Data Structures & Algorithm Analysis

Asymptotic Analysis

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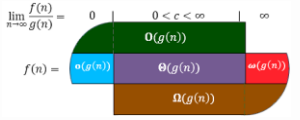

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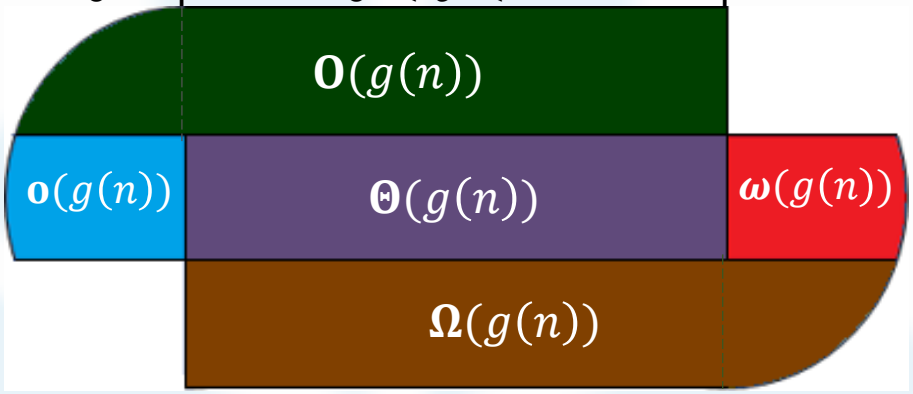
$$\lim_{n\to\infty}\frac{f(n)}{g(n)} = \quad 0 \quad | \quad 0 < c < \infty \quad | \quad \infty$$

$f(n) =$



$$if \quad 0 \le \lim_{n\to\infty}\frac{f(n)}{g(n)} < \infty \quad \Rightarrow \quad f(n) = \mathbf{O}(g(n))$$

$$if \quad \lim_{n\to\infty}\frac{f(n)}{g(n)} = 0 \quad \Rightarrow \quad f(n) = \mathbf{o}(g(n))$$

$$if \quad 0 < \lim_{n\to\infty}\frac{f(n)}{g(n)} < \infty \quad \Rightarrow \quad f(n) = \mathbf{\Theta}(g(n))$$

$$if \quad \lim_{n\to\infty}\frac{f(n)}{g(n)} = \infty \quad \Rightarrow \quad f(n) = \boldsymbol{\omega}(g(n))$$

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Big-Oh Examples

- $2n^3 + 3n^2 + n$

$= 2n^3 + 3n^2 + O(n)$

$= 2n^3 + O(n^2 + n)$

$= 2n^3 + O(n^2)$

$= O(n^3) = O(n^4)$

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Big-Oh Examples

Suppose a program P is $\mathbf{O(n^3)}$, and a program **Q** is $\mathbf{O(3^n)}$, and that currently both can solve problems of size 50 in 1 hour. If the programs are run on another system that executes exactly 729 times as fast as the original system, what size problems will they be able to solve (in 1 hour)?

$n^3 = 50^3 * 729$

$n = \sqrt[3]{50^3} * \sqrt[3]{729}$

$n = 50*9$

$n = 50*9 = 450$

$3^n = 3^{50} * 729$

$n = \log_3 (729 * 3^{50})$

$n = \log_3(729) + \log_3 3^{50}$

$n = 6 + \log_3 3^{50}$

$n = 6 + 50 = 56$

- ***Improvement**: problem size increased by **9** times for $\mathbf{n^3}$ algorithm but only a slight improvement in problem size (+6) for **exponential** algorithm.*

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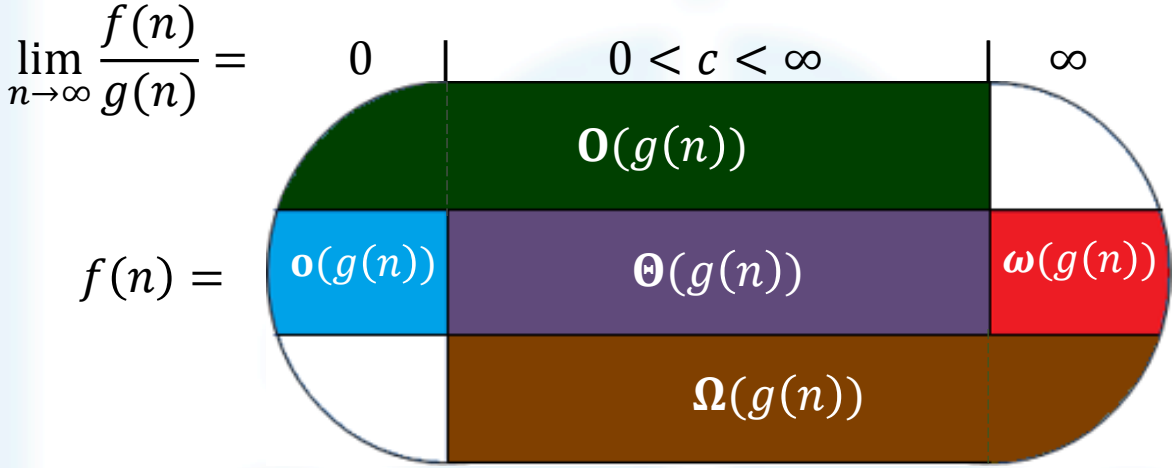

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