j16.667$$

$$Y_{34} = Y_{43} = 0.000$$

$$Y_{44} = \frac{1}{j0.1} + \frac{1}{j0.15} = -j16.667$$

$$Y_{BUS} = \begin{bmatrix} -j28.333 & j5.000 & j6.667 & j10.000 \\ j5.000 & -j28.333 & j10.000 & j6.667 \\ j6.667 & j10.000 & -j16.667 & j0.000 \\ j10.000 & j6.667 & j0.000 & -j16.667 \end{bmatrix}$$

By inversion we get Z_{BUS} as

$$Z_{BUS} = \begin{bmatrix} j0.0903 & j0.0597 & j0.0719 & j0.0780 \\ j0.0597 & j0.0903 & j0.0780 & j0.0719 \\ j0.0719 & j0.0780 & j0.1356 & j0.0743 \\ j0.0780 & j0.0719 & j0.0743 & j0.1356 \end{bmatrix}$$

Now, the postfault bus voltages can be obtained using Eq. (9.27) as

$$V_1^f = V_1^0 - \frac{Z_{14}}{Z_{44}} V_4^0$$

The prefault condition being no load, $V_1^0 = V_2^0 = V_3^0 = V_4^0 = 1$ pu

$$V_1^f = 1.0 - \frac{j0.0780}{j0.1356} \times 1.0 = 0.4248 \text{ pu}$$

$$V_2^f = V_2^0 - \frac{Z_{24}}{Z_{44}} V_4^0$$

$$= 1.0 - \frac{j0.0719}{j0.1356} \times 1.0 = 0.4698 \text{ pu}$$

$$V_3^f = V_3^0 - \frac{Z_{34}}{Z_{44}} V_4^0$$

$$= 1.0 - \frac{j0.0743}{j0.1356} 1.0 = 0.4521 \text{ pu}$$

$$V_4^f = 0.0$$

Using Eq. (9.25) we can obtain the fault current as

$$I^f = \frac{1.000}{j0.1356} = -j7.37463 \text{ pu}$$

These values agree with those obtained earlier in Example 9.5. Let us also calculate the short circuit current in lines 1-3, 1-2, 1-4, 2-4 and 2-3.

$$I_{12}^f = \frac{V_1^f - V_3^f}{z_{13}} = \frac{0.4248 - 0.4521}{j0.15} = j0.182 \text{ pu}$$

$$I_{12}^f = \frac{V_1^f - V_2^f}{z_{12}} = \frac{0.4248 - 0.4698}{j0.2} = j0.225 \text{ pu}$$

$$I_{14}^f = \frac{V_1^f - V_4^f}{z_{14}} = \frac{0.4248 - 0}{j0.1} = -j4.248 \text{ pu}$$

$$I_{24}^f = \frac{V_2^f - V_4^f}{z_{24}} = \frac{0.4698 - 0}{j0.15} = -j3.132 \text{ pu}$$

$$I_{23}^f = \frac{V_2^f - V_3}{z_{23}} = \frac{0.4698 - 0.4521}{j.01} = -j0.177 \text{ pu}$$

For the example on hand this method may appear more involved compared to the heuristic network reduction method employed in Example 9.5. This, however, is a systematic method and can be easily adopted on the digital computer for practical networks of large size. Further, another important feature of the method is that having computed Z_{BUS}, we can at once obtain all the required short circuit data for a fault on any bus. For example, in this particular system, the fault current for a fault on bus 1 (or bus 2) will be

$$I^f = \frac{1.000}{Z_{11}(\text{or } Z_{22})} = \frac{1.00}{j0.0903} = -j11.074197 \text{ pu}$$

9.7 Z_{BUS} FORMULATION

By Inventing Y_{BUS}

$$J_{BUS} = Y_{BUS} V_{BUS}$$

or $$V_{BUS} = [Y_{BUS}]^{-1} J_{BUS} = Z_{BUS} J_{BUS} \quad (9.33)$$

or $$Z_{BUS} = [Y_{BUS}]^{-1}$$

The sparsity of Y_{BUS} may be retained by using an efficient inversion technique [1] and nodal impedance matrix can then be calculated directly from the factorized admittance matrix. This is beyond the scope of this book.

Current Injection Technique

Equation (9.33) can be written in the expanded form

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + \ldots + Z_{1n}I_n \quad (9.34)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 + \ldots + Z_{2n}I_n$$

$$\text{-----------------------------}$$

$$V_n = Z_{n1}I_1 + Z_{n2}I_1 + \ldots + Z_{nn}I_n$$

It immediately follows from Eq. (9.34) that

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{\substack{I_1 = I_2 = \cdots = I_n = 0 \\ I_j \neq 0}} \quad (9.35)$$

Also $Z_{ij} = Z_{ji}$; (Z_{BUS} is a symmetrical matrix).

As per Eq. (9.35) if a unit current is injected at bus (node) j, while the other buses are kept open circuited, the bus voltages yield the values of the jth column of Z_{BUS}. However, no organized computerizable techniques are possible for finding the bus voltages. The technique had utility in AC Network Analyzers where the bus voltages could be read by a voltmeter.

Example 9.7

Consider the network of Fig. 9.23(a) with three buses one of which is a reference. Evaluate Z_{BUS}.

Solution Inject a unit current at bus 1 keeping bus 2 open circuit, i.e., $I_1 = 1$ and $I_2 = 0$ as in Fig. 9.22(b). Calculating voltages at buses 1 and 2, we have

$$Z_{11} = V_1 = 7$$

$$Z_{21} = V_2 = 4$$

Now let $I_1 = 0$ and $I_2 = 1$. It similarly follows that

$$Z_{12} = V_1 = 4 = Z_{12}$$
$$Z_{22} = V_2 = 6$$

Collecting the above values

$$Z_{BUS} = \begin{bmatrix} 7 & 4 \\ 4 & 6 \end{bmatrix}$$

Because of the above computational procedure, the Z_{BUS} matrix is referred to as the 'open-circuit impedance matrix'.

Z_{BUS} Building Algorithm

It is a step-by-step programmable technique which proceeds branch by branch. It has the advantage that any modification of the network does not require complete rebuilding of Z_{BUS}.

Consider that Z_{BUS} has been formulated upto a certain stage and another branch is now added. Then

$$Z_{BUS}\text{ (old)} \xrightarrow{Z_b = \text{branch impedance}} Z_{BUS}\text{ (new)}$$

Upon adding a new branch, one of the following situations is presented.

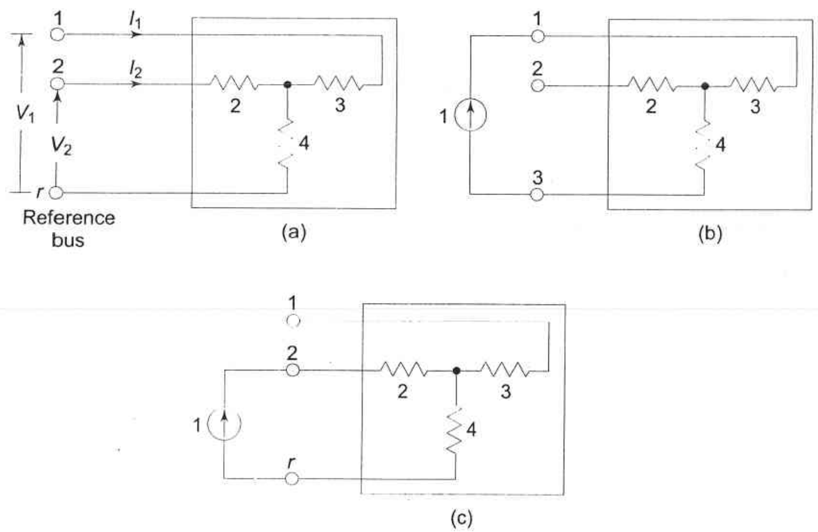


Fig. 9.23 Current injection method of computing Z_{BUS}

1. Z_b is added from a new bus to the reference bus (i.e. a new branch is added and the dimension of Z_{BUS} goes up by one). This is *type-1 modification.*

2. Z_b is added from a new bus to an old bus (i.e., a new branch is added and the dimension of Z_{BUS} goes up by one). This is *type-2 modification.*
3. Z_b connects an old bus to the reference branch (i.e., a new loop is formed but the dimension of Z_{BUS} does not change). This is *type-3 modification.*
4. Z_b connects two old buses (i.e., new loop is formed but the dimension of Z_{BUS} does not change). This is *type-4 modification.*
5. Z_b connects two new buses (Z_{BUS} remains unaffected in this case). This situation can be avoided by suitable numbering of buses and from now onwards will be ignored.

Notation: i, j—old buses; *r*—reference bus; *k*—new bus.

Type-1 Modification

Figure 9.24 shows a passive (linear) n-bus network in which branch with impedance Z_b is added to the new bus k and the reference bus r. Now

$$V_k = Z_b I_k$$
$$Z_{ki} = Z_{ik} = 0;\ i = 1, 2, \ldots, n$$
$$\therefore \quad Z_{kk} = Z_b$$

Hence

$$Z_{BUS}\text{ (new)} = \left[\begin{array}{ccc|c} & & & 0 \\ & Z_{Bus}\text{(old)} & & \vdots \\ & & & 0 \\ \hline 0 & & \ldots 0 & Z_b \end{array}\right] \tag{9.36}$$

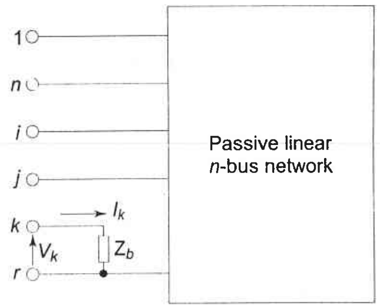


Fig. 9.24 Type-1 modification

Type-2 Modification

Z_b is added from new bus k to the old bus j as in Fig. 9.25. It follows from this figure that

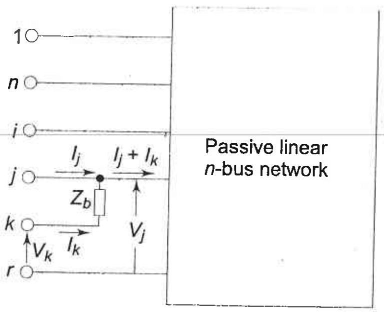


Fig. 9.25 Type-2 modification

$$V_k = Z_b I_k + V_j$$
$$= Z_b I_k + Z_{j1} I_1 + Z_{j2} I_2 + \ldots + Z_{jj}(I_j + I_k) + \ldots + Z_{jn} I_n$$

Rearranging,

$$V_k = Z_{j1} I_1 + Z_{j2} I_2 + \ldots + Z_{jj} I_j + \ldots + Z_{jn} I_n + (Z_{jj} + Z_b) I_k$$

Consequently

$$Z_{BUS}\text{ (new)} = \left[\begin{array}{c|c} & Z_{1j} \\ & Z_2 j \\ Z_{BUS}(\text{old}) & \vdots \\ & Z_{nj} \\ \hline Z_{ji} Z_{j2} \ldots Z_{jn} & Z_{jj} + Z_b \end{array}\right] \tag{9.37}$$

Type-3 Modification

Z_b connects an old bus (j) to the reference bus (r) as in Fig. 9.26. This case follows from Fig. 9.25 by connecting bus k to the reference bus r, i.e. by setting $V_k = 0$.

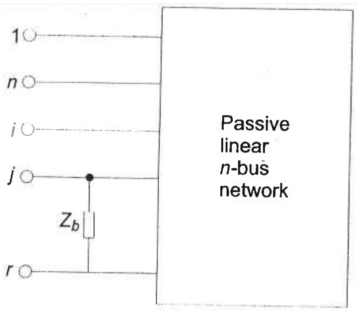


Fig. 9.26 Type-3 modification

Thus

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} & Z_{1j} \\ & Z_2 j \\ Z_{BUS}(\text{old}) & \vdots \\ & Z_{nj} \\ \hline Z_{j1} Z_{j2} \ldots Z_{jn} & Z_{jj} + Z_b \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_k \end{bmatrix} \tag{9.38}$$

Eliminate I_k in the set of equations contained in the matrix operation (9.38),

$$0 = Z_{j1} I_1 + Z_{j2} I_2 + \ldots + Z_{jn} I_n + (Z_{jj} + Z_b) I_k$$

or

$$I_k = -\frac{1}{Z_{jj} - Z_b}\,(Z_{j1} I_1 + Z_{j2} I_2 + \ldots + Z_{jn} I_n) \tag{9.39}$$

Now

$$V_i = Z_{i1} I_1 + Z_{i2} I_2 + \ldots + Z_{in} I_n + Z_{ij} I_k \tag{9.40}$$

Substituting Eq. (9.40) in Eq. (9.39)

$$V_i = \left[Z_{i1} - \frac{1}{Z_{jj} + Z_b}(Z_{ij} Z_{j1})\right] I_1 + \left[Z_{i2} - \frac{1}{Z_{jj} + Z_b}(Z_{ij} Z_{j2})\right] I_2$$
$$+ \ldots + \left[Z_{in} - \frac{1}{Z_{jj} + Z_b}(Z_{ij}\ Z_{jn})\right] I_n \tag{9.41}$$

Equation (9.37) can be written in matrix form as

$$Z_{BUS}\text{ (new)} = Z_{BUS}\text{ (old} - \frac{1}{Z_{jj} + Z_b} \begin{bmatrix} Z_{1j} \\ \vdots \\ Z_{nj} \end{bmatrix} [Z_{j1} \ldots Z_{jn}] \tag{9.42}$$

Type-4 Modification

Z_b connects two old buses as in Fig. 9.27. Equations can be written as follows for all the network buses.

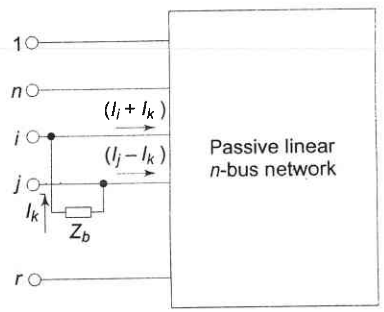


Fig. 9.27 Type-4 modification

$$V_i = Z_{i1} I_1 + Z_{i2} I_2 + \ldots + Z_{1i}(I_i + I_k) + Z_{ij}(I_j - I_k) + \ldots + Z_{in} I_n \tag{9.43}$$

Similar equations follow for other buses.

The voltages of the buses i and j are, however, constrained by the equation (Fig. 9.27)

$$V_j = Z_b I_k + V_i \tag{9.44}$$

or $\quad Z_{j1}I_1 + Z_{j2}I_2 + \ldots + Z_{ji}(I_i + I_k) + Z_{jj}(I_j - I_k) + \ldots + Z_{jn}I_n$

$$= Z_b I_k + Z_{i1}I_1 + Z_{i2}I_2 + \ldots + Z_{ii}(I_i + I_k) + Z_{ij}(I_j - I_k) + \ldots + Z_{in}I_n$$

Rearranging

$$0 = (Z_{i1} - Z_{j1})\, I_1 + \ldots + (Z_{ii} - Z_{ji})\, I_i + (Z_{ij} - Z_{jj})\, I_j$$
$$+\ldots+ (Z_{in} - Z_{jn})\, I_n + (Z_b + Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji})\, I_k \tag{9.45}$$

Collecting equations similar to Eq. (9.43) and Eq. (9.45) we can write

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ - \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} & (Z_{1i} - Z_{1j}) \\ Z_{\mathrm{BUS}} & | \\ & (Z_{ni} - Z_{nj}) \\ \hline (Z_{i1} - Z_{j1})\ldots(Z_{in} - Z_{jn}) & Z_b + Z_{ii} + Z_{jj} - 2Z_{ij} \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ - \\ I_j \end{bmatrix} \tag{9.46}$$

Eliminating I_k in Eq. (9.46) on lines similar to what was done in Type-2 modification, it follows that

$$Z_{\mathrm{BUS}}\,(\mathrm{new}) = Z_{\mathrm{BUS}}\,(\mathrm{old}) - \frac{1}{Z_b + Z_{ii} + Z_{jj} - 2Z_{ij}} \begin{bmatrix} Z_{1i} & - & Z_{1j} \\ & \vdots & \\ Z_{ni} & - & Z_{nj} \end{bmatrix}$$

$$[Z_{i1} - Z_{j1}] \ldots (Z_{in} - Z_{jn})] \tag{9.47}$$

With the use of four relationships Eqs (9.36), (9.37), (9.42) and (9.47) bus impedance matrix can be built by a step-by-step procedure (bringing in one branch at a time) as illustrated in Example 9.8. This procedure being a mechanical one can be easily computerized.

When the network undergoes changes, the modification procedures can be employed to revise the bus impedance matrix of the network. The opening of a line (Z_{ij}) is equivalent to adding a branch in parallel to it with impedance $-Z_{ij}$ (see Example 9.8).

Example 9.8

For the 3-bus network shown in Fig. 9.28 build Z_{BUS}.

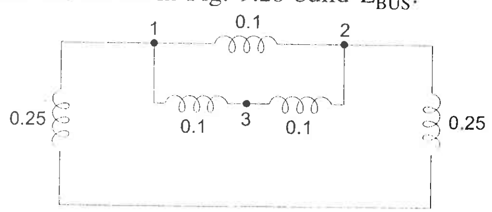


Fig. 9.28

Solution

Step 1: Add branch $z_{1r} = 0.25$ (from bus 1 (new) to bus r)

$$z_{\mathrm{BUS}} = [0.25] \tag{i}$$

Step 2: Add branch $z_{21} = 0.1$ (from bus 2 (new) to bus 1 (old)); type-2 modification

$$Z_{\mathrm{BUS}} = \begin{array}{c} 1 \\ 2 \end{array}\begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.35 \end{bmatrix} \tag{ii}$$

Step 3: Add branch $z_{13} = 0.1$ (from bus 3 (new) to bus 1 (old)); type-2 modification

$$Z_{\mathrm{BUS}} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} \tag{iii}$$

Step 4: Add branch z_{2r} (from bus 2 (old) to bus r); type-3 modification

$$Z_{\mathrm{BUS}} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} - \frac{1}{0.35 + 0.25} \begin{bmatrix} 0.25 \\ 0.35 \\ 0.25 \end{bmatrix} [0.25\ 0.35\ 0.25]$$

$$= \begin{bmatrix} 0.1458 & 0.1042 & 0.1458 \\ 0.1042 & 0.1458 & 0.1042 \\ 0.1458 & 0.1042 & 0.2458 \end{bmatrix}$$

Step 5: Add branch $z_{23} = 0.1$ (from bus 2 (old) to bus 3 (old)); type-4 modification

$$Z_{\mathrm{BUS}} = \begin{bmatrix} 0.1458 & 0.1042 & 0.1458 \\ 0.1042 & 0.1458 & 0.1042 \\ 0.1458 & 0.1042 & 0.2458 \end{bmatrix} - \frac{1}{0.1 + 0.1458 + 0.2458 - 2 \times 0.1042}$$

$$= \begin{bmatrix} -0.1042 \\ 0.0417 \\ -0.0417 \end{bmatrix} [-0.1042 \quad 0.0417 \quad -0.0417]$$

$$= \begin{bmatrix} 0.1397 & 0.1103 & 0.1250 \\ 0.1103 & 0.1397 & 0.1250 \\ 0.1250 & 0.1250 & 0.1750 \end{bmatrix}$$

Opening a line (line 3-2): This is equivalent to connecting an impedance -0.1 between bus 3 (old) and bus 2 (old) i.e. type-4 modification.

$$Z_{\mathrm{BUS}} = Z_{\mathrm{BUS}}\,(\mathrm{old}) - \frac{1}{(-0.1) + 0.175 + 0.1397 - 2 \times 0.125}$$

$$\begin{bmatrix} 0.0147 \\ -0.0147 \\ 0.0500 \end{bmatrix} [0.0147 \;\; -0.0147 \;\; 0.0500]$$

$$= \begin{bmatrix} 0.1458 & 0.1042 & 0.1458 \\ 0.1042 & 0.1458 & 0.1042 \\ 0.1458 & 0.1042 & 0.2458 \end{bmatrix}; \text{ (same as in step 4)}$$

Example 9.9

For the power system shown in Fig. 9.29 the pu reactances are shown therein. For a solid 3-phase fault on bus 3, calculate the following

(a) Fault current
(b) V_1^f and V_2^f
(c) I_{12}^f, I_{13}^f and I_{23}^f
(d) I_{G1}^f, and I_{G2}^f

Assume prefault voltage to be 1 pu.

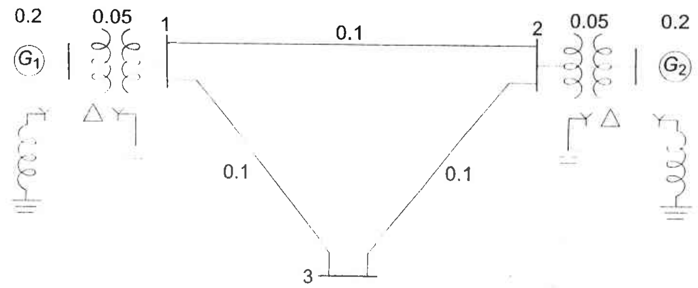


Fig. 9.29

Solution The Thevenin passive network for this system is drawn in Fig. 9.28 with its Z_{BUS} given in Eq. (iv) of Example 9.8.

(a) As per Eq. (9.25)

$$I^f = \frac{V_r^0}{Z_{rr} + Z^f}$$

or $$I^f = \frac{V_3^0}{Z_{33}} = \frac{1}{j0.175} = -j5.71$$

(b) As per Eq. (9.26)

$$V_i^f = V_i^0 - \frac{Z_{ir}}{Z_{rr} + Z^f} V_r^0$$

Now $$V_1^f = \left(1 - \frac{Z_{13}}{Z_{33}}\right) = 1 - \frac{0.125}{0.175}$$
$$= 0.286$$

and $$V_2^f = \left(1 - \frac{Z_{23}}{Z_{33}}\right) = 0.286$$

These two voltages are equal because of the symmetry of the given power network

(c) From Eq. (9.29)

$$I_{ij}^f = Y_{ij}\,(V_i^f - V_j^f)$$

$$I_{12}^f = \frac{1}{j0.1}(0.286 - 0.286) = 0$$

and $$I_{13}^f = I_{31}^f = \frac{1}{j0.1}(0.286 - 0)$$
$$= -j2.86$$

(d) As per Eq. (9.32)

$$I_{G1}^f = \frac{E'_{G1} - V_1^f}{jX'_{iG} + jX_T}$$

But $E'_{G1} = 1$ pu (prefault no load)

$$\therefore \quad I_{G1}^f = \frac{1 - 0.286}{j0.2 + j0.05} = -j2.86$$

Similarly

$$I_{G2}^f = j2.86$$

PROBLEMS

9.1 A transmission line of inductance 0.1 H and resistance 5 ohms is suddenly short circuited at $t = 0$ at the bar end as shown in Fig. P-9.1. Write the expression for short circuit current $i(t)$. Find approximately the value of the first current maximum (maximum momentary current).

[**Hint:** Assume that the first current maximum occurs at the same time as the first current maximum of the symmetrical short circuit current.)

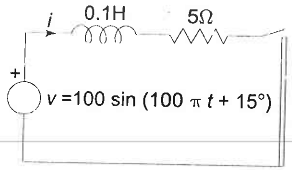


Fig. P-9.1

9.2 (a) What should the instant of short circuit be in Fig. P-9.1 so that the DC off-set current is zero?

(b) What should the instant of short circuit be in Fig. P-9.1 so that the DC off-set current is maximum?

9.3 For the system of Fig. 9.8 (Example 9.2) find the symmetrical currents to be interrupted by circuit breakers A and B for a fault at (i) P and (ii) Q.

9.4 For the system in Fig. P-9.4 the ratings of the various components are:

Generator:	25 MVA, 12.4 kV, 10% subtransient reactance
Motor:	20 MVA, 3.8 kV, 15% subtransient reactance
Transformer T_1:	25 MVA, 11/33 kV, 8% reactance
Transformer T_2:	20 MVA, 33/3.3 kV, 10% reactance
Line:	20 ohms reactance

The system is loaded so that the motor is drawing 15 MW at 0.9 loading power factor, the motor terminal voltage being 3.1 kV. Find the subtransient current in generator and motor for a fault at generator bus. [**Hint:** Assume a suitable voltage base for the generator. The voltage base for transformers, line and motor would then be given by the transformation ratios. For example, if we choose generator voltage base as 11 kV, the line voltage base is 33 kV and motor voltage base is 3.3 kV. Per unit reactances are calculated accordingly.]

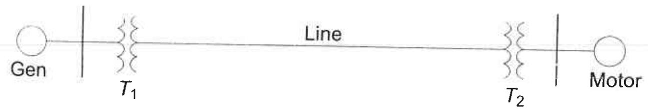


Fig. P-9.4

9.5 Two synchronous motors are connected to the bus of a large system through a short transmission line as shown in Fig. P-9.5. The ratings of various components are:

Motors (each):	1 MVA, 440 V, 0.1 pu transient reactance
Line:	0.05 ohm reactance
Large system:	Short circuit MVA at its bus at 440 V is 8.

When the motors are operating at 440 V, calculate the short circuit current (symmetrical) fed into a three-phase fault at motor bus.

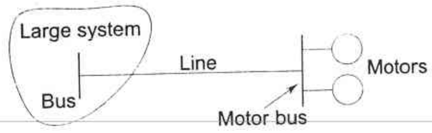


Fig. P-9.5

9.6 A synchronous generator rated 500 kVA, 440 V, 0.1 pu subtransient reactance is supplying a passive load of 400 kW at 0.8 lagging power factor. Calculate the initial symmetrical rms current for a three-phase fault at generator terminals.

9.7 A generator-transformer unit is connected to a line through a circuit breaker. The unit ratings are:

Generator:	10 MVA, 6.6 kV; $X''_d = 0.1$ pu, $X'_d = 0.20$ pu and $X_d = 0.80$ pu
Transformer:	10 MVA, 6.9/33 kV, reactance 0.08 pu

The system is operating no load at a line voltage of 30 kV, when a three-phase fault occurs on the line just beyond the circuit breaker. Find

(a) the initial symmetrical rms current in the breaker,
(b) the maximum possible DC off-set current in the breaker,
(c) the momentary current rating of the breaker,
(d) the current to be interrupted by the breaker and the interrupting kVA, and
(e) the sustained short circuit current in the breaker.

9.8 The system shown in Fig. P-9.8 is delivering 50 MVA at 11 kV, 0.8 lagging power factor into a bus which may be regarded as infinite. Particulars of various system components are:

Generator:	60 MVA, 12 kV, $X'_d = 0.35$ pu
Transformers (each):	80 MVA, 12/66 kV, $X = 0.08$ pu
Line:	Reactance 12 ohms, resistance negligible.

Calculate the symmetrical current that the circuit breakers A and B will be called upon to interrupt in the event of a three-phase fault occurring at F near the circuit breaker B.

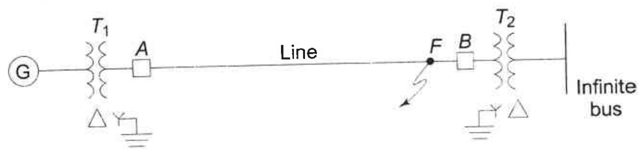


Fig. P-9.8

9.9 A two generator station supplies a feeder through a bus as shown in Fig. P-9.9. Additional power is fed to the bus through a transformer from a large system which may be regarded as infinite. A reactor X is included between the transformer and the bus to limit the SC rupturing capacity of the feeder circuit breaker B to 333 MVA (fault close to breaker). Find the inductive reactance of the reactor required. System data are:

Generator G_1:	25 MVA, 15% reactance
Generator G_2:	50 MVA, 20% reactance
Transformer T_1:	100 MVA; 8% reactance
Transformer T_2:	40 MVA; 10% reactance.

Assume that all reactances are given on appropriate voltage bases. Choose a base of 100 MVA.

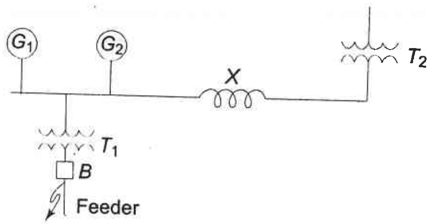


Fig. P-9.9

9.10 For the three-phase power network shown in Fig. P-9.10, the ratings of the various components are:

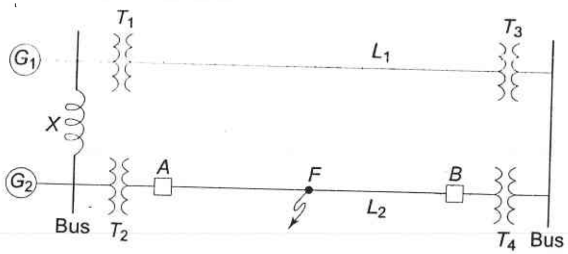


Fig. P-9.10

Generators G_1: 100 MVA, 0.30 pu reactance
G_2: 60 MVA, 0.18 pu reactance
Transformers (each): 50 MVA, 0.10 pu reactance
Inductive reactor X: 0.20 pu on a base of 100 MVA
Lines (each): 80 ohms (reactive); neglect resistance.

With the network initially unloaded and a line voltage of 110 kV, a symmetrical short circuit occurs at mid point F of line L_2.

Calculate the short circuit MVA to be interrupted by the circuit breakers A and B at the ends of the line. What would these values be, if the reactor X were eliminated? Comment.

9.11 A synchronous generator feeds bus 1 of a system. A power network feeds bus 2 of the system. Buses 1 and 2 are connected through a transformer and a transmission line. Per unit reactances of the various components are:

Generator (connected to bus bar 1)	0.25
Transformer	0.12
Transmission line	0.28

The power network can be represented by a generator with a reactance (unknown) in series.

With the generator on no load and with 1.0 pu voltage at each bus under operating condition, a three-phase short circuit occurring on bus 1 causes a current of 5.0 pu to flow into the fault. Determine the equivalent reactance of the power network.

9.12 Consider the 3-bus system of Fig. P-9.12. The generators are 100 MVA, with transient reactance 10% each. Both the transformers are 100 MVA with a leakage reactance of 5%. The reactance of each of the lines to a base of 100 MVA, 110 KV is 10%. Obtain the short circuit solution for a three-phase solid short circuit on bus 3.

Assume prefault voltages to be 1 pu and prefault currents to be zero.

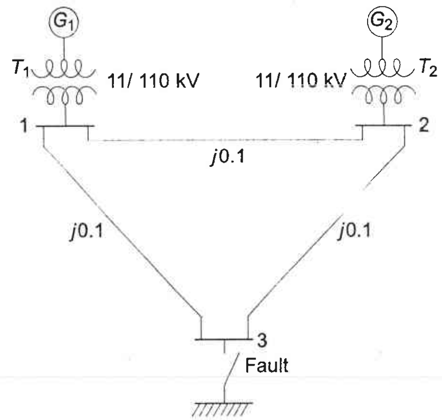


Fig. P-9.12

9.13 In the system configuration of Fig. P-9.12, the system impedance data are given below:

Transient reactance of each generator = 0.15 pu
Leakage reactance of each transformer = 0.05 pu
$z_{12} = j0.1$, $z_{13} = j0.12$, $z_{23} = j0.08$ pu

For a solid 3-phase fault on bus 3, find all bus voltages and sc currents in each component.

9.14 For the fault (solid) location shown in Fig. P-9.14, find the sc currents in lines 1.2 and 1.3. Prefault system is on no-load with 1 pu voltage and prefault currents are zero. Use Z_{BUS} method and compute its elements by the current injection technique.

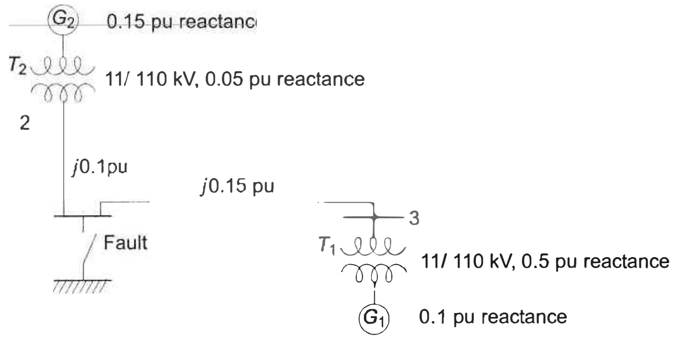


Fig. P-9.14

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