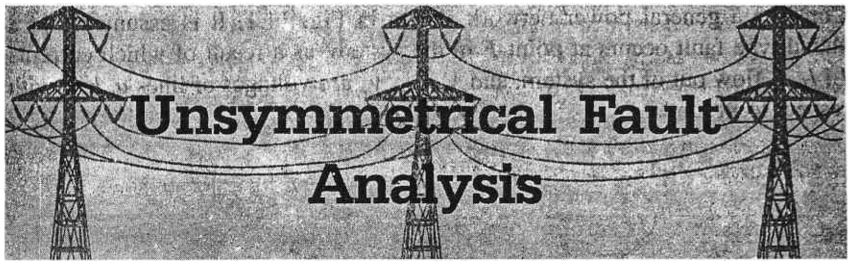
11



Unsymmetrical Fault Analysis

11.1 INTRODUCTION

Chapter 9 was devoted to the treatment of symmetrical (three-phase) faults in a power system. Since the system remains balanced during such faults, analysis could conveniently proceed on a single-phase basis. In this chapter, we shall deal with unsymmetrical faults. Various types of unsymmetrical faults that occur in power systems are:

Shunt Type Faults

(i) Single line-to-ground (LG) fault
(ii) Line-to-line (LL) fault
(iii) Double line-to-ground (LLG) fault

Series Type Faults

(i) Open conductor (one or two conductors open) fault.

It was stated in Chapter 9, that a three-phase (3L) fault being the most severe must be used to calculate the rupturing capacity of circuit breakers, even though this type of fault has a low frequency of occurrence, when compared to the unsymmetrical faults listed above. There are, however, situations when an LG fault can cause greater fault current than a three-phase fault (this may be so when the fault location is close to large generating units). Apart from this, unsymmetrical fault analysis is important for relay setting, single-phase switching and system stability studies (Chapter 12).

The probability of two or more simultaneous faults (cross-country faults) on a power system is remote and is therefore ignored in system design for abnormal conditions.

The method of symmetrical components presented in Chapter 10, is a powerful tool for study of unsymmetrical faults and will be fully exploited in this chapter.

11.2 SYMMETRICAL COMPONENT ANALYSIS OF UNSYMMETRICAL FAULTS

Consider a general power network shown in Fig. 11.1. It is assumed that a shund type fault occurs at point F in the system, as a result of which currents I_a, I_b, I_c flow out of the system, and V_a, V_b, V_c are voltages of lines a, b, c with respect to ground.

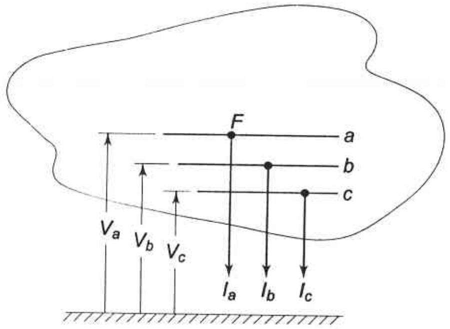


Fig. 11.1 A general power network

Let us also assume that the system is operating at no load before the occurrence of a fault. Therefore, the positive sequence voltages of all synchronous machines will be identical and will equal the prefault voltage at F. Let this voltage be labelled as E_a.

As seen from F, the power system will present positive, negative and zero sequence networks, which are schematically represented by Figs. 11.2a, b and c. The reference bus is indicated by a thick line and the point F is identified on each sequence network. Sequence voltages at F and sequence currents flowing out of the networks at F are also shown on the sequence networks. Figures 11.3a, b, and c respectively, give the Thevenin equivalents of the three sequence networks.

Recognizing that voltage E_a is present only in the positive sequence network and that there is no coupling between sequence networks, the sequence voltages at F can be expressed in terms of sequence currents and Thevenin sequence impedances as

$$\begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \begin{bmatrix} E_a \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_2 & 0 \\ 0 & 0 & Z_0 \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} \tag{11.1}$$

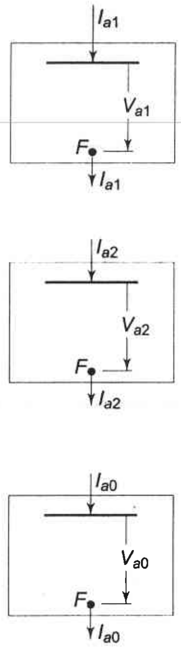


Fig. 11.2 Sequence networks as seen from the fault point F

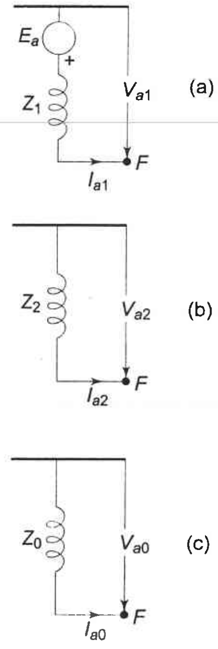


Fig. 11.3 Thevenin equivalents of the sequence networks as seen from the fault point F

Depending upon the type of fault, the sequence currents and voltages are constrained, leading to a particular connection of sequence networks. The sequence currents and voltages and fault currents and voltages can then be easily computed. We shall now consider the various types of faults enumerated earlier.

11.3 SINGLE LINE-TO-GROUND (LG) FAULT

Figure 11.4 shows a line-to-ground fault at F in a power system through a fault impedance Z^f. The phases are so labelled that the fault occurs on phase a.

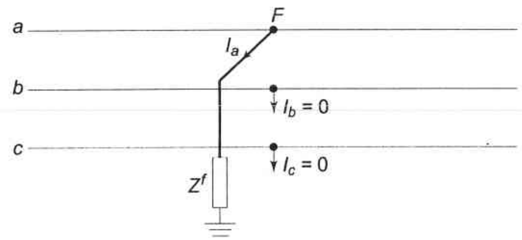


Fig. 11.4 Single line-to-ground (LG) fault at F

At the fault point F, the currents out of the power system and the line to ground voltages are constrained as follows:

$$I_b = 0 \tag{11.2}$$

$$I_c = 0 \tag{11.3}$$

$$V_a = Z^f I_a \tag{11.4}$$

The symmetrical components of the fault currents are

$$\begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

from which it is easy to see that

$$I_{a1} = I_{a2} = I_{a0} = \frac{1}{3} I_a \tag{11.5}$$

Expressing Eq. (11.4) in terms of symmetrical components, we have

$$V_{a1} + V_{a2} + V_{a0} = Z^f I_a = 3Z^f I_{a1} \tag{11.6}$$

As per Eqs. (11.5) and (11.6) all sequence currents are equal and the sum of sequence voltages equals $3Z^f I_{a1}$. Therefore, these equations suggest a series connection of sequence networks through an impedance $3Z^f$ as shown in Figs. 11.5a and b.

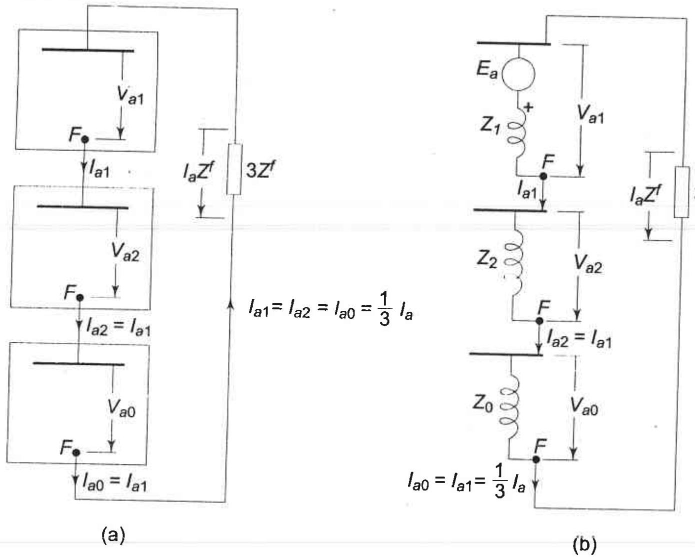


Fig. 11.5 Connection of sequence network for a single line-to-ground (LG) fault

In terms of the Thevenin equivalent of sequence networks, we can write from Fig. 11.5b.

$$I_{a1} = \frac{E_a}{(Z_1 + Z_2 + Z_0) + 3Z^f} \tag{11.7}$$

Fault current I_a is then given by

$$I_a = 3I_{a1} = \frac{3E_a}{(Z_1 + Z_2 + Z_0) + 3Z^f} \tag{11.8}$$

The above results can also be obtained directly from Eqs. (11.5) and (11.6) by using V_{a1}, V_{a2} and V_{a0} from Eq. (11.1). Thus

$$(E_a - I_{a1}Z_1) + (-I_{a2}Z_2) + (-I_{a0}Z_0) = 3Z^f I_{a1}$$

or

$$[(Z_1 + Z_2 + Z_0) + 3Z^f]I_{a1} = E_a$$

or

$$I_{a1} = \frac{E_a}{(Z_1 + Z_2 + Z_0) + 3Z^f}$$

The voltage of line b to ground under fault condition is

$$V_b = \alpha^2 V_{a1} + \alpha V_{a2} + V_{a0}$$

$$= \alpha^2\left(E_a - Z_1\frac{I_a}{3}\right) + \alpha\left(-Z_2\frac{I_a}{3}\right) + \left(-Z_0\frac{I_a}{3}\right)$$

Substituting for I_a from Eq. (11.8) and reorganizing, we get

$$V_b = E_a\frac{3\alpha^2 Z^f + Z_2(\alpha^2 - \alpha) + Z_0(\alpha^2 - 1)}{(Z_1 + Z_2 + Z_0) + 3Z^f} \tag{11.9}$$

The expression for V_c can be similarly obtained.

Fault Occurring Under Loaded Conditions

When a fault occurs under balanced load conditions, positive sequence currents alone flow in power system before the occurrence of the fault. Therefore, negative and zero sequence networks are the same as without load. The positive sequence network must of course carry the load current. To account for load current, the synchronous machines in the positive sequence network are replaced by subtransient, transient or synchronous reactances (depending upon the time after the occurrence of fault, when currents are to be determined) and voltages behind appropriate reactances. This change does not disturb the flow of prefault positive sequence currents (*see* Chapter 9). This positive sequence network would then be used in the sequence network connection of Fig. 11.5a for computing sequence currents under fault.

In case the positive sequence network is replaced by its Thevenin equivalent as in Fig. 11.5b, the Thevenin voltage equals the prefault voltage V_f^o at the fault point F (under loaded conditions). The Thevenin impedance is the impedance between F and the reference bus of the passive positive sequence network (with voltage generators short circuited).

This is illustrated by a two machine system in Fig. 11.6. It is seen from this figure that while the prefault currents flow in the actual positive sequence network of Fig. 11.6a, the same do not exist in its Thevenin equivalent network of Fig. 11.6b. Therefore, when the Thevenin equivalent of positive sequence network is used for calculating fault currents, the positive sequence currents within the network are those due to fault alone and we must superimpose on these the prefault currents. Of course, the positive sequence current into the fault is directly the correct answer, the prefault current into the fault being zero.

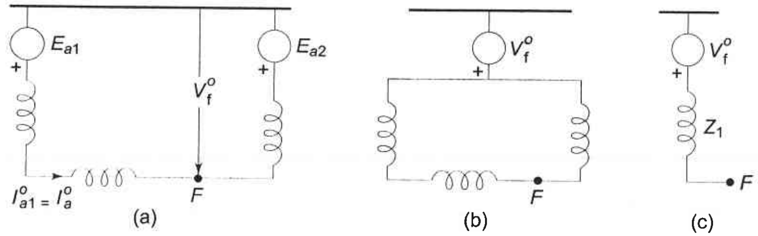


Fig. 11.6 Positive sequence network and its Thevenin equivalent before occurrence of a fault

The above remarks are valid for the positive sequence network, independent of the type of fault.

11.4 LINE-TO-LINE (LL) FAULT

Figure 11.7 shows a line-to-line fault at F in a power system on phases b and c through a fault impedance Z^f. The phases can always be relabelled, such that the fault is on phases b and c.

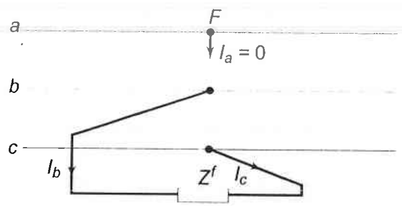


Fig. 11.7 Line-to-line (*LL*) fault through impedance Z^f

The currents and voltages at the fault can be expressed as

$$I_p = \begin{bmatrix} I_a = 0 \\ I_b \\ I_c = -I_b \end{bmatrix}; \; V_b - V_c = I_b Z^f \tag{11.10}$$

The symmetrical components of the fault currents are

$$\begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

from which we get

$$I_{a2} = -\,I_{a1} \tag{11.11}$$

$$I_{a0} = 0 \tag{11.12}$$

The symmetrical components of voltages at F under fault are

$$\begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b - Z^f I_b \end{bmatrix} \tag{11.13}$$

Writing the first two equations, we have

$$3V_{a1} = V_a + (\alpha + \alpha^2)\, V_b - \alpha^2 Z^f I_b$$

$$3V_{a2} = V_a + (\alpha + \alpha^2)\, V_b - \alpha Z^f I_b$$

from which we get

$$3(V_{a1} - V_{a2}) = (\alpha - \alpha^2) Z^f I_b = j\sqrt{3}\, Z^f I_b \tag{11.14}$$

Now

$$I_b = (\alpha^2 - \alpha)\, I_{a1} \;\; (\because\; I_{a2} = -\,I_{a1};\; I_{a0} = 0)$$

$$= -j\sqrt{3} I_{a1} \tag{11.15}$$

Substituting I_b from Eq. (11.15) in Eq. (11.14), we get

$$V_{a1} - V_{a2} = Z^f I_{a1} \tag{11.16}$$

Equations (11.11) and (11.16) suggest parallel connection of positive and negative sequence networks through a series impedance Z^f as shown in Figs. 11.8a and b. Since $I_{a0} = 0$ as per Eq. (11.12), the zero sequence network is unconnected.

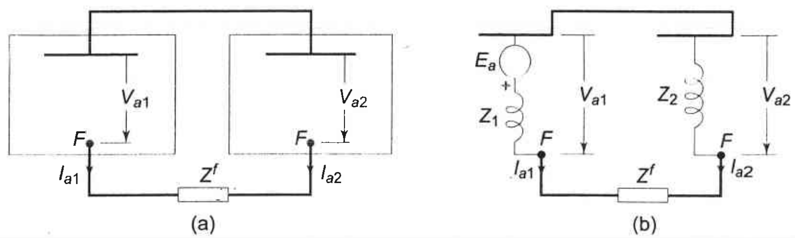


Fig. 11.8 Connection of sequence networks for a line-to-line (*LL*) fault

In terms of the Thvenin equivalents, we get from Fig. 11.8b

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z^f} \tag{11.17}$$

From Eq. (11.15), we get

$$I_b = -I_c = \frac{-j\sqrt{3}E_a}{Z_1 + Z_2 + Z^f} \tag{11.18}$$

Knowing I_{a1}, we can calculate V_{a1} and V_{a2} from which voltages at the fault can be found.

If the fault occurs from loaded conditions, the positive sequence network can be modified on the lines of the later portion of Sec. 11.3.

11.5 DOUBLE LINE-TO-GROUND (LLG) FAULT

Figure 11.9 shows a double line-to-ground fault at F in a power system. The fault may in general have an impedance Z^f as shown.

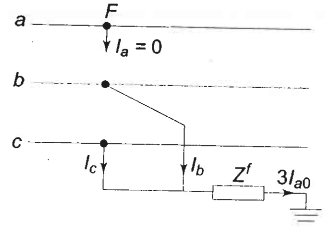


Fig. 11.9 Double line-to-ground (LLG) fault through impedance Z^f

The current and voltage (to ground) conditions at the fault are expressed as

$$\left.\begin{aligned} I_a &= 0 \\ \text{or} \quad I_{a1} + I_{a2} + I_{a0} &= 0 \end{aligned}\right\} \tag{11.19}$$

$$V_b = V_c = Z^f(I_b + I_c) = 3Z^f I_{a0} \tag{11.20}$$

The symmetrical components of voltages are given by

$$\begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix} \tag{11.21}$$

from which it follows that

$$V_{a1} = V_{a2} = \frac{1}{3}[V_a + (\alpha + \alpha^2)V_b] \tag{11.22a}$$

$$V_{a0} = \frac{1}{3}(V_a + 2V_b) \tag{11.22b}$$

From Eqs. (11.22a) and (11.22b)

$$V_{a0} - V_{a1} = \frac{1}{3}(2 - \alpha - \alpha^2)\, V_b = V_b = 3Z^f I_{a0}$$

or

$$V_{a0} = V_{a1} + 3Z^f I_{a0} \tag{11.23}$$

From Eqs. (11.19), (11.22a) and (11.23), we can draw the connection of sequence networks as shown in Figs. 11.10a and b. The reader may verify this by writing mesh and nodal equations for these figures.

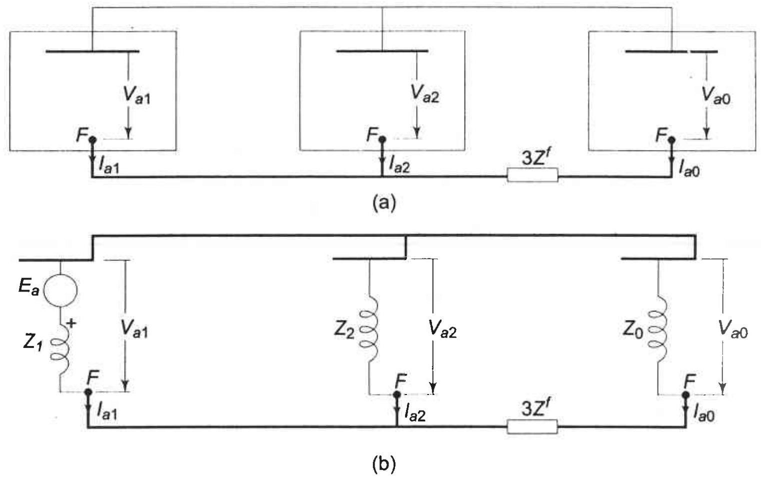


Fig. 11.10 Connection of sequence networks for a double line-to-ground (LLG) fault

In terms of the Thevenin equivalents, we can write from Fig. 11.10b

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 \| (Z_0 + 3Z^f)}$$

$$= \frac{E_a}{Z_1 + Z_2(Z_0 + 3Z^f)/(Z_2 + Z_0 + 3Z^f)} \tag{11.24}$$

The above result can be obtained analytically as follows:

Substituting for V_{a1}, V_{a2} and V_{a0} in terms of E_a in Eq. (11.1) and premultiplying both sides by Z^{-1} (inverse of sequence impedance matrix), we get

$$\begin{bmatrix} Z_1^{-1} & 0 & 0 \\ 0 & Z_2^{-1} & 0 \\ 0 & 0 & Z_0^{-1} \end{bmatrix}\begin{bmatrix} E_a - Z_1 I_{a1} \\ E_a - Z_1 I_{a1} \\ E_a - Z_1 I_{a1} + 3Z^f I_{a0} \end{bmatrix}$$

$$= \begin{bmatrix} Z_1^{-1} & 0 & 0 \\ 0 & Z_2^{-1} & 0 \\ 0 & 0 & Z_0^{-1} \end{bmatrix}\begin{bmatrix} E_a \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} \tag{11.25}$$

Premultiplying both sides by row matrix [1 1 1] and using Eqs. (11.19) and (11.20), we get

$$-\frac{3Z^f}{Z_0}I_{a0}+\left(1+\frac{Z_1}{Z_0}+\frac{Z_1}{Z_2}\right)I_{a1}=\left(\frac{1}{Z_2}+\frac{1}{Z_0}\right)E_a \qquad (11.26)$$

From Eq. (11.22a), we have

$$E_a - Z_1 I_{a1} = - Z_2 I_{a2}$$

Substituting $I_{a2} = -(I_{a1} + I_{a0})$ [*see* Eq. (11.19)]

$$E_a - Z_1 I_{a1} = Z_2(I_{a1} + I_{a0})$$

or

$$I_{a0} = \frac{E_a}{Z_2} - \left(\frac{Z_1+Z_2}{Z_2}\right)I_{a1}$$

Substituting this value of I_{a0} in Eq. (11.26) and simplifying, we finally get

$$I_{a1} = \frac{E_a}{Z_1+Z_2(Z_0+3Z^f)/(Z_2+Z_0+3Z^f)}$$

If the fault takes place from loaded conditions, the positive sequence network will be modified as discussed in Sec. 11.3.

Example 11.1

Figure 11.11 shows a synchronous generator whose neutral is grounded through a reactance X_n. The generator has balanced emfs and sequence reactances X_1, X_2 and X_0 such that $X_1 = X_2 \gg X_0$.

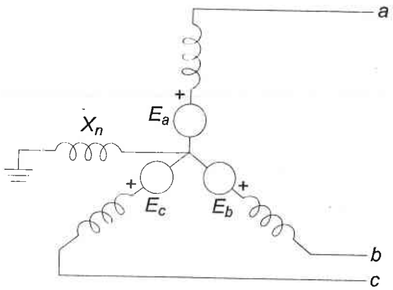


Fig. 11.11 Synchronous generator grounded through neutral reactance

(a) Draw the sequence networks of the generator as seen from the terminals.

(b) Derive expression for fault current for a solid line-to-ground fault on phase *a*.

(c) Show that, if the neutral is grounded solidly, the LG fault current would be more than the three-phase fault current.

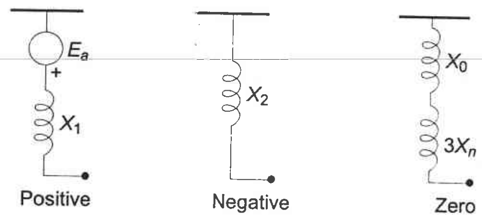


Fig. 11.12 Sequence networks of synchronous generator grounded through neutral impedance

(d) Write expression for neutral grounding reactance, such that the LG fault current is less than the three-phase fault current.

Solution (a) Figure 11.12 gives the sequence networks of the generator. As stated earlier voltage source is included in the positive sequence network only.

(b) Connection of sequence networks for a solid LG fault ($Z^f = 0$) is shown in Fig. 11.13, from which we can write the fault current as

$$|I_a|_{LG} = \frac{3|E_a|}{2X_1+X_0+3X_n} \qquad \text{(i)}$$

(c) If the neutral is solidly grounded

$$|I_a|_{LG} = \frac{3|E_a|}{2X_1+X_0} \qquad \text{(ii)}$$

For a solid three-phase fault (*see* Fig. 11.14)

$$|I_a|_{3L} = \frac{|E_a|}{X_1} = \frac{3|E_a|}{3X_1} \qquad \text{(iii)}$$

Comparing (ii) and (iii), it is easy to see that

$$|I_a|_{LG} > |I_a|_{3L}$$

An important observation is made here that, when the generator neutral is solidly grounded, LG fault is more severe than a 3L fault. It is so because, $X_0 \ll X_1 = X_2$ in generator. However, for a line $X_0 \gg X_1 = X_2$, so that for a fault on a line sufficiently away from generator, 3L fault will be more severe than an LG fault.

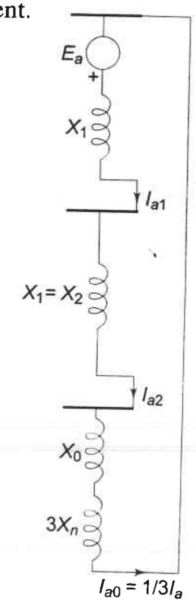


Fig. 11.13 LG fault

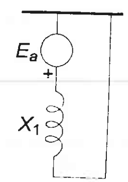


Fig. 11.14 Three-phase fault

(d) With generator neutral grounded through reactance, comparing Eqs. (i) and (iii), we have for LG fault current to be less than 3L fault

$$\frac{3|E_a|}{2X_1 + X_0 + 3X_n} < \frac{3|E_a|}{3X_1}$$

or

$$2X_1 + X_0 + 3X_n > 3X_1$$

or

$$X_n > \frac{1}{3}(X_1 - X_0) \quad \text{(iv)}$$

Example 11.2

Two 11 kV, 20 MVA, three-phase, star connected generators operate in parallel as shown in Fig. 11.15; the positive, negative and zero sequence reactances of each being, respectively, $j0.18$, $j0.15$, $j0.10$ pu. The star point of one of the generators is isolated and that of the other is earthed through a 2.0 ohm resistor. A single line-to-ground fault occurs at the terminals of one of the generators. Estimate (i) the fault current, (ii) current in grounding resistor, and (iii) the voltage across grounding resistor.

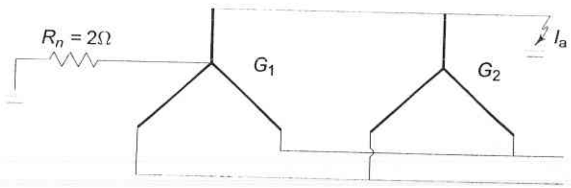


Fig. 11.15

Solution (*Note*: All values are given in per unit.)
Since the two identical generators operate in parallel,

$$X_{1eq} = \frac{j0.18}{2} = j0.09,\ X_{2eq} = \frac{j0.15}{2} = j0.075$$

Since the star point of the second generator is isolated, its zero sequence reactance does not come into picture. Therefore,

$$Z_{0eq} = j0.10 + 3R_n = j0.10 + 3 \times \frac{2 \times 20}{(11)^2} = 0.99 + j0.1$$

For an LG fault, using Eq. (11.18), we get

$$I_f \text{ (fault current for LG fault)} = I_a = 3I_{a1} = \frac{3E_a}{X_{1eq} + X_{2eq} + Z_{0eq}}$$

(a) $$I_f = \frac{3 \times 1}{j0.09 + j0.075 + j0.1 + 0.99} = \frac{3}{0.99 + j0.265}$$

$$= 2.827 - j0.756$$

(b) Current in the grounding resistor = $I_f = 2.827 - j0.756$

$$|I_f| = 2.926 \times \frac{20}{\sqrt{3} \times 11} = 3.07 \text{ kA}$$

(c) Voltage across grounding resistor = $\frac{40}{121}(2.827 - j0.756)$

$$= 0.932 - j0.249$$

$$= 0.965 \times \frac{11}{\sqrt{3}} = 6.13 \text{ kV}$$

Example 11.3

For the system of Example 10.3 the one-line diagram is redrawn in Fig. 11.16. On a base of 25 MVA and 11 kV in generator circuit, the positive, negative and zero sequence networks of the system have been drawn already in Figs. 10.23, 10.24 and 10.27. Before the occurrence of a solid LG at bus g, the motors are loaded to draw 15 and 7.5 MW at 10 kV, 0.8 leading power factor. If prefault current is neglected, calculate the fault current and subtransient current in all parts of the system.

What voltage behind subtransient reactances must be used in a positive sequence network if prefault current is to be accounted for?

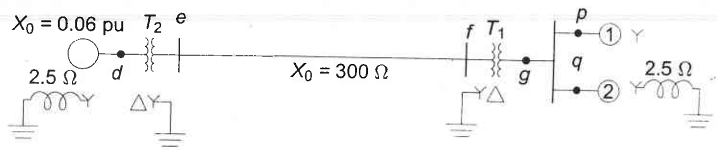


Fig. 11.16 One-line diagram of the system of Example 11.3

Solution The sequence networks given in Figs. 10.23, 10.24 and 10.27 are connected in Fig. 11.17 to simulate a solid LG fault at bus g (*see* Fig. 11.16). If prefault currents are neglected

$$E''_g = E''_{m1} = E''_{m2} = V_f^{\circ} \text{ (prefault voltage at } g)$$

$$= \frac{10}{11} = 0.909 \text{ pu}$$

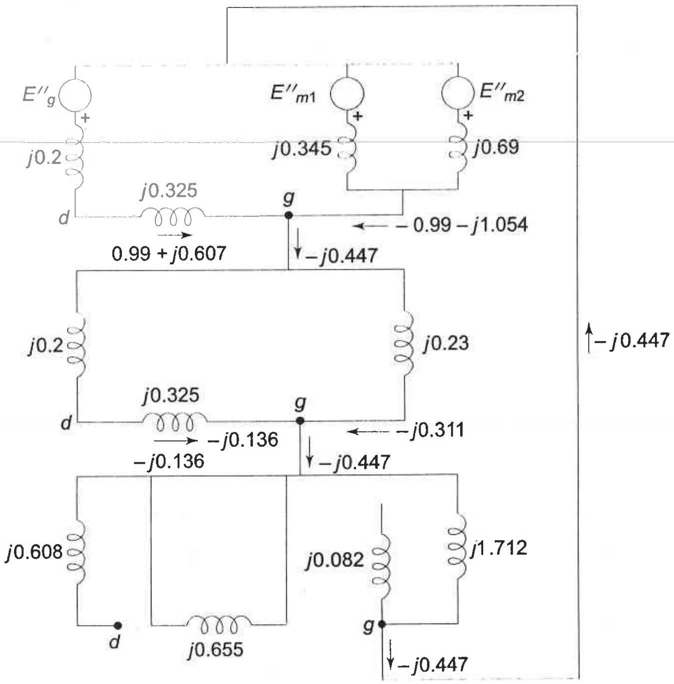


Fig. 11.17 Connection of the sequence networks of Example 11.3. Subtransient currents are shown on the diagram in pu for a solid line-to-ground fault at g

The positive sequence network can now be easily replaced by its Thevenin equivalent as shown in Fig. 11.18.

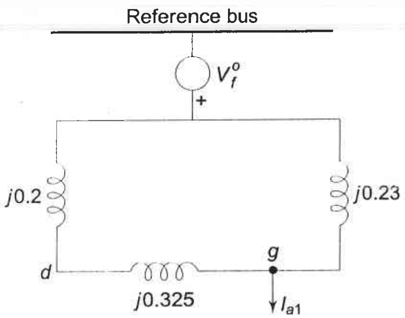


Fig. 11.18

Now

$$Z_1 = \frac{j0.525 \times j0.23}{j0.755} = j0.16 \text{ pu}$$

$$Z_2 = Z_1 = j0.16 \text{ pu}$$

From the sequence network connection

$$I_{a1} = \frac{V_f^o}{Z_1 + Z_2 + Z_0}$$

$$= \frac{0.909}{j2.032} = -j0.447 \text{ pu}$$

$$I_{a2} = I_{a0} = I_{a1} = -j0.447 \text{ pu}$$

$$\text{Fault current} = 3I_{a0} = 3 \times (-j0.447) = -j1.341 \text{ pu}$$

The component of I_{a1} flowing towards g from the generator side is

$$-j0.447 \times \frac{j0.23}{j0.755} = -j0.136 \text{ pu}$$

and its component flowing towards g from the motors side is

$$-j0.447 \times \frac{j0.525}{j0.755} = -j0.311 \text{ pu}$$

Similarly, the component of I_{a2} from the generator side is $-j0.136$ pu and its component from the motors side is $-j0.311$. All of I_{a0} flows towards g from motor 2.

Fault currents from the generator towards g are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \begin{bmatrix} -j0.136 \\ -j0.136 \\ 0 \end{bmatrix} = \begin{bmatrix} -j0.272 \\ j0.136 \\ j0.136 \end{bmatrix} \text{pu}$$

and to g from motors are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \begin{bmatrix} -j0.311 \\ -j0.311 \\ -j0.447 \end{bmatrix} = \begin{bmatrix} -j1.069 \\ -j0.136 \\ -j0.136 \end{bmatrix} \text{pu}$$

The positive and negative sequence components of the transmission line currents are shifted $-90°$ and $+90°$ respectively, from the corresponding components on the generator side of T_2, i.e.

Positive sequence current $= -j(-j0.136) = -0.136$ pu

Negative sequence current $= j(-j0.136) = 0.136$ pu

Zero sequence current $= 0$ ($\because$ there are no zero sequence currents on the transmission line, *see* Fig. 11.17)

$\therefore$ Line a current on the transmission line

$$= -0.136 + 0.136 + 0 = 0$$

I_b and I_c can be similarly calculated.

Let us now calculate the voltages behind subtransient reactances to be used if the load currents are accounted for. The per unit motor currents are:

$$\text{Motor 1: } \frac{15}{25 \times 0.909 \times 0.8} \angle 36.86^\circ = 0.825 \angle 36.86^\circ = 0.66 + j0.495 \text{ pu}$$

$$\text{Motor 2: } \frac{7.5}{25 \times 0.909 \times 0.8} \angle 36.86^\circ = 0.4125 \angle 36.86^\circ = 0.33 + j0.248 \text{ pu}$$

Total current drawn by both motors = $0.99 + j0.743$ pu

The voltages behind subtransient reactances are calculated below:

$$\text{Motor 1: } E''_{m1} = 0.909 - j0.345 \times 0.825\angle 36.86^\circ$$
$$= 1.08 - j0.228 = 1.104\angle -11.92^\circ \text{ pu}$$

$$\text{Motor 2: } E''_{m2} = 0.909 - j0.69 \times 0.4125\angle 36.86^\circ$$
$$= 1.08 - j0.228 = 1.104\angle -11.92^\circ \text{ pu}$$

$$\text{Generator: } E''_{g} = 0.909 + j0.525 \times 1.2375\angle 36.86^\circ$$
$$= 0.52 + j0.52 = 0.735\angle 45^\circ \text{ pu}$$

It may be noted that with these voltages behind subtransient reactances, the Thevenin equivalent circuit will still be the same as that of Fig. 11.18. Therefore, in calculating fault currents taking into account prefault loading condition, we need not calculate E''_{m1}, E''_{m2} and E''_{g}. Using the Thevenin equivalent approach, we can first calculate currents caused by fault to which the load currents can then be added.

Thus, the actual value of positive sequence current from the generator towards the fault is

$$0.99 + -j0.743 - j0.136 = 0.99 + j0.607$$

and the actual value of positive sequence current from the motors to the fault is

$$-0.99 - j0.743 - j0.311 = -0.99 - j1.054$$

In this problem, because of large zero sequence reactance, load current is comparable with (in fact, more than) the fault current. In a large practical system, however, the reverse will be the case, so that it is normal practice to neglect load current without causing an appreciable error.

Example 11.4

For Example 11.2, assume that the grounded generator is solidly grounded. Find the fault current and voltage of the healthy phase for a line-to-line fault on terminals of the generators. Assume solid fault ($Z^f = 0$).

Solution For the *LL* fault, using Eq. (11.17) and substituting the values of X_{1eq} and X_{2eq} from Example 11.2, we get

$$I_{a1} = \frac{E_a}{X_{1eq} + X_{2eq}} = \frac{1}{j0.09 + j0.075} = -j6.06$$

Using Eq. (11.15), we have

$$I_f \text{ (fault current)} = I_b = -j\sqrt{3}I_{a1} = (-j\sqrt{3})(-j6.06) = -10.496$$

Now

$$V_{a1} = V_{a2} = E_a - I_{a1}X_{1eq} = 1.0 - (-j6.06)(j0.09)$$
$$= 0.455$$

$$V_{a0} = -I_{a0}Z_0 = 0 \qquad (\because I_{a0} = 0)$$

Voltage of the healthy phase,

$$V_a = V_{a1} + V_{a2} + V_{a0} = 0.91$$

Example 11.5

For Example 11.2, assume that the grounded generator is solidly grounded. Find the fault current in each phase and voltage of the healthy phase for a double line-to-ground fault on terminals of the generator. Assume solid fault ($Z^f = 0$).

Solution Using Eq. (11.24) and substituting the values of Z_{1eq}, Z_{2eq} and Z_{0eq} from Example 11.2, we get (note $Z^f = 0$, $Z_{0eq} = j0.1$)

$$I_{a1} = \frac{1 + j0}{j0.09 + \dfrac{j0.075 \times j0.10}{j0.075 + j0.10}} = -j7.53$$

$$V_{a1} = V_{a2} = V_{a0} = E_a - I_{a1}Z_{1eq} = 1 - (-j7.53)(j0.09)$$
$$= 0.323$$

$$I_{a2} = -\frac{V_{a2}}{Z_{2eq}} = -\frac{0.323}{j0.075} = j4.306$$

$$I_{a0} = -\frac{V_{a0}}{Z_{0eq}} = -\frac{0.323}{j0.10} = j3.23$$

Now

$$I_b = \alpha^2 I_{a1} + \alpha I_{a2} + I_{a0}$$
$$= (-0.5 - j0.866)(-j7.53) + (-0.5 + j0.866)(j4.306) + j3.23$$
$$= -10.248 + j4.842 = 11.334 \angle 154.74^\circ$$

$$I_c = \alpha I_{a1} + \alpha^2 I_{a2} + I_{a0}$$
$$= (-0.5 + j0.866)(-j7.53) + (-0.5 - j0.866)(j4.306) + j3.23$$

$$= 10.248 + j4.842 = 11.334\angle 25.28°$$

Voltage of the healthy phase,

$$V_a = 3V_{a1} = 3 \times 0.323 = 0.969$$

11.6 OPEN CONDUCTOR FAULTS

An open conductor fault is in series with the line. Line currents and series voltages between broken ends of the conductors are required to be determined.

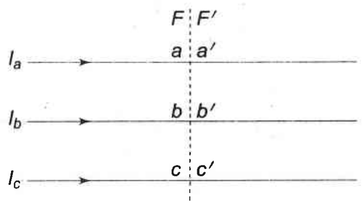


Fig. 11.19 Currents and voltages in open conductor fault

Figure 11.19 shows currents and voltages in an open conductor fault. The ends of the system on the sides of the fault are identified as F, F', while the conductor ends are identified as aa', bb' and cc'. The set of series currents and voltages at the fault are

$$I_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}; \; V_p = \begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix}$$

The symmetrical components of currents and voltages are

$$I_s = \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix}; \; V_s = \begin{bmatrix} V_{aa'1} \\ V_{aa'2} \\ V_{aa'0} \end{bmatrix}$$

The sequence networks can be drawn for the power system as seen from FF' and are schematically shown in Fig. 11.20. These are to be suitably connected depending on the type of fault (one or two conductors open).

Two Conductors Open

Figure 11.21 represents the fault at FF' with conductors b and c open. The currents and voltages due to this fault are expressed as

$$V_{aa'} = 0 \tag{11.27}$$

$$I_b = I_c = 0 \tag{11.28}$$

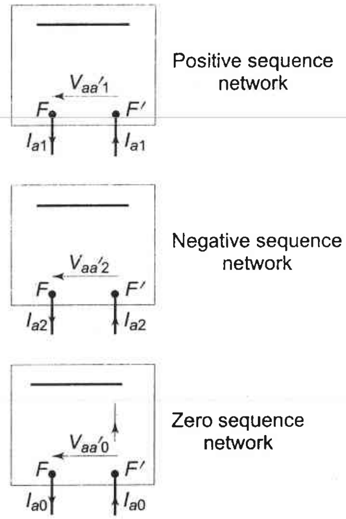


Fig. 11.20 Sequence networks for open conductor fault at FF'

In terms of symmetrical components, we can write

$$V_{aa'1} + V_{aa'2} + V_{aa'0} = 0 \tag{11.29}$$

$$I_{a1} = I_{a2} = I_{a0} = \frac{1}{3} I_a \tag{11.30}$$

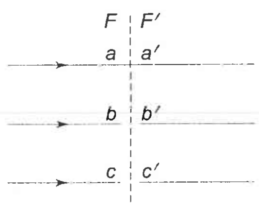


Fig 11.21 Two conductors open

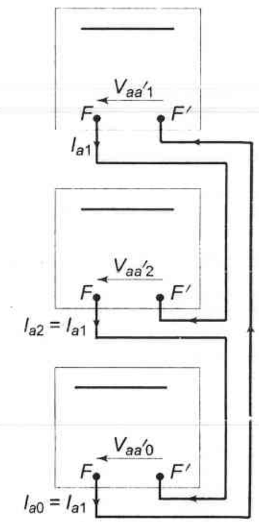


Fig. 11.22 Connection of sequence networks for two conductors open

Equations (11.29) and (11.30) suggest a series connection of sequence networks as shown in Fig. 11.22. Sequence currents and voltages can now be computed.

One Conductor Open

For one conductor open as in Fig. 11.23, the circuit conditions require

$$V_{bb'} = V_{cc'} = 0 \tag{11.31}$$

$$I_a = 0 \tag{11.32}$$

In terms of symmetrical components these conditions can be expressed as

$$V_{aa'1} = V_{aa'2} = V_{aa'0} = \frac{1}{3} V_{aa'} \tag{11.33}$$

$$I_{a1} + I_{a2} + I_{a0} = 0 \tag{11.34}$$

Equations (11.33) and (11.34) suggest a parallel connection of sequence networks as shown in Fig. 11.24.

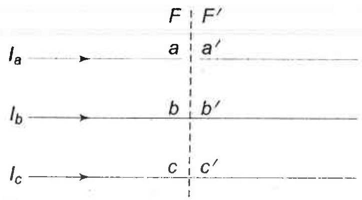


Fig. 11.23 One conductor open

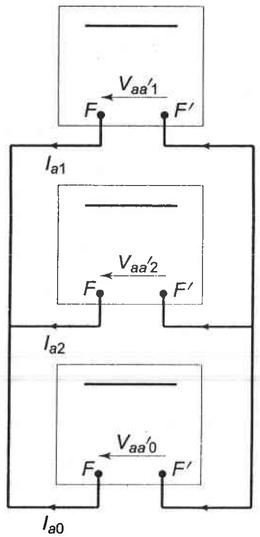


Fig. 11.24 Connection of sequence networks for one conductor open

11.7 BUS IMPEDANCE MATRIX METHOD FOR ANALYSIS OF UNSYMMETRICAL SHUNT FAULTS

Bus impedance method of fault analysis, given for symmetrical faults in Chapter 9, can be easily extended to the case of unsymmetrical faults. Consider for example an LG fault on the rth bus of a n-bus system. The connection of sequence networks to simulate the fault is shown in Fig. 11.25. The positive sequence network has been replaced here by its Thevenin equivalent, i.e.

prefault voltage V^{o}_{1-r} of bus r in series with the passive positive sequence network (all voltage sources short circuited). Since negative and zero sequence prefault voltages are zero, both these are passive networks only.

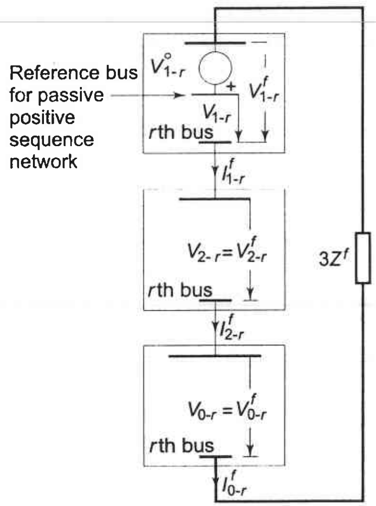


Fig. 11.25 Connection of sequence networks for LG fault on the rth bus (positive sequence network represented by its Thevenin equivalent)

It may be noted that subscript a has been dropped in sequence currents and voltages, while integer subscript is introduced for bus identification. Superscripts o and f respectively, indicate prefault and postfault values.

For the passive positive sequence network

$$\mathbf{V}_{1\text{-BUS}} = \mathbf{Z}_{1\text{-BUS}}\, \mathbf{J}_{1\text{-BUS}} \tag{11.35}$$

where

$$\mathbf{V}_{1\text{-BUS}} = \begin{bmatrix} V_{1-1} \\ V_{1-2} \\ \vdots \\ V_{1-n} \end{bmatrix} = \text{positive sequence bus voltage vector} \tag{11.36}$$

$$\mathbf{Z}_{1\text{-BUS}} = \begin{bmatrix} Z_{1-11} & \cdots & Z_{1-1n} \\ \vdots & & \vdots \\ Z_{1-n1} & \cdots & Z_{1-nn} \end{bmatrix} = \text{positive sequence bus impedance matrix} \tag{11.37}$$

and

$$\mathbf{J}_{1\text{-BUS}} = \begin{bmatrix} J_{1-1} \\ J_{1-2} \\ \vdots \end{bmatrix} = \text{positive sequence bus current injection vector} \tag{11.38}$$

As per the sequence network connection, current $-I^f_{1-r}$ is injected only at the faulted rth bus of the positive sequence network, we have therefore

$$\mathbf{J}_{1\text{-BUS}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I^f_{1-r} \\ \vdots \\ 0 \end{bmatrix} \quad (11.39)$$

Substituting Eq. (11.39) in Eq. (11.35), we can write the positive sequence voltage at the rth bus of the passive positive sequence network as

$$\mathbf{V}_{1-r} = -\ \mathbf{Z}_{1-rr}\mathbf{I}^f_{1-r} \quad (11.40)$$

Thus the passive positive sequence network presents an impedance $\mathbf{Z}_{1-rr}$ to the positive sequence current $\mathbf{I}^f_{1-r}$.

For the negative sequence network

$$\mathbf{V}_{2\text{-BUS}} = \mathbf{Z}_{2\text{-BUS}}\ \mathbf{J}_{2\text{-BUS}} \quad (11.41)$$

The negative sequence network is injected with current $\mathbf{I}^f_{2-r}$ at the rth bus only. Therefore,

$$\mathbf{J}_{2\text{-BUS}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I^f_{2-r} \\ \vdots \\ 0 \end{bmatrix} \quad (11.42)$$

The negative sequence voltage at the rth bus is then given by

$$\mathbf{V}_{2-r} = -\ \mathbf{Z}_{2-rr}\mathbf{I}^f_{2-r} \quad (11.43)$$

Thus, the negative sequence network offers an impedance $\mathbf{Z}_{2-rr}$ to the negative sequence current $\mathbf{I}^f_{2-r}$.

Similarly, for the zero sequence network

$$\mathbf{V}_{0\text{-BUS}} = \mathbf{Z}_{0\text{-BUS}}\ \mathbf{J}_{0\text{-BUS}} \quad (11.44)$$

$$\mathbf{J}_{0\text{-BUS}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I^f_{0-r} \\ \vdots \\ 0 \end{bmatrix} \quad (11.45)$$

and

$$\mathbf{V}_{0-r} = -\ \mathbf{Z}_{0-rr}\mathbf{I}^f_{0-rr} \quad (11.46)$$

That is, the zero sequence network offers an impedance $\mathbf{Z}_{0-rr}$ to the zero sequence current $\mathbf{I}^f_{0-r}$.

From the sequence network connection of Fig. 11.25, we can now write

$$I^f_{1-r} = I^f_{2-r} = I^f_{0-r} = \frac{V^0_{1-r}}{Z_{1-rr} + Z_{2-rr} + Z_{0-rr} + 3Z^f} \quad (11.47)$$

Sequence currents for other types of faults can be similarly computed using Z_{1-rr}, Z_{2-rr} and Z_{0-rr} in place of Z_1, Z_2 and Z_0 in Eqs. (11.7), (11.17) and (11.24) with $E_a = V^o_{1-r}$.

Postfault sequence voltages at any bus can now be computed by superposing on prefault bus voltage, the voltage developed owing to the injection of appropriate sequence current at bus r.

For passive positive sequence network, the voltage developed at bus i owing to the injection of $-\ I^f_{1-r}$ at bus r is

$$V_{1-i} = -\ Z_{1-ir}I^f_{1-r} \quad (11.48)$$

Hence postfault positive sequence voltage at bus i is given by

$$V^f_{1-i} = V^o_{1-i} - Z_{1-ir}I^f_{1-r};\ i = 1, 2, \ldots, n \quad (11.49)$$

where

V^o_{1-i} = prefault positive sequence voltage at bus i

Z_{1-ir} = irth component of $Z_{1-\text{BUS}}$

Since the prefault negative sequence bus voltages are zero, the postfault negative sequence bus voltages are given by

$$\begin{aligned} V^f_{2-i} &= 0 + V_{2-i} \\ &= -\ Z_{2-ir}I^f_{2-r} \end{aligned} \quad (11.50)$$

where

Z_{2-ir} = irth component of $Z_{2-\text{BUS}}$

Similarly, the postfault zero sequence bus voltages are given by

$$V^f_{0-i} = -\ Z_{0-ir}I^f_{0-r};\ i = 1, 2, \ldots, n \quad (11.51)$$

where

Z_{0-ir} = irth component of $Z_{0-\text{BUS}}$

With postfault sequence voltages known at the buses, sequence currents in lines can be computed as:

For line uv, having sequence admittances y_{1-uv}, y_{2-uv} and y_{0-uv}

$$\begin{aligned} I^f_{1-uv} &= y_{1-uv}\ (V^f_{1-u} - V^f_{1-v}) \\ I^f_{2-uv} &= y_{2-uv}\ (V^f_{2-u} - V^f_{2-v}) \\ I^f_{0-uv} &= y_{0-uv}\ (V^f_{0-u} - V^f_{0-v}) \end{aligned} \quad (11.52)$$

Knowing sequence voltages and currents, phase voltages and currents can be easily computed by the use of the symmetrical component transformation

$$\mathbf{V}_p = \mathbf{A}\mathbf{V}_s$$

$$\mathbf{I}_p = \mathbf{A}\mathbf{I}_s$$

It appears at first, as if this method is more laborious than computing fault currents from Thevenin impedances of the sequence networks, as it requires computation of bus impedance matrices of all the three sequence networks. It must, however, be pointed out here that once the bus impedance matrices have been assembled, fault analysis can be conveniently carried out for all the buses, which, in fact, is the aim of a fault study. Moreover, bus impedance matrices can be easily modified to account for changes in power network configuration.

Example 11.6

For Example 10.3, positive, negative and zero sequence networks have been drawn in Figs. 10.23, 10.24 and 10.27. Using the bus impedance method of fault analysis, find fault currents for a solid LG fault at (i) bus e and (ii) bus f. Also find bus voltages and line currents in case (i). Assume the prefault currents to be zero and the prefault voltages to be 1 pu.

Solution Figure 11.26 shows the connection of the sequence networks of Figs. 10.23, 10.24 and 10.27 for a solid LG fault at bus e.

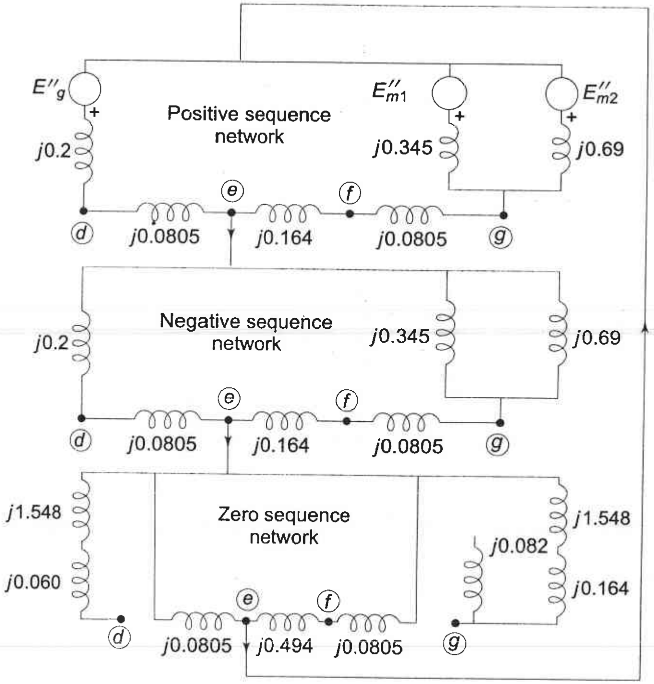


Fig. 11.26 Connection of the sequence networks of Example 11.6 for an LG fault at bus e

Refer to Fig. 11.26 to find the elements of the bus admittance matrices of the three sequence networks, as follows:

$$Y_{1-dd} = \frac{1}{j0.2} + \frac{1}{j0.0805} = -\,j17.422$$

$$Y_{1-fg} = Y_{1-de} = \frac{-1}{j0.0805} = j12.422$$

$$Y_{1-ff} = Y_{1-ee} = \frac{1}{j0.0805} + \frac{1}{j0.164} = -j18.519$$

$$Y_{1-ef} = \frac{-1}{j0.164} = j6.097$$

$$Y_{1-gg} = \frac{1}{j0.085} + \frac{1}{j0.345} + \frac{1}{j0.69} = -\,j16.769$$

$$Y_{1\text{-BUS}} = Y_{2\text{-BUS}} = j\;\begin{array}{c c} & \begin{array}{cccc} d & e & f & g \end{array} \\ \begin{array}{c} d \\ e \\ f \\ g \end{array} & \begin{bmatrix} -17.422 & 12.422 & 0 & 0 \\ 12.422 & -18.519 & 6.097 & 0 \\ 0 & 6.097 & -18.519 & 12.422 \\ 0 & 0 & 12.422 & -16.769 \end{bmatrix} \end{array}$$

$$Y_{0-dd} = \frac{1}{j1.608} = -\,j0.621$$

$$Y_{0-ee} = Y_{0-ff} = \frac{1}{j0.0805} + \frac{1}{j0.494} = -\,j14.446$$

$$Y_{0-gg} = \frac{1}{j1.712} = -\,j0.584$$

$$Y_{0-de} = 0.0$$

$$Y_{0-ef} = \frac{-1}{j0.494} = j2.024$$

$$Y_{0-fg} = 0.0$$

$$Y_{0\text{-BUS}} = j\;\begin{array}{c c} & \begin{array}{cccc} d & e & f & g \end{array} \\ \begin{array}{c} d \\ e \\ f \\ g \end{array} & \begin{bmatrix} -0.621 & 0 & 0 & 0 \\ 0 & -14.446 & 2.024 & 0 \\ 0 & 2.024 & -14.446 & 0 \\ 0 & 0 & 0 & -0.584 \end{bmatrix} \end{array}$$

Inverting the three matrices above renders the following three bus impedance matrices

$$Z_{1\text{-BUS}} = Z_{2\text{-BUS}} = j\begin{bmatrix} 0.14706 & 0.12575 & 0.08233 & 0.06102 \\ 0.12575 & 0.17636 & 0.11547 & 0.08558 \\ 0.08233 & 0.11547 & 0.18299 & 0.13563 \\ 0.06102 & 0.08558 & 0.13563 & 0.16019 \end{bmatrix}$$

$$Z_{0\text{-BUS}} = j\begin{bmatrix} 1.61031 & 0 & 0 & 0 \\ 0 & 0.07061 & 0.00989 & 0 \\ 0 & 0.00989 & 0.07061 & 0 \\ 0 & 0 & 0 & 1.71233 \end{bmatrix}$$

The fault current with LG fault on bus e is

$$I_e^f = \frac{3\times 1}{j0.17636 + j0.17636 + j0.07061} = -j7.086 \text{ pu} \qquad \text{(i)}$$

The fault current with LG fault on bus f is

$$I_f^f = \frac{3\times 1}{j0.18299 + j0.18299 + j0.07061} = \frac{3}{j0.43659}$$

$$= -j6.871 \text{ pu} \qquad \text{(ii)}$$

Bus voltages and line currents in case (i) can easily be computed using Eqs. (11.49)–(11.52). Given below is a sample calculation for computing voltage at bus f and current in line ef.

From Eq. (11.49)

$$V^f_{1-d} = V^o_{1-d} - Z_{1-de} - I^f_{1-e}$$

$$= 1.0 - j0.12575\left(-j\frac{7.086}{3}\right) = 0.703 \text{ pu}$$

$$V^f_{1-f} = V^o_{1-f} - Z_{1-fe} - I^f_{1-e}$$

$$= 1.0 - j0.11547\left(-j\frac{7.086}{3}\right) = 0.728 \text{ pu}$$

$$V^f_{1-e} = V^o_{1-e} - Z_{1-ee} - I^f_{1-e}$$

$$= 1.0 - j0.17638\ (-j2.363) = 0.584 \text{ pu}$$

$$V^f_{1-g} = V^o_{1-g} - Z_{1-ge} - I^f_{1-e}$$

$$= 1.0 - j0.08558\ (-j2.363) = 0.798 \text{ pu}$$

$$V^f_{2-f} = -Z_{2-fe} I^f_{2-e}$$

$$= -j0.11547 \times (-j2.362) = -0.272 \text{ pu}$$

$$V^f_{0-f} = -Z_{0-fe} I^f_{0-e} = -j0.00989 \times (-j2.362)$$

$$= -0.023 \text{ pu}$$

$$V^f_{2-e} = -Z_{2-ee} I^f_{2-e} = -j0.17636 \times (-j2.362)$$

$$= -0.417 \text{ pu}$$

$$V^f_{0-e} = -Z_{0-ee} I^f_{0-e} = -j0.0706 \times (-j2.362)$$

$$= -0.167 \text{ pu}$$

$$V^f_{2-g} = -Z_{2-ge} I^f_{2-e} = -j0.08558 \times (-j2.362)$$

$$= -0.202 \text{ pu}$$

$$V^f_{0-g} = -Z_{0-ge} I^f_{0-e} = 0$$

Using Eq. (11.52), the currents in various parts of Fig. 11.26 can be computed as follows:

$$I^f_{1-fe} = Y_{1-fe}\ (V^f_{1-f} - V^f_{1-fe})$$

$$= -j6.097\ (0.728 - 0.584)$$

$$= -j0.88$$

$$I^f_{1-de} = Y_{1-de}\ (V^f_{1-d} - V^f_{1-e})$$

$$= -j12.422\ (0.703 - 0.584) = -j1.482$$

$$\therefore \quad I_{a1} = I^f_{1-fe} + I^f_{1-de} = -j0.88 + (-j1.482)$$

$$= -j2.362$$

which is the same as obtained earlier [see Eq. (i)] where $I^f_e = 3I_{a1}$.

$$I^f_{1-gf} = Y_{1-gf}\ (V^f_{1-g} - V^f_{1-f})$$

$$= j12.422\ (-0.798 - 0.728) = -j0.88$$

Notice that as per Fig. 11.26, it was required to be the same as I^f_{1-fe}.

$$I_{2-fe} = Y_{2-fe}\ (V^f_{2-f} - V^f_{2-e})$$

$$= -j6.097\ (-0.272 + 0.417) = -j0.884$$

$$I^f_{0-fe} = \gamma_{0-fe}\ (V^f_{0-f} - V^f_{0-e})$$

$$= -j2.024\ (-0.023 + 0.167) = -j0.291 \text{ pu}$$

$$\therefore \quad I^f_{fe}\ (a) = I^f_{1-fe} + I^f_{2-fe} + I^f_{0-fe}$$

$$= -j0.88 + (-j0.88) + (-j0.291)$$

$$= -j2.05$$

Similarly, other currents can be computed.

Example 11.7

A single line to ground fault (on phase a) occurs on the bus 1 of the system of Fig. 11.27. Find

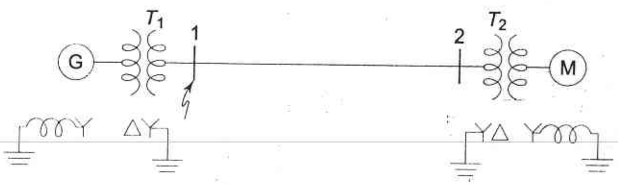


Fig. 11.27

(a) Current in the fault.
(b) SC current on the transmission line in all the three phases.
(c) SC current in phase a of the generator.
(d) Voltage of the healthy phases of the bus 1.

Given: Rating of each machine 1200 kVA, 600 V with $X' = X_2 = 10\%$, $X_0 = 5\%$. Each three-phase transformer is rated 1200 kVA, 600 V – Δ/3300 V–Y with leakage reactance of 5%. The reactances of the transmission line are $X_1 = X_2 = 20\%$ and $X_0 = 40\%$ on a base of 1200 kVA, 3300 V. The reactances of the neutral grounding reactors are 5% on the kVA and voltage base of the machine.

Note: Use Z_{BUS} method.

Solution Figure 11.28 shows the passive positive sequence network of the system of Fig.11.27. This also represents the negative sequence network for the system. Bus impedance matrices are computed below:

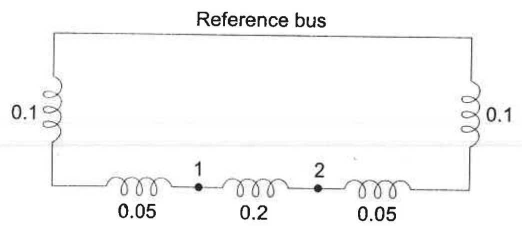


Fig. 11.28

Bus 1 to reference bus

$$Z_{1\text{-BUS}} = j[0.15]$$

Bus 2 to Bus 1

$$Z_{1\text{-BUS}} = j\begin{bmatrix} 0.15 & 0.15 \\ 0.15 & 0.35 \end{bmatrix}$$

Bus 2 to reference bus

$$Z_{1\text{-BUS}} = j\begin{bmatrix} 0.15 & 0.15 \\ 0.15 & 0.35 \end{bmatrix} - \frac{j}{0.35+0.15}\begin{bmatrix} 0.15 \\ 0.35 \end{bmatrix}[0.15 \quad 0.35]$$

or

$$Z_{1\text{-BUS}} = j\begin{bmatrix} 0.105 & 0.045 \\ 0.045 & 0.105 \end{bmatrix} = Z_{2\text{-BUS}} \qquad \text{(i)}$$

Zero sequence network of the system is drawn in Fig. 11.29 and its bus impedance matrix is computed below.

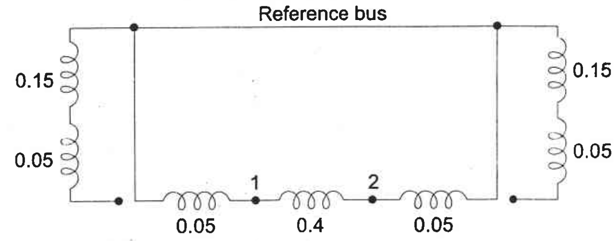


Fig. 11.29

Bus 1 to reference bus

$$Z_{0\text{-BUS}} = j\,[0.05]$$

Bus 2 to bus 1

$$Z_{0\text{-BUS}} = j\begin{bmatrix} 0.05 & 0.05 \\ 0.05 & 0.45 \end{bmatrix}$$

Bus 2 to reference bus

$$Z_{0\text{-BUS}} = j\begin{bmatrix} 0.05 & 0.05 \\ 0.05 & 0.45 \end{bmatrix} - \frac{j}{0.45+0.05}\begin{bmatrix} 0.05 \\ 0.45 \end{bmatrix}[0.05 \quad 0.45]$$

or

$$Z_{0\text{-BUS}} = j\frac{1}{2}\begin{bmatrix} 0.045 & 0.005 \\ 0.005 & 0.045 \end{bmatrix} \qquad \text{(ii)}$$

As per Eq. (11.47)

$$I^f_{1\text{-}1} = \frac{V^0_1}{Z_{1\text{-}11} + Z_{2\text{-}11} + Z_{0\text{-}11} + 3Z^f}$$

But $V^o_1 = 1$ pu (system unloaded before fault)

Then

$$I^f_{1\text{-}1} = \frac{-j1.0}{0.105 + 0.105 + 0.045} = -\,j3.92 \text{ pu}$$

$$I^f_{1\text{-}1} = I^f_{2\text{-}1} = I^f \quad \text{[illegible]2 pu}$$

(a) Fault current, $I^f_1 = 3I^f_{1\text{-}1} =$

(b) $V^f_{1\text{-}1} = V^o_{1\text{-}1} = Z_{1\text{-}11}\, I^f_{1\text{-}1}$

$= 1.0 - j0.105 \times -\,j3.92 = 0.588;\; V^o_{1\text{-}1} = 1$ pu

$V^f_{1-2} = V^o_{1-2} - Z_{1-21} I^f_{2-1}$; $V^o_{1-2} = 1.0$ (system unloaded before fault)

$= 1.0 - j0.045 \times - j3.92 = 0.824$

$V^f_{2-1} = - Z_{2-11} I^f_{2-1}$

$= - j0.105 \times - j3.92 = 0.412$

$V^f_{2-2} = - Z_{2-21}\, I^f_{2-1}$

$= - j0.045 \times - j3.92 = - 0.176$

$V^f_{0-1} = - Z_{0-11} I^f_{0-1}$

$= - j0.045 \times - j3.92 = - 0.176$

$V^f_{0-2} = - Z_{0-21}\, I^f_{0-1}$

$= - j0.005 \times - j3.92 = - 0.02$

$I^f_{1-12} = y_{1-12}\,(V^f_{1-1} - V^f_{1-3})$

$$= \frac{1}{j0.2}\,(0.588 - 0.824) = j1.18$$

$I^f_{2-12} = y_{2-12}\,(V^f_{2-1} - V^f_{2-2})$

$$= \frac{1}{j0.2}(- 0.412 + 0.176) = j1.18$$

$I^f_{0-12} = y_{0-12}\,(V^f_{0-1} - V^f_{0-2})$

$$= \frac{1}{j0.4}\,(- 0.176 + 0.020) = j0.39$$

$$\begin{bmatrix} I^f_{a-12} \\ I^f_{b-12} \\ I^f_{c-12} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \begin{bmatrix} I^f_{1-12} \\ I^f_{2-12} \\ I^f_{0-12} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \begin{bmatrix} j1.18 \\ j1.18 \\ j0.39 \end{bmatrix}$$

$I^f_{a-12} = j1.18 + j1.18 + j0.39 = j2.75$

$I^f_{b-12} = j1.18\ \angle 240° + j1.18\ \angle 120° + j0.39$

$= - j079$

$I^f_{c-12} = j1.18\ \angle 120° + j1.18\ \angle 240° + j0.39$

$= j0.79$

(c) $I^f_{1-G} = \dfrac{1}{j0.15}\ (1 - 0.588)\ \angle -33°$

$= - 1.37 - j2.38$

$I^f_{0-G} = \dfrac{1}{j0.15}\ [0 - (- 0.412)]\ \angle 30°$

$= 1.37 - j2.38$

$I^f_{0-G} = 0$ (*see* Fig. 11.29)

$\therefore \quad I^f_{a-G} = (-1.37 - j2.38) + (1.37 - j2.38)$

$= - j4.76$

Current in phases b and c of the generator can be similarly calculated.

(d) $V^f_{b-1} = 2V^f_{1-1} + V^f_{2-1} + V^f_{0-1}$

$= 0.588\ \angle 240° - 0.412\ \angle 120° - 0.176$

$= - 0.264 - j0.866 = 0.905\ \angle -107°$

$V^f_{c-1} = V^f_{1-1} + V^f_{2-1} + V^f_{0-1}$

$= 0.588\ \angle 120° - 0.412\ \angle 240° - 0.176$

$= - 0.264 + j0.866 = 0.905\ \angle 107°$

PROBLEMS

11.1 A 25 MVA, 11 kV generator has a $X''_d = 0.2$ pu. Its negative and zero sequence reactances are respectively 0.3 and 0.1 pu. The neutral of the generator is solidly grounded. Determine the subtransient current in the generator and the line-to-line voltages for subtransient conditions when an LG fault occurs at the generator terminals. Assume that before the occurrence of the fault, the generator is operating at no load at rated voltage. Ignore resistances.

11.2 Repeat Problem 11.1 for (a) an LL fault; and (b) an LLG fault.

11.3 A synchronous generator is rated 25 MVA, 11 kV. It is star-connected with the neutral point solidly grounded. The generator is operating at no load at rated voltage. Its reactances are $X'' = X_2 = 0.20$ and $X_0 = 0.08$ pu. Calculate the symmetrical subtransient line currents for (i) single line-to-ground fault; (ii) double line fault; (iii) double line-to-ground fault; and (iv) symmetrical three-phase fault. Compare these currents and comment.

11.4 For the generator of Problem 11.3, calculate the value of reactance to be included in the generator neutral and ground, so that line-to-ground fault current equals the three-phase fault current. What will be the value of the grounding resistance to achieve the same condition?

With the reactance value (as calculated above) included between neutral and ground, calculate the double line fault current and also double line-to-ground fault current.

11.5 Two 25 MVA, 11 kV synchronous generators are connected to a common bus bar which supplies a feeder. The star point of one of the generators is grounded through a resistance of 1.0 ohm, while that of the other generator is isolated. A line-to-ground fault occurs at the far end of the feeder. Determine: (a) the fault current; (b) the voltage to ground of the sound phases of the feeder at the fault point; and (c) voltage of the star point of the grounded generator with respect to ground.

The impedances to sequence currents of each generator and feeder are given below:

	Generator (per unit)	Feeder (ohms/phase)
Positive sequence	$j0.2$	$j0.4$
Negative sequence	$j0.15$	$j0.4$
Zero sequence	$j0.08$	$j0.8$

11.6 Determine the fault currents in each phase following a double line-to-ground short circuit at the terminals of a star-connected synchronous generator operating initially on an open circuit voltage of 1.0 pu. The positive, negative and zero sequence reactance of the generator are, respectively, $j0.35$, $j0.25$ and $j0.20$, and its star point is isolated from ground.

11.7 A three-phase synchronous generator has positive, negative and zero sequence reactances per phase respectively, of 1.0, 0.8 and 0.4 ohm. The winding resistances are negligible. The phase sequence of the generator is RYB with a no load voltage of 11 kV between lines. A short circuit occurs between lines Y and B and earth at the generator terminals.

Calculate sequence currents in phase R and current in the earth return circuit, (a) if the generator neutral is solidly earthed; and (b) if the generator neutral is isolated.

Use R phase voltage as reference.

11.8 A generator supplies a group of identical motors as shown in Fig. P-11.8. The motors are rated 600 V, 90% efficiency at full load unity power factor with sum of their output ratings being 5 MW. The motors are sharing equally a load of 4 MW at rated voltage, 0.8 power factor lagging and 90% efficiency when an LG fault occurs on the low voltage side of the transformer.

Specify completely the sequence networks to simulate the fault so as to include the effect of prefault current. The group of motors can be treated as a single equivalent motor.

Find the subtransient line currents in all parts of the system with prefault current ignored.

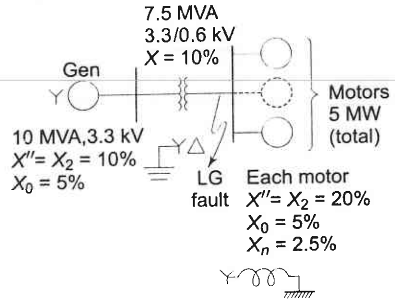


Fig. P-11.8

11.9 A double line-to-ground fault occurs on lines b and c at point F in the system of Fig. P-11.9. Find the subtransient current in phase c of machine 1, assuming prefault currents to be zero. Both machines are rated 1,200 kVA, 600 V with reactances of $X'' = X_2 = 10\%$ and $X_0 = 5\%$. Each three-phase transformer is rated 1,200 kVA, 600 V–Δ/3,300 V–Y with leakage reactance of 5%. The reactances of the transmission line are $X_1 = X_2 = 20\%$ and $X_0 = 40\%$ on a base of 1,200 kVA, 3,300 V. The reactances of the neutral grounding reactors are 5% on the kVA base of the machines.

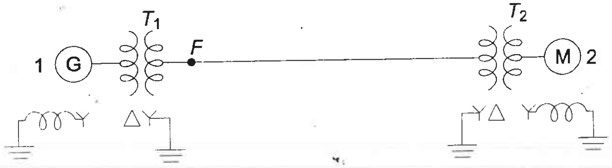


Fig. P-11.9

11.10 A synchronous machine 1 generating 1 pu voltage is connected through a Y/Y transformer of reactance 0.1 pu to two transmission lines in parallel. The other ends of the lines are connected through a Y/Y transformer of reactance 0.1 pu to a machine 2 generating 1 pu voltage. For both transformers $X_1 = X_2 = X_0$.

Calculate the current fed into a double line-to-ground fault on the line side terminals of the transformer fed from machine 2. The star point of machine 1 and of the two transformers are solidly grounded. The reactances of the machines and lines referred to a common base are

	X_1	X_2	X_0
Machine 1	0.35	0.25	0.05
Machine 2	0.30	0.20	0.04
Line (each)	0.40	0.40	0.80

11.11 Figure P-11.11 shows a power network with two generators connected in parallel to a transformer feeding a transmission line. The far end of the line is connected to an infinite bus through another transformer. Star point of each transformer, generator 1 and infinite bus are solidly grounded. The positive, negative and zero sequence reactances of various components in per unit on a common base are:

	Positive	*Negative*	*Zero*
Generator 1	0.15	0.15	0.08
Generator 2	0.25	0.25	∞ (i.e. neutral isolated)
Each transformer	0.15	0.15	0.15
Infinite bus	0.15	0.15	0.05
Line	0.20	0.20	0.40

(a) Draw the sequence networks of the power system.

(b) With both generators and infinite bus operating at 1.0 pu voltage on no load, a line-to-ground fault occurs at one of the terminals of the star-connected winding of the transformer *A*. Calculate the currents flowing (i) in the fault; and (ii) through the transformer *A*.

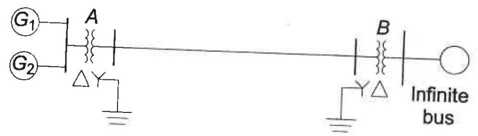


Fig. P-11.11

11.12 A star connected synchronous generator feeds bus bar 1 of a power system. Bus bar 1 is connected to bus bar 2 through a star/delta transformer in series with a transmission line. The power network connected to bus bar 2 can be equivalently represented by a star-connected generator with equal positive and negative sequences reactances. All star points are solidly connected to ground. The per unit sequence reactances of various components are given below:

	Positive	*Negative*	*Zero*
Generator	0.20	0.15	0.05
Transformer	0.12	0.12	0.12
Transmission Line	0.30	0.30	0.50
Power Network	X	X	0.10

Under no load condition with 1.0 pu voltage at each bus bar, a current of 4.0 pu is fed to a three-phase short circuit on bus bar 2. Determine the positive sequence reactance X of the equivalent generator of the power network.

For the same initial conditions, find the fault current for single line-to-ground fault on bus bar 1.

11.13 The reactance data for the three-phase system of Fig. P-11.13 is:

Generator: $X_1 = X_2 = 0.1$ pu; $X_0 = 0.05$ pu

X_g (grounding reactance) = 0.02 pu

Transformer: $X_1 = X_2 = X_0 = 0.1$ pu

X_g (grounding reactance) = 0.04 pu

Form the positive, negative and zero sequence bus impedance matrices. For a solid LG fault at bus 1, calculate the fault current and its contributions from the generator and transformer.

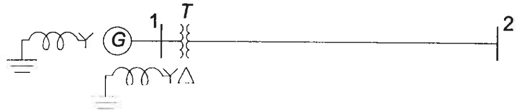


Fig. P-11.13

Hint: Notice that the line reactances are not given. Therefore it is convenient to obtain $Z_{1,\,\mathrm{BUS}}$ directly rather than by inverting $Y_{1,\,\mathrm{BUS}}$. Also $Y_{0,\,\mathrm{BUS}}$ is singular and $Z_{0,\,\mathrm{BUS}}$ cannot be obtained from it. In such situations the method of unit current injection outlined below can be used.

For a two-bus case

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix},$$

Injecting unit current at bus 1 (i.e. $I_1 = 1$, $I_2 = 0$), we get

$$Z_{11} = V_1$$

$$Z_{21} = V_2$$

Similarly injecting unit current at bus 2 (i.e. $I_1 = 0$, $I_2 = 1$), we get

$$Z_{12} = V_1$$

$$Z_{22} = V_2$$

Z_{BUS} could thus be directly obtained by this technique.

11.14 Consider the 2-bus system of Example 11.3. Assume that a solid LL fault occurs on bus *f*. Determine the fault current and voltage (to ground) of the healthy phase.

11.15 Write a computer programme to be employed for studying a solid LG fault on bus 2 of the system shown in Fig. 9.17. Our aim is to find the fault current and all bus voltages and the line currents following the fault. Use the impedance data given in Example 9.5. Assume all transformers to be *Y/Δ* type with their neutrals (on HV side) solidly grounded.

Assume that the positive and negative sequence reactances of the generators are equal, while their zero sequence reactance is one-fourth of their positive sequence reactance. The zero sequence reactances of the lines are to be taken as 2.5 times their positive sequence reactances. Set all prefault voltages = 1 pu.

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