# CONTENTS

# To the Student

This book contains precisely 2500 completely solved problems in the areas of fluid mechanics and hydraulics. Virtually all types of problems ordinarily encountered in study and practice in these areas are covered. Not only you, but teachers, practitioners, and graduates reviewing for engineering licensing examinations should find these problems valuable.

To acquaint you with our "approach," particular steps taken in presenting the problems and their solutions are itemized below.

- First and most important of all, each problem and its solution are essentially independent and self-contained. That is to say, each contains all the data, equations, and computations necessary to find the answers. Thus, you should be able to pick a problem anywhere and follow its solution without having to review whatever precedes it. The exception to this is the occasional problem that specifically refers to, and carries over information from, a previous problem.
- In the solutions, our objective has been to present any needed equation first and then clearly to evaluate each term in the equation in order to find the answer. The terms may be evaluated separately or within the equation itself. For example, when solving an equation that has the parameter "area" as one of its terms, the area term ($A$) may be evaluated separately and its value substituted into the equation [as in Prob. 14.209], or it may be evaluated within the equation itself [as in Prob. 14.94].
- Virtually every number appearing in a solution is either "given" information (appearing as data in the statement of the problem or on an accompanying illustration), a previously computed value within the problem, a conversion factor (obtainable from the List of Conversion Factors), or a physical property (obtainable from a table or illustration in the Appendix). For example, in Prob. 1.77, the number 1.49, which does not appear elsewhere in the problem, is the dynamic viscosity ($\mu$) of glycerin; it was obtained from Table A-3 in the Appendix.
- We have tried to include all but the most familiar items in the List of Abbreviations and Symbols. Hence, when an unknown sign is encountered in a problem or its solution, a scan of that list should prove helpful. Thus, the infrequently used symbol $\psi$ is encountered in Prob. 25.6. According to the list, $\psi$ represents the stream function, and you are quickly on your way to a solution.

Every problem solution in this book has been checked, but, with 2500 in all, it is inevitable that some mistakes will slip through. We would appreciate it if you would take the time to communicate any mistakes you find to us, so that they may be corrected in future printings. We wish to thank Bill Langley, of The University of North Carolina at Charlotte, who assisted us with some of the problem selection and preparation.

# Abbreviations and Symbols

| | |
|---|---|
| $a$ | acceleration or area |
| $A$ | area |
| abs | absolute |
| $\alpha$ (alpha) | angle between absolute velocity of fluid in hydraulic machine and linear velocity of a point on a rotating body or coefficient of thermal expansion or dimensionless ratio of similitude |
| atm | atmosphere |
| atmos | atmospheric |
| $\beta$ (beta) | angle between relative velocity in hydraulic machines and linear velocity of a point on a rotating body or coefficient of compressibility or ratio of obstruction diameter to duct diameter |
| $b$ | surface width or other width |
| $B$ | surface width or other width |
| bhp | brake horsepower |
| bp | brake power |
| Btu | British thermal unit |
| $c$ | speed of sound or wave speed (celerity) |
| C | Celsius or discharge coefficient or speed of propagation |
| cal | calorie |
| c.b. or CB | center of buoyancy |
| $C_c$ | coefficient of contraction |
| $C_d$ | coefficient of discharge |
| $C_D$ | drag coefficient |
| $C_f$ | friction-drag coefficient |
| $C_F$ | force coefficient |
| cfs | cubic foot per second |
| c.g. or CG | center of gravity |
| $C_I$ | Pitot tube coefficient |
| $C_L$ | lift coefficient |
| cm | centimeter ($10^{-2}$ m) |
| cP | centipoise |
| c.p. | center of pressure |
| $c_P$ | specific heat at constant pressure |
| $c_v$ | specific heat at constant volume |
| $C_v$ | coefficient of velocity |
| $C_w$ | weir coefficient |
| $d$ | depth or diameter |
| $D$ | depth or diameter or drag force |
| $\delta$ (delta) | thickness of boundary layer |
| $\delta_1$ (delta) | thickness of the viscous sublayer |
| $\Delta$ (Delta) | change in (or difference between) |
| $d_c$ | critical depth |
| $D_{\text{eff}}$ | effective diameter |
| $D_h$ | hydraulic diameter |
| $d_m$ | mean depth |
| $d_n$ | normal depth |
| $d_N$ | normal depth |
| $E$ | modulus of elasticity or specific energy or velocity approach factor |
| $e_h$ | hydraulic efficiency |
| el | elevation |
| $\eta$ (eta) | pump or turbine efficiency |
| $\epsilon$ (epsilon) | height or surface roughness |
| $E_p$ | pump energy |
| $E_t$ | turbine energy |
| exp | exponential |
| $f$ | frequency of oscillation (cycles per second) or friction factor |

| | |
|---|---|
| F | Fahrenheit or force |
| $F_b$ | buoyant force |
| $F_D$ | drag force |
| $F_H$ | horizontal force |
| $F_L$ | lift force |
| fps | foot per second |
| F.S. | factor of safety |
| ft | foot |
| $F_U$ | uplift force on a dam |
| $F_V$ | vertical force |
| $g$ | acceleration due to gravity or gage height or gram |
| $G$ | weight flow rate |
| gal | gallon |
| $\gamma$ (gamma) | specific (or unit) weight |
| $\Gamma$ (Gamma) | circulation |
| GN | giganewton ($10^9$ N) |
| GPa | gigapascal ($10^9$ Pa) |
| gpm | gallons per minute |
| $h$ | enthalpy per unit mass or height or depth or pressure head or hour |
| $\bar{h}$ | average height or depth or head |
| $\hat{h}$ | enthalpy per unit weight |
| $H$ | energy head or total energy head |
| $h_1$ | unit head loss |
| $h_{cg}$ | vertical depth to center of gravity |
| $h_{cp}$ | vertical depth to center of pressure |
| $h_f$ | head loss due to friction |
| Hg | mercury |
| HGL | hydraulic grade line |
| $h_L$ | total head loss |
| $h_m$ | head loss due to minor losses |
| hp | horsepower |
| Hz | hertz (cycles per second) |
| $I$ | inflow or moment of inertia |
| ID | inside diameter |
| in | inch |
| $\infty$ (infinity) | sometimes used as a subscript to indicate upstream |
| J | joule |
| K | bulk modulus of elasticity or Kelvin or minor loss coefficient |
| $k$ | specific heat ratio |
| kcal | kilocalorie ($10^3$ cal) |
| kg | kilogram ($10^3$ g) |
| kJ | kilojoule ($10^3$ J) |
| km | kilometer ($10^3$ m) |
| kN | kilonewton ($10^3$ N) |
| kPa | kilopascal ($10^3$ Pa) |
| kW | kilowatt ($10^3$ W) |
| $L$ | length or lift force or liter |
| $\lambda$ (lambda) | model ratio or wave length |
| lb | pound |
| $lb_m$ | pound mass |
| $L_e$ | equivalent length |
| $L_m$ | linear dimension in model |
| $L_p$ | linear dimension in prototype |
| m | mass or meter |
| $\dot{m}$ | mass flow rate |
| $M$ | mass flow rate or molecular weight or moment or torque |
| $\overline{MB}$ | distance from center of buoyancy to metacenter |
| mbar | millibar ($10^{-3}$ bar) |
| mc | metacenter |
| mgd | million gallons per day |

| | |
|---|---|
| ml | milliliter ($10^{-3}$ L) |
| min | minute |
| mm | millimeter ($10^{-3}$ meter) |
| MN | meganewton ($10^6$ N) |
| MPa | megapascal ($10^6$ Pa) |
| mph | mile per hour |
| MR | manometer reading |
| $\mu$ (mu) | absolute or dynamic viscosity |
| MW | megawatt ($10^6$ W) |
| $n$ | Manning roughness coefficient or number of moles |
| $N$ | newton or rotational speed |
| $N_B$ | Brinkman number |
| $N_F$ | Froude number |
| $N_M$ | Mach number |
| NPSH | net positive suction head |
| $N_R$ | Reynolds number |
| $N_s$ | specific speed of pump or turbine |
| $\nu$ (nu) | kinematic viscosity |
| $N_W$ | Weber number |
| O | outflow |
| OD | outside diameter |
| $\Omega$ (ohm) | rotational rate |
| $\omega$ (omega) | angular velocity |
| $p$ | pressure or poise |
| $P$ | force (usually resulting from an applied pressure) or power |
| Pa | pascal |
| $\phi$ (phi) | peripheral-velocity factor |
| $\pi$ (pi) | constant = 3.14159265 |
| $\Pi$ (pi) | dimensionless parameter |
| $P_r$ | power ratio |
| $p_s$ | stagnation pressure |
| psi | pound per square inch |
| $\psi$ (psi) | stream function |
| psia | pound per square inch absolute |
| psig | pound per square inch gage |
| $p^{*t}$ | pressure for condition at $N_M = 1/\sqrt{k}$ |
| $p_v$ | vapor pressure |
| $p_w$ | wetted perimeter |
| $q$ | flow rate per unit width or heat per unit mass |
| $Q$ | discharge or heat or volume flow rate |
| $Q_H$ | heat transferred per unit weight of fluid |
| $Q/w$ | volume flow rate per unit width of channel |
| qt | quart |
| $r$ | radius |
| $R$ | gas constant or Rankine or resultant force or hydraulic radius |
| $R'$ | manometer reading |
| rad | radian |
| $R_c$ | critical hydraulic radius |
| $R_h$ | hydraulic radius |
| $\rho$ (rho) | mass density |
| $r_i$ | inside radius |
| $r_o$ | outside radius |
| rpm | revolutions per minute |
| $R_u$ | universal gas constant |
| $s$ | entropy of a substance or second or slope |
| $S$ | slope or storage |
| $s_c$ | critical slope |
| s.g. | specific gravity |
| s.g.$_M$ | specific gravity of manometer fluid |
| s.g.$_F$ | specific gravity of flowing fluid |

| | |
|---|---|
| $\sigma$ (sigma) | pump cavitation parameter or stress or surface tension |
| $\sigma'$ | cavitation index |
| $\Sigma$ (sigma) | summation |
| $S$ | specific gravity of flowing fluid |
| $S_0$ | specific gravity of manometer fluid |
| $t$ | thickness or time |
| $T$ | surface width or temperature or torque or tension |
| $\tau$ (tau) | shear stress |
| $\tau_0$ (tau) | shear stress at the wall |
| $T_s$ | stagnation temperature |
| $u$ | velocity |
| $u_{CL}$ | centerline velocity |
| $U$ | velocity |
| $v$ | velocity |
| $v_c$ | critical velocity |
| $V$ | velocity or volume |
| $v_{av}$ | average velocity |
| $V_{CL}$ | centerline velocity |
| $V_d$ | volume of fluid displaced |
| $V_m$ | velocity in model |
| $V_p$ | velocity in prototype |
| $V_s$ | specific volume |
| $v_*$ | shear velocity |
| $v_t$ | tangential velocity |
| $v_T$ | terminal velocity |
| $w$ | width |
| $W$ | watt or weight or weight flow rate or work |
| $x_{cp}$ | distance from center of gravity to center of pressure in $x$ direction |
| $\xi$ (xi) | vorticity |
| $y$ | depth |
| $y_c$ | critical depth |
| $y_{cp}$ | distance from center of gravity to center of pressure in $y$ direction |
| $y_n$ | normal depth |
| $y_N$ | normal depth |
| $z_{cg}$ | inclined distance from liquid surface to center of gravity |
| $z_{cp}$ | inclined distance from liquid surface to center of pressure |

# Conversion Factors

0.00001667 $m^3/s$ = 1 L/min
0.002228 $ft^3/s$ = 1 gal/min
0.0145 $lb/in^2$ = 1 mbar
0.3048 m = 1 ft
2.54 cm = 1 in
3.281 ft = 1 m
4 qt = 1 gal
4.184 kJ = 1 kcal
4.448 N = 1 lb
6.894 $kN/m^2$ = 1 $lb/in^2$
7.48 gal = 1 $ft^3$
12 in = 1 ft
14.59 kg = 1 slug
25.4 mm = 1 in
60 min = 1 h
60 s = 1 min
100 cm = 1 m
100 kPa = 1 bar

101.3 kPa = 1 atm
144 $in^2$ = 1 $ft^2$
550 ft-lb/s = 1 hp
778 ft-lb = 1 Btu
1000 N = 1 kN
1000 L = 1 $m^3$
1000 mm = 1 m
1000 Pa = 1 kPa
1728 $in^3$ = 1 $ft^3$
2000 lb = 1 ton
3600 s = 1 h
4184 J = 1 kcal
5280 ft = 1 mile
86 400 s = 1 day
1 000 000 N = 1 MN
1 000 000 Pa = 1 MPa
1 000 000 000 N = 1 GN
1 000 000 000 Pa = 1 GPa

# CHAPTER 1
# Properties of Fluids

**Note:** For many problems in this chapter, values of various physical properties of fluids are obtained from Tables A-1 through A-8 in the Appendix.

**1.1** A reservoir of glycerin (glyc) has a mass of 1200 kg and a volume of 0.952 m³. Find the glycerin's weight ($W$), mass density ($\rho$), specific weight ($\gamma$), and specific gravity (s.g.).

$$F = W = ma = (1200)(9.81) = 11\,770\text{ N} \quad \text{or} \quad 11.77\text{ kN}$$
$$\rho = m/V = 1200/0.952 = 1261\text{ kg/m}^3$$
$$\gamma = W/V = 11.77/0.952 = 12.36\text{ kN/m}^3$$
$$\text{s.g.} = \gamma_{\text{glyc}}/\gamma_{\text{H}_2\text{O at }4\,^\circ\text{C}} = 12.36/9.81 = 1.26$$

**1.2** A body requires a force of 100 N to accelerate it at a rate of 0.20 m/s². Determine the mass of the body in kilograms and in slugs.

$$F = ma$$
$$100 = (m)(0.20)$$
$$m = 500\text{ kg} = 500/14.59 = 34.3\text{ slugs}$$

**1.3** A reservoir of carbon tetrachloride ($CCl_4$) has a mass of 500 kg and a volume of 0.315 m³. Find the carbon tetrachloride's weight, mass density, specific weight, and specific gravity.

$$F = W = ma = (500)(9.81) = 4905\text{ N} \quad \text{or} \quad 4.905\text{ kN}$$
$$\rho = m/V = 500/0.315 = 1587\text{ kg/m}^3$$
$$\gamma = \text{W/V} = 4.905/0.315 = 15.57\text{ kN/m}^3$$
$$\text{s.g.} = \gamma_{\text{CCl}_4}/\gamma_{\text{H}_2\text{O at }4\,^\circ\text{C}} = 15.57/9.81 = 1.59$$

**1.4** The weight of a body is 100 lb. Determine (*a*) its weight in newtons, (*b*) its mass in kilograms, and (*c*) the rate of acceleration [in both feet per second per second (ft/s²) and meters per second per second (m/s²)] if a net force of 50 lb is applied to the body.

(*a*) $$W = (100)(4.448) = 444.8\text{ N}$$

(*b*) $$F = W = ma \qquad 444.8 = (m)(9.81) \qquad m = 45.34\text{ kg}$$

(*c*) $$m = 45.34/14.59 = 3.108\text{ slugs}$$

$$F = ma \qquad 50 = 3.108a \qquad a = 16.09\text{ ft/s}^2 = (16.09)(0.3048) = 4.904\text{ m/s}^2$$

**1.5** The specific gravity of ethyl alcohol is 0.79. Calculate its specific weight (in both pounds per cubic foot and kilonewtons per cubic meter) and mass density (in both slugs per cubic foot and kilograms per cubic meter).

$$\gamma = (0.79)(62.4) = 49.3\text{ lb/ft}^3 \qquad \gamma = (0.79)(9.79) = 7.73\text{ kN/m}^3$$
$$\rho = (0.79)(1.94) = 1.53\text{ slugs/ft}^3 \qquad \rho = (0.79)(1000) = 790\text{ kg/m}^3$$

**1.6** A quart of water weights about 2.08 lb. Compute its mass in slugs and in kilograms.

$$F = W = ma \qquad 2.08 = (m)(32.2)$$
$$m = 0.0646\text{ slug} \qquad m = (0.0646)(14.59) = 0.943\text{ kg}$$

**1.7** One cubic foot of glycerin has a mass of 2.44 slugs. Find its specific weight in both pounds per cubic foot and kilonewtons per cubic meter.

$F = W = ma = (2.44)(32.2) = 78.6$ lb. Since the glycerin's volume is 1 ft³, $\gamma = 78.6\text{ lb/ft}^3 = (78.6)(4.448)/(0.3048)^3 = 12\,350\text{ N/m}^3$, or $12.35\text{ kN/m}^3$.

**1.8** A quart of SAE 30 oil at 68 °F weighs about 1.85 lb. Calculate the oil's specific weight, mass density, and specific gravity.

▌
$$V = 1/[(4)(7.48)] = 0.03342\ \text{ft}^3$$
$$\gamma = W/V = 1.85/0.03342 = 55.4\ \text{lb/ft}^3$$
$$\rho = \gamma/g = 55.4/32.2 = 1.72\ \text{slugs/ft}^3$$
$$\text{s.g.} = \gamma_{\text{oil}}/\gamma_{\text{H}_2\text{O at 4°C}} = 55.4/62.4 = 0.888$$

**1.9** The volume of a rock is found to be 0.00015 m³. If the rock's specific gravity is 2.60, what is its weight?

▌ $\gamma_{\text{rock}} = (2.60)(9.79) = 25.5\ \text{kN/m}^3$ $\qquad W_{\text{rock}} = (25.5)(0.00015) = 0.00382\ \text{kN}$ or $3.82\ \text{N}$

**1.10** A certain gasoline weighs 46.5 lb/ft³. What are its mass density, specific volume, and specific gravity?

▌ $\rho = \gamma/g = 46.5/32.2 = 1.44\ \text{slugs/ft}^3$ $\qquad V_s = 1/\rho = 1/1.44 = 0.694\ \text{ft}^3/\text{slug}$

$\text{s.g.} = 1.44/1.94 = 0.742$

**1.11** If the specific weight of a substance is 8.2 kN/m³, what is its mass density?

▌
$$\rho = \gamma/g = 8200/9.81 = 836\ \text{kg/m}^3$$

**1.12** An object at a certain location has a mass of 2.0 kg and weighs 19.0 N on a spring balance. What is the acceleration due to gravity at this location?

▌
$$F = W = ma \qquad 19.0 = 2.0a \qquad a = 9.50\ \text{m/s}^2$$

**1.13** If an object has a mass of 2.0 slugs at sea level, what would its mass be at a location where the acceleration due to gravity is 30.00 ft/s²?

▌ Since the mass of an object does not change, its mass will be 2.0 slugs at that location.

**1.14** What would be the weight of a 3-kg mass on a planet where the acceleration due to gravity is 10.00 m/s²?

▌
$$F = W = ma = (3)(10.00) = 30.00\ \text{N}$$

**1.15** Determine the weight of a 5-slug boulder at a place where the acceleration due to gravity is 31.7 ft/s².

▌
$$F = W = ma = (5)(31.7) = 158\ \text{lb}$$

**1.16** If 200 ft³ of oil weighs 10 520 lb, calculate its specific weight, density, and specific gravity.

▌ $\gamma = W/V = 10\,520/200 = 52.6\ \text{lb/ft}^3$ $\qquad \rho = \gamma/g = 52.6/32.2 = 1.63\ \text{slugs/ft}^3$

$\text{s.g.} = \gamma_{\text{oil}}/\gamma_{\text{H}_2\text{O at 4°C}} = 52.6/62.4 = 0.843$

**1.17** Find the height of the free surface if 0.8 ft³ of water is poured into a conical tank (Fig. 1-1) 20 in high with a base radius of 10 in. How much additional water is required to fill the tank?

▌ $V_{\text{cone}} = \pi r^2 h/3 = \pi(10)^2(20)/3 = 2094\ \text{in}^3$ $\qquad V_{\text{H}_2\text{O}} = 0.8\ \text{ft}^3 = 1382\ \text{in}^3$

Additional water needed $= 2095 - 1382 = 713\ \text{in}^3$. From Fig. 1-1, $r_o/10 = h_o/20$, or $r_o = h_o/2.0$; $V_{\text{empty (top) cone}} = \pi(h_o/2.0)^2 h_o/3 = 713$; $h_o = 13.96$ in. Free surface will be $20 - 13.96$, or 6.04 in above base of tank.

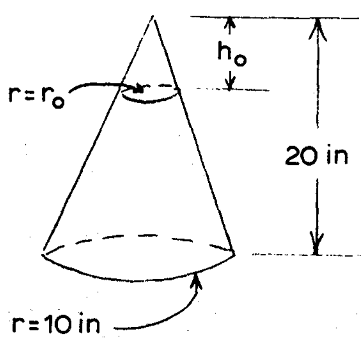



Fig. 1-1

**1.18** If the tank of Prob. 1.17 holds 30.5 kg of salad oil, what is the density of the oil?

$$V_{\text{cone}} = 2094\ \text{in}^3 \qquad \text{(from Prob. 1.17)}$$
$$= \tfrac{2094}{1728}(0.3048)^3 = 0.03431\ \text{m}^3$$
$$\rho = m/V = 30.5/0.03431 = 889\ \text{kg/m}^3$$

**1.19** Under standard conditions a certain gas weighs 0.14 lb/ft³. Calculate its density, specific volume, and specific gravity relative to air weighing 0.075 lb/ft³.

$$\rho = \gamma/g = 0.14/32.2 = 0.00435\ \text{slug/ft}^3 \qquad V_s = 1/\rho = 1/0.00435 = 230\ \text{ft}^3/\text{slug}$$
$$\text{s.g.} = 0.14/0.075 = 1.87$$

**1.20** If the specific volume of a gas is 360 ft³/slug, what is its specific weight?

$$\rho = 1/V_s = \tfrac{1}{360} = 0.002778\ \text{slug/ft}^3 \qquad \gamma = \rho g = (0.002778)(32.2) = 0.0895\ \text{lb/ft}^3$$

**1.21** A vertical glass cylinder contains 900.00 mL of water at 10 °C; the height of the water column is 90.00 cm. The water and its container are heated to 80 °C. Assuming no evaporation, what will be the height of the water if the coefficient of thermal expansion ($\alpha$) for the glass is $3.6 \times 10^{-6}\ {}^\circ\text{C}^{-1}$?

Mass of water $= \rho V = \rho_{10} V_{10} = \rho_{80} V_{80}$ $\qquad (1000)(900 \times 10^{-6}) = 971 V_{80} \qquad V_{80} = 926.9 \times 10^{-6}\ \text{m}^3 = 926.9\ \text{cm}^3$

$$A_{10} = V_{10}/h_{10} = 900.00/90.00 = 10.000\ \text{cm}^2$$
$$A_{10} = \pi r_{10}^2 \qquad 10.000 = \pi r_{10}^2 \qquad r_{10} = 1.7841\ \text{cm}$$
$$r_{80} = r_{10}[1 + (\Delta T)(\alpha)] = (1.7841)[1 + (80 - 10)(3.6 \times 10^{-6})] = 1.7845\ \text{cm}$$
$$A_{80} = \pi r_{80}^2 = \pi(1.7845)^2 = 10.004\ \text{cm}^2 \qquad h_{80} = V_{80}/A_{80} = 926.9/10.004 = 92.65\ \text{cm}$$

**1.22** If a vessel that contains 3.500 ft³ of water at 50 °F and atmospheric pressure is heated to 160 °F, what will be the percentage change in its volume? What weight of water must be removed to maintain the original volume?

$$\text{Weight of water} = \gamma V = \gamma_{50} V_{50} = \gamma_{160} V_{160} \qquad (62.4)(3.500) = 61.0 V_{160} \qquad V_{160} = 3.5803\ \text{ft}^3$$

Change in volume $= (3.5803 - 3.500)/3.000 = 0.027$, or 2.7% (increase). Must remove $(3.5803 - 3.500)(61.0)$, or 4.90 lb.

**1.23** A vertical, cylindrical tank with a diameter of 12.00 m and a depth of 4.00 m is filled to the top with water at 20 °C. If the water is heated to 50 °C, how much water will spill over?

$$V_{\text{tank}} = (V_{H_2O})_{20} = \pi(12.00/2)^2(4.00) = 452.4\ \text{m}^3$$
$$W_{H_2O} = (9.79)(452.4) = 4429\ \text{kN} \qquad (V_{H_2O})_{50} = 4429/9.69 = 457.1\ \text{m}^3$$
$$\text{Volume of water spilled} = 457.1 - 452.4 = 4.7\ \text{m}^3$$

**1.24** A thick, closed, steel chamber is filled with water at 50 °F and atmospheric pressure. If the temperature of water and chamber is raised to 100 °F, find the new pressure of the water. The coefficient of thermal expansion of steel is $6.5 \times 10^{-6}$ per °F.

The volume of water would attempt to increase as the cube of the linear dimension; hence, $V_{90} = V_{50}[1 + (100 - 50)(6.5 \times 10^{-6})]^3 = 1.000975 V_{50}$; weight of water $= \gamma V = \gamma_{50} V_{50} = \gamma_{90} V_{90}$, $62.4 V_{50} = \gamma_{90}(1.000975 V_{50})$, $\gamma_{90} = 62.34\ \text{lb/ft}^3$. From Fig. A-3, $p_{90} = 1300$ psia (approximately).

**1.25** A liquid compressed in a cylinder has a volume of 1000 cm³ at 1 MN/m² and a volume of 995 cm³ at 2 MN/m². What is its bulk modulus of elasticity ($K$)?

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{2 - 1}{(995 - 1000)/1000} = 200\ \text{MPa}$$

**1.26** Find the bulk modulus of elasticity of a liquid if a pressure of 150 psi applied to 10 ft³ of the liquid causes a volume reduction of 0.02 ft³.

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{(150 - 0)(144)}{-0.02/10} = 10\,800\,000\ \text{lb/ft}^2 \quad \text{or} \quad 75\,000\ \text{psi}$$

**1.27** If $K = 2.2$ GPa is the bulk modulus of elasticity for water, what pressure is required to reduce a volume by 0.6 percent?

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 2.2 = -\frac{p_2 - 0}{-0.006} \qquad p_2 = 0.0132\ \text{GPa} \quad \text{or} \quad 13.2\ \text{MPa}$$

**1.28** Find the change in volume of 1.00000 ft³ of water at 80 °F when subjected to a pressure increase of 300 psi. Water's bulk modulus of elasticity at this temperature is 325 000 psi.

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 325\,000 = -\frac{300 - 0}{\Delta V/1.00000} \qquad \Delta V = -0.00092\ \text{ft}^3$$

**1.29** From the following test data, determine the bulk modulus of elasticity of water: at 500 psi the volume was 1.000 ft³, and at 3500 psi the volume was 0.990 ft³.

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{500 - 3500}{(1.000 - 0.990)/1.000} = 300\,000\ \text{psi}$$

**1.30** A rigid steel container is partially filled with a liquid at 15 atm. The volume of the liquid is 1.23200 L. At a pressure of 30 atm, the volume of the liquid is 1.23100 L. Find the average bulk modulus of elasticity of the liquid over the given range of pressure if the temperature after compression is allowed to return to its initial value. What is the coefficient of compressibility ($\beta$)?

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{(30 - 15)(101.3)}{(1.23100 - 1.23200)/1.23200} = 1.872 \times 10^6\ \text{kPa} \quad \text{or} \quad 1.872\ \text{GPa}$$

$$\beta = 1/K = 1/1.872 = 0.534\ \text{GPa}^{-1}$$

**1.31** A heavy tank contains oil ($A$) and water ($B$) subject to variable air pressure; the dimensions shown in Fig. 1-2 correspond to 1 atm. If air is slowly added from a pump to bring pressure $p$ up to 1 MPa gage, what will be the total downward movement of the free surface of oil and air? Take average values of bulk moduli of elasticity of the liquids as 2050 MPa for oil and 2075 MPa for water. Assume the container does not change volume. Neglect hydrostatic pressures.

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 2050 = -\frac{1 - 0}{\Delta V_{\text{oil}}/[600\pi(300)^2/4]} \qquad \Delta V_{\text{oil}} = -20\,690\ \text{mm}^3$$

$$2075 = -\frac{1 - 0}{\Delta V_{H_2O}/[700\pi(300)^2/4]} \qquad \Delta V_{H_2O} = -23\,850\ \text{mm}^3$$

$$\Delta V_{\text{total}} = -44\,540\ \text{mm}^3$$

Let $x$ = distance the upper free surface moves. $-44\,540 = -[\pi(300)^2/4]x$, $x = 0.630$ mm.

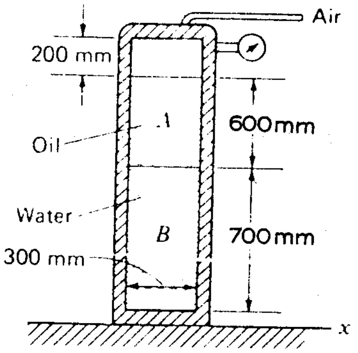



**Fig. 1-2**

**1.32** A thin-walled spherical tank is filled with water at a pressure of 4666 psig; the tank's volume is then 805.407 in³. If the water is released from the tank, how many pounds will be collected at atmospheric pressure? 805.4069 in³ when the pressure is 4666 psig. Use 305 000 psi as an average value of the bulk modulus of elasticity.

▮ $$K = -\frac{\Delta p}{\Delta V/V} \qquad 305\,000 = -\frac{0-4666}{(V_2-805.407)/805.407} \qquad V_2 = 817.73\ \text{in}^3$$

$$W = (62.4)(817.73/1728) = 29.5\ \text{lb}$$

**1.33** Water in a hydraulic press, initially at 20 psia, is subjected to a pressure of 17 000 psia at 68 °F. Determine the percentage decrease in specific volume if the average bulk modulus of elasticity is 365 000 psi.

▮ $$K = -\frac{\Delta p}{\Delta V/V} \qquad 365\,000 = -\frac{17\,000-20}{\Delta V/V_1} \qquad \frac{\Delta V}{V_1} = -0.0465 \quad \text{or} \quad 4.65\%\ \text{decrease}$$

**1.34** At a depth of 7 km in the ocean, the pressure is 71.6 MPa. Assume a specific weight at the surface of 10.05 kN/m³ and an average bulk modulus of elasticity of 2.34 GPa for that pressure range. Find ***(a)*** the change in specific volume between the surface and 7 km; ***(b)*** the specific volume at 7 km; ***(c)*** the specific weight at 7 km.

▮ ***(a)*** $$(V_s)_1 = 1/\rho_1 = g/\gamma_1 = 9.81/10\,050 = 0.0009761\ \text{m}^3/\text{kg}$$

$$K = -\frac{\Delta p}{\Delta V_s/V_s} \qquad 2.34\times10^9 = -\frac{71.6\times10^6-0}{\Delta V_s/0.0009761} \qquad \Delta V_s = -0.0000299\ \text{m}^3/\text{kg}$$

***(b)*** $$(V_s)_2 = (V_s)_1 + \Delta V_s = 0.0009761 - 0.0000299 = 0.000946\ \text{m}^3/\text{kg}$$

***(c)*** $$\gamma_2 = g/V_2 = 9.81/0.000946 = 10\,370\ \text{N/m}^3$$

**1.35** Approximately what pressure must be applied to water at 60 °F to reduce its volume 2.5 percent?

▮ $$K = -\frac{\Delta p}{\Delta V/V} \qquad 311\,000 = -\frac{p_2-0}{0.025} \qquad p_2 = 7775\ \text{psi}$$

**1.36** A gas at 20 °C and 0.21 MPa abs has a volume of 41 L and a gas constant ($R$) of 210 m · N/(kg · K). Determine the density and mass of the gas.

▮ $$\rho = p/RT = 0.21\times10^6/[(210)(20+273)] = 3.41\ \text{kg/m}^3 \qquad m = \rho V = (3.41)(0.041) = 0.140\ \text{kg}$$

**1.37** What is the specific weight of air at 70 psia and 70 °F?

▮ $\gamma = p/RT$. From Table A-6, $R = 53.3$ ft/°R; $\gamma = (70)(144)/[(53.3)(70+460)] = 0.357\ \text{lb/ft}^3$.

**Note:** $p/RT$ gives $\rho$ (Prob. 1.36) or $\gamma$ (Prob. 1.37), depending on the value of $R$ used. Corresponding values of $R$ in Table A-6 differ by a factor of $g$.

**1.38** Calculate the density of water vapor at 350 kPa abs and 20 °C if its gas constant ($R$) is 0.462 kPa · m³/kg · K.

▮ $$\rho = p/RT = 350/[(0.462)(20+273)] = 2.59\ \text{kg/m}^3$$

**1.39** Nitrogen gas (molecular weight 28) occupies a volume of 4.0 ft³ at 2500 lb/ft² abs and 750 °R. What are its specific volume and specific weight?

▮ $$R = R_u/M = 49\,709/28 = 1775\ \text{ft}\cdot\text{lb}/(\text{slug}\cdot{}^\circ\text{R})$$

[where $R_u$, the universal gas constant, = 49 709 ft · lb/(slug · °R)]

$$\rho = 1/V_s = p/RT = 2500/[(1775)(750)] \qquad V_s = 532.5\ \text{ft}^3/\text{slug}$$

$$\gamma = \rho g = (1/V_s)(g) = (1/532.5)(32.2) = 0.0605\ \text{lb/ft}^3$$

**1.40** One kilogram of hydrogen is confined in a volume of 200 L at −45 °C. What is the pressure if $R$ is 4.115 kJ/kg · K?

▮ $$p = \rho RT = (m/V)RT = (1/0.200)(4115)(-45+273) = 4.691\times10^6\ \text{Pa} \quad \text{or} \quad 4.691\ \text{MPa abs}$$

**1.41** What is the specific weight of air at a temperature of 30 °C and a pressure of 470 kPa abs?

▮ $$\gamma = p/RT = 470/[(29.3)(30+273)] = 0.0529\ \text{kN/m}^3$$

**1.42** Find the mass density of helium at a temperature of 39 °F and a pressure of 26.9 psig, if atmospheric pressure is 14.9 psia.

▮
$$\rho = p/RT = (14.9 + 26.9)(144)/[(12\,420)(39 + 460)]$$
$$= 0.000971 \text{ lb}\cdot\text{s}^2/\text{ft}^4 \quad \text{or} \quad 0.000971 \text{ slug/ft}^3$$

**1.43** The temperature and pressure of nitrogen in a tank are 28 °C and 600 kPa abs, respectively. Determine the specific weight of the nitrogen.

▮
$$\gamma = p/RT = 600/[(30.3)(28 + 273)] = 0.0658 \text{ kN/m}^3$$

**1.44** The temperature and pressure of oxygen in a container are 60 °F and 20.0 psig, respectively. Determine the oxygen's mass density if atmospheric pressure is 14.7 psia.

▮
$$\rho = p/RT = (20.0 + 14.7)(144)/[(1552)(60 + 460)] = 0.00619 \text{ slug/ft}^3$$

**1.45** Calculate the specific weight and density of methane at 100 °F and 120 psia.

▮
$$\gamma = p/RT = (120)(144)/[(96.2)(100 + 460)] = 0.321 \text{ lb/ft}^3$$
$$\rho = \gamma/g = 0.321/32.2 = 0.00997 \text{ slug/ft}^3$$

**1.46** At 90 °F and 30.0 psia, the specific weight of a certain gas was 0.0877 lb/ft³. Determine the gas constant and density of this gas.

▮
$$\gamma = p/RT \qquad 0.0877 = (30.0)(144)/[(R)(90 + 460)] \qquad R = 89.6 \text{ ft/°R}$$
$$\rho = \gamma/g = 0.0877/32.2 = 0.00272 \text{ slug/ft}^3$$

**1.47** A cylinder contains 12.5 ft³ of air at 120 °F and 40 psia. The air is then compressed to 2.50 ft³. ***(a)*** Assuming isothermal conditions, what are the pressure at the new volume and the bulk modulus of elasticity? ***(b)*** Assuming adiabatic conditions, what are the final pressure and temperature and the bulk modulus of elasticity?

▮ ***(a)***
$$p_1 V_1 = p_2 V_2 \qquad \text{(for isothermal conditions)}$$
$$(40)(12.5) = (p_2')(2.50)$$
$$p_2' = 200 \text{ psia}$$
$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{40 - 200}{(12.5 - 2.5)/12.5} = 200 \text{ psi}$$

***(b)*** $p_1 V_1^k = p_2 V_2^k$ (for adiabatic conditions). From Table A-6, $k = 1.40$. $(40)(12.5)^{1.40} = (p_2')(2.50)^{1.40}$, $p_2' = 381$ psia; $T_2/T_1 = (p_2/p_1)^{(k-1)/k}$, $T_2/(120 + 460) = (\frac{381}{40})^{(1.40-1)/1.40}$, $T_2 = 1104$ °R, or 644 °F; $K = kp' = (1.40)(381) = 533$ psi.

**1.48** Air is kept at a pressure of 200 kPa and a temperature of 30 °C in a 500-L container. What is the mass of the air?

▮ $\rho = p/RT = [(200)(1000)]/[(287)(30 + 273)] = 2.300 \text{ kg/m}^3 \qquad m = (2.300)(\frac{500}{1000}) = 1.15 \text{ kg}$

**1.49** An ideal gas has its pressure doubled and its specific volume decreased by two-thirds. If the initial temperature is 80 °F, what is the final temperature?

▮
$$\rho = 1/V_s = p/RT \qquad pV_s = RT \qquad p_1(V_s)_1 = RT_1 \qquad p_2(V_s)_2 = RT_2$$
$$(p_2/p_1)[(V_s)_2/(V_s)_1] = (R/R)(T_2/T_1) \qquad (2)(\tfrac{1}{3}) = T_2/(80 + 460) \qquad T_2 = 360 \text{ °R} \quad \text{or} \quad -100 \text{ °F}$$

**1.50** The tank of a leaky air compressor originally holds 90 L of air at 33 °C and 225 kPa. During a compression process, 4 grams of air is lost; the remaining air occupies 42 L at 550 kPa. What is the temperature of the remaining air?

$$\rho_1 = p_1/RT_1 = (225\times 10^3)/[(287)(33+273)] = 2.562\ \text{kg/m}^3 \qquad m = (2.562)(0.090) = 0.2306\ \text{kg}$$

$$\rho_2 = p_2/RT_2 \qquad (0.2306-0.004)/0.042 = (550\times 10^3)/(287T_2) \qquad T_2 = 355\ \text{K} \quad \text{or} \quad 82\ ^\circ\text{C}$$

**1.51** In a piston-and-cylinder apparatus the initial volume of air is 90 L at a pressure of 130 kPa and temperature of 26 °C. If the pressure is doubled while the volume is decreased to 56 L, compute the final temperature and density of the air.

$$\rho_1 = p_1/RT_1 = (130\times 10^3)/[(287)(26+273)] = 1.515\ \text{kg/m}^3 \qquad m = (1.515)(0.090) = 0.1364\ \text{kg}$$

$$\rho_2 = p_2/RT_2 \qquad 0.1364/\tfrac{56}{1000} = (2)(130\times 10^3)/(287T_2) \qquad T_2 = 372\ \text{K} \quad \text{or} \quad 99\ ^\circ\text{C}$$

$$\rho = 0.1364/(0.056) = 2.44\ \text{kg/m}^3$$

**1.52** For 2 lb mol of air with a molecular weight of 29, a temperature of 90 °F, and a pressure of 2.5 atm, what is the volume?

$$pV/nM = RT \qquad [(2.5)(14.7)(144)]\{V/[(2)(29)]\} = (53.3)(90+460) \qquad V = 321\ \text{ft}^3$$

**1.53** If nitrogen has a molecular weight of 28, what is its density according to the perfect gas law when $p = 0.290$ MPa and $T = 30$ °C?

$$R = R_u/M = 8312/28 = 297\ \text{J/(kg}\cdot\text{K)} \qquad [\text{where } R_u = 8312\ \text{J/(kg}\cdot\text{K)}]$$

$$\rho = p/RT = 290\,000/[(297)(30+273)] = 3.22\ \text{kg/m}^3$$

**1.54** If a gas occupies 1 m$^3$ at 1 atm pressure, what pressure is required to reduce the volume of the gas by 2 percent under isothermal conditions if the fluid is (*a*) air, (*b*) argon, and (*c*) hydrogen?

$pV = nRT =$ constant for isothermal conditions. Therefore, if $V$ drops to $0.98V_o$, $p$ must rise to $(1/0.98)p_o$, or $1.020p_o$. This is true for any perfect gas.

**1.55** (*a*) Calculate the density, specific weight, and specific volume of oxygen at 100 °F and 15 psia. (*b*) What would be the temperature and pressure of this gas if it were compressed isentropically to 40 percent of its original volume? (*c*) If the process described in (*b*) had been isothermal, what would the temperature and pressure have been?

(*a*)
$$\rho = p/RT = (15)(144)/[(1552)(100+460)] = 0.00248\ \text{slug/ft}^3$$

$$\gamma = \rho g = (0.00248)(32.2) = 0.0799\ \text{lb/ft}^3 \qquad V_s = 1/\rho = 1/0.00248 = 403\ \text{ft}^3/\text{slug}$$

(*b*)
$$p_1(V_s)_1^k = p_2(V_s)_2^k \quad [(15)(144)](403)^{1.40} = [(p_2)(144)][(0.40)(403)]^{1.40} \quad p_2 = 54.1\ \text{psia}$$

$$p_2 = \rho_2 RT_2 \qquad (54.1)(144) = (0.00248/0.40)(1552)(T_2+460) \qquad T_2 = 350\ ^\circ\text{F}$$

(*c*) If isothermal, $T_2 = T_1 = 100$ °F and $pV =$ constant.

$$[(15)(144)](403) = [(p_2)(144)][(0.40)(403)] \qquad p_2 = 37.5\ \text{psia}$$

**1.56** Calculate the density, specific weight, and volume of chloride gas at 25 °C and pressure of 600 000 N/m$^2$ abs.

$$\rho = p/RT = 600\,000/[(118)(25+273)] = 17.1\ \text{kg/m}^3$$

$$\gamma = \rho g = (17.1)(9.81) = 168\ \text{N/m}^3 \qquad V_s = 1/\rho = 1/17.1 = 0.0585\ \text{m}^3/\text{kg}$$

**1.57** If methane gas has a specific gravity of 0.55 relative to air at 14.7 psia and 68 °F, what are its specific weight and specific volume at that same pressure and temperature? What is the value of $R$ for the gas?

$$\gamma_{\text{air}} = p/RT = (14.7)(144)/[(53.3)(68+460)] = 0.07522\ \text{lb/ft}^3$$

$$\gamma_{\text{gas}} = (0.55)(0.07522) = 0.0414\ \text{lb/ft}^3$$

$$V_s = 1/\rho = g/\gamma \qquad (V_s)_{\text{gas}} = 32.2/0.0414 = 778\ \text{ft}^3/\text{slug}$$

Since $R$ varies inversely with density for fixed pressure and temperature, $R_{\text{gas}} = 53.3/0.55 = 96.9$ ft/°R.

**1.58** A gas at 40 °C under a pressure of 21.868 bar abs has a unit weight of 362 N/m³. What is the value of $R$ for this gas? What gas might this be?

$$\gamma = p/RT \qquad 362 = (21.868 \times 10^5)/[(R)(40+273)] \qquad R = 19.3 \text{ m/K}$$

This gas might be carbon dioxide, since its gas constant is 19.3 m/K (from Table A-6).

**1.59** If water vapor ($R = 85.7$ ft/°R) in the atmosphere has a partial pressure of 0.60 psia and the temperature is 80 °F, what is its specific weight?

$$\gamma = p/RT = (0.60)(144)/[(85.7)(80+460)] = 0.00187 \text{ lb/ft}^3$$

**1.60** Refer to Prob. 1.59. If the barometer reads 14.60 psia, calculate the partial pressure of the air, its specific weight, and the specific weight of the atmosphere (air plus water vapor).

$$p_{\text{air}} = 14.60 - 0.60 = 14.00 \text{ psia} \qquad \gamma = p/RT$$

$$\gamma_{\text{air}} = (14.00)(144)/[(53.3)(80+460)] = 0.0700 \text{ lb/ft}^3 \qquad \gamma_{\text{atm}} = \gamma_{\text{air}} + \gamma_{H_2O(\text{vap})}$$

$$\gamma_{H_2O(\text{vap})} = 0.00187 \text{ lb/ft}^3 \quad \text{(from Prob. 1.59)} \qquad \gamma_{\text{atm}} = 0.0700 + 0.00187 = 0.0719 \text{ lb/ft}^3$$

**1.61** (*a*) Calculate the density, specific weight, and specific volume of oxygen at 20 °C and 40 kPa abs. (*b*) If the oxygen is enclosed in a rigid container, what will be the pressure if the temperature is reduced to −100 °C?

(*a*)

$$\rho = p/RT = (40)(1000)/[(260)(20+273)] = 0.525 \text{ kg/m}^3$$

$$\gamma = \rho g = (0.525)(9.81) = 5.15 \text{ N/m}^3 \qquad V_s = 1/\rho = 1/0.525 = 1.90 \text{ m}^3\text{/kg}$$

(*b*) $\rho = 1/V_s = p/RT$. Since $V_s$ and $R$ are constants, $V_s/R = T/p$ = constant, $(20+273)/40 = (-100+273)/p_2$, $p_2 = 23.6$ kPa.

**1.62** Helium at 149 kPa abs and 10 °C is isentropically compressed to one-fourth of its original volume. What is its final pressure?

$$p_1V_1^k = p_2V_2^k \qquad 149V_1^{1.66} = (p_2)(V_1/4)^{1.66} \qquad p_2 = 1488 \text{ kPa abs}$$

**1.63** (*a*) If 9 ft³ of an ideal gas at 75 °F and 22 psia is compressed isothermally to 2 ft³, what is the resulting pressure? (*b*) What would the pressure and temperature have been if the process had been isentropic?

(*a*)

$$p_1V_1 = p_2V_2 \qquad (22)(9) = (p_2)(2) \qquad p_2 = 99 \text{ psia}$$

(*b*)

$$p_1V_1^k = p_2V_2^k \qquad (22)(9)^{1.30} = (p_2)(2)^{1.30} \qquad p_2 = 155 \text{ psia}$$

$$T_2/T_1 = (p_2/p_1)^{(k-1)/k} \qquad T_2/(75+460) = (\tfrac{155}{22})^{(1.30-1)/1.30} \qquad T_2 = 840\,°\text{R} \quad \text{or} \quad 380\,°\text{F}$$

**1.64** (*a*) If 12 m³ of nitrogen at 30 °C and 125 kPa abs is permitted to expand isothermally to 30 m³, what is the resulting pressure? (*b*) What would the pressure and temperature have been if the process had been isentropic?

(*a*)

$$p_1V_1 = p_2V_2 \qquad (125)(12) = (p_2)(30) \qquad p_2 = 50.0 \text{ kPa abs}$$

(*b*)

$$p_1V_1^k = p_2V_2^k \qquad (125)(12)^{1.40} = (p_2)(30)^{1.40} \qquad p_2 = 34.7 \text{ kPa abs}$$

$$T_2/T_1 = (p_2/p_1)^{(k-1)/k} \qquad T_2/(30+273) = (34.7/125)^{(1.40-1)/1.40} \qquad T_2 = 210 \text{ K} \quad \text{or} \quad -63\,°\text{C}$$

**1.65** If the viscosity of water at 68 °F is 0.01008 poise, compute its absolute viscosity ($\mu$) in pound-seconds per square foot. If the specific gravity at 68 °F is 0.998, compute its kinematic viscosity ($\nu$) in square feet per second.

The poise is measured in dyne-seconds per square centimeter. Since 1 lb = 444 800 dynes and 1 ft = 30.48 cm, 1 lb · s/ft² = 444 800 dyne · s/(30.48 cm)² = 478.8 poises

$$\mu = \frac{0.01008}{478.8} = 2.11 \times 10^{-5} \text{ lb}\cdot\text{s/ft}^2 \qquad \nu = \frac{\mu}{\rho} = \frac{\mu}{\gamma/g} = \frac{\mu g}{\gamma} = \frac{(2.11\times 10^{-5})(32.2)}{(0.998)(62.4)} = 1.09 \times 10^{-5} \text{ ft}^2\text{/s}$$

**1.66** Convert 15.14 poises to kinematic viscosity in square feet per second if the liquid has a specific gravity of 0.964.

$$1 \text{ lb}\cdot\text{s/ft}^2 = 478.8 \text{ poises} \qquad \text{(from Prob. 1.65)}$$

$$\mu = 15.14/478.8 = 0.03162 \text{ lb}\cdot\text{s/ft}^2 \qquad \nu = \mu g/\gamma = (0.03162)(32.2)/[(0.964)(62.4)] = 0.0169 \text{ ft}^2\text{/s}$$

**1.67** The fluid flowing in Fig. 1-3 has an absolute viscosity ($\mu$) of 0.0010 lb · s/ft$^2$ and specific gravity of 0.913. Calculate the velocity gradient and intensity of shear stress at the boundary and at points 1 in, 2 in, and 3 in from the boundary, assuming **(a)** a straight-line velocity distribution and **(b)** a parabolic velocity distribution. The parabola in the sketch has its vertex at $A$ and origin at $B$.

**(a)** For the straight-line assumption, the relation between velocity $v$ and distance $y$ is $v = 15y$, $dv = 15dy$. The velocity gradient $= dv/dy = 15$. Since $\mu = \tau/(dv/dy)$, $\tau = \mu\,(dv/dy)$. For $y = 0$ (i.e., at the boundary), $v = 0$ and $dv/dy = 15\ \text{s}^{-1}$; $\tau = (0.0010)(15) = 0.015\ \text{lb/ft}^2$. For $y = 1$ in, 2 in, and 3 in, $dv/dy$ and $\tau$ are also $15\ \text{s}^{-1}$ and $0.015\ \text{lb/ft}^2$, respectively. **(b)** For the parabolic assumption, the parabola passes through the points $v = 0$ when $y = 0$ and $v = 45$ when $y = 3$. The equation of this parabola is $v = 45 - 5(3 - y)^2$, $dv/dy = 10(3 - y)$, $\tau = 0.0010\,(dv/dy)$. For $y = 0$ in, $v = 0$ in/s, $dv/dy = 30\ \text{s}^{-1}$, and $\tau = 0.030\ \text{lb/ft}^2$. For $y = 1$ in, $v = 25$ in/s, $dv/dy = 20\ \text{s}^{-1}$, and $\tau = 0.020\ \text{lb/ft}^2$. For $y = 2$ in, $v = 40$ in/s, $dv/dy = 10\ \text{s}^{-1}$, and $\tau = 0.010\ \text{lb/ft}^2$. For $y = 3$ in, $v = 45$ in/s, $dv/dy = 0\ \text{s}^{-1}$, and $\tau = 0\ \text{lb/ft}^2$.

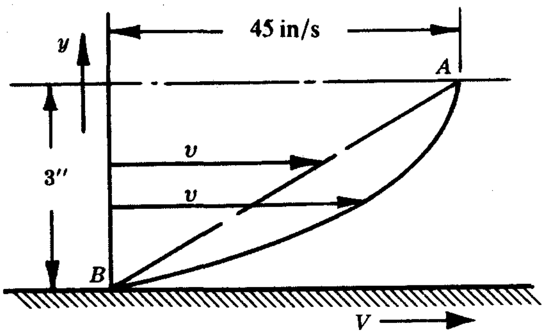



**Fig. 1-3**

**1.68** A cylinder of 0.40-ft radius rotates concentrically inside a fixed cylinder of 0.42-ft radius. Both cylinders are 1.00 ft long. Determine the viscosity of the liquid that fills the space between the cylinders if a torque of 0.650 lb · ft is required to maintain an angular velocity of 60 rpm.

The torque is transmitted through the field layers to the outer cylinder. Since the gap between the cylinders is small, the calculations may be made without integration. The tangential velocity $v_t$ of the inner cylinder $= r\omega$, where $r = 0.40$ ft and $\omega = 2\pi$ rad/s. Hence, $v_t = (0.40)(2\pi) = 2.51$ ft/s. For the small space between cylinders, the velocity gradient may be assumed to be a straight line and the mean radius can be used. Then, $dv/dy = (2.51 - 0)/(0.42 - 0.40) = 125.5\ \text{s}^{-1}$. Since applied torque equals resisting torque, applied torque $=$ $(\tau)(\text{area})(\text{arm})$, $0.650 = \tau[(1.00)(2\pi)(0.40 + 0.42)/2][(0.40 + 0.42)/2]$, $\tau = 0.615\ \text{lb/ft}^2 = \mu\,(dv/dy)$, $0.615 = (\mu)(125.5)$, $\mu = 0.00490\ \text{lb}\cdot\text{s/ft}^2$.

**1.69** Water is moving through a pipe. The velocity profile at some section is shown in Fig. 1-4 and is given mathematically as $v = (\beta/4\mu)(d^2/4 - r^2)$, where $v$ = velocity of water at any position $r$, $\beta$ = a constant, $\mu$ = viscosity of water, $d$ = pipe diameter, and $r$ = radial distance from centerline. What is the shear stress at the wall of the pipe due to the water? What is the shear stress at a position $r = d/4$? If the given profile persists a distance $L$ along the pipe, what drag is induced on the pipe by the water in the direction of flow over this distance?

$$v = (\beta/4\mu)(d^2/4 - r^2) \qquad dv/dr = (\beta/4\mu)(-2r) = -2\beta r/4\mu$$

$$\tau = \mu\,(dv/dr) = \mu(-2\beta r/4\mu) = -2\beta r/4$$

At the wall, $r = d/2$. Hence,

$$\tau_{\text{wall}} = \frac{-2\beta(d/2)}{4} = -\frac{\beta d}{4} \qquad \tau_{r=d/4} = \frac{-2\beta(d/4)}{4} = -\frac{\beta d}{8}$$

$$\text{Drag} = (\tau_{\text{wall}})(\text{area}) = (\tau_{\text{wall}})(\pi dL) = (\beta d/4)(\pi dL) = \beta d^2\pi L/4$$

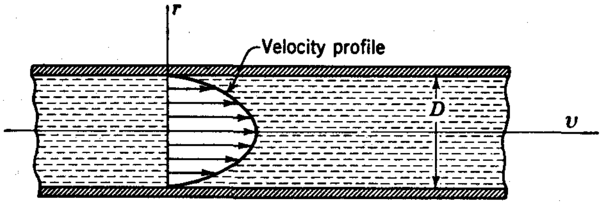



**Fig. 1-4**

**1.70** A large plate moves with speed $v_0$ over a stationary plate on a layer of oil (see Fig. 1-5). If the velocity profile is that of a parabola, with the oil at the plates having the same velocity as the plates, what is the shear stress on the moving plate from the oil? If a linear profile is assumed, what is the shear stress on the upper plate?

▌ For a parabolic profile, $v^2 = ay$. When $y = d$, $v = v_0$. Hence, $v_0^2 = ad$, $a = v_0^2/d$. Therefore,

$$v^2 = (v_0^2/d)(y) = (v_0^2)(y/d) \qquad v = v_0\sqrt{y/d} \qquad dv/dy = [(v_0)(1/\sqrt{d})(\tfrac{1}{2})(y^{-1/2})]$$

$$\tau = \mu\,(dv/dy) = \mu[(v_0)(1/\sqrt{d})(\tfrac{1}{2})(y^{-1/2})]$$

For $y = d$, $\tau = \mu[(v_0)(1/\sqrt{d})(\tfrac{1}{2})(d^{-1/2})] = \mu v_0/(2d)$. For a linear profile, $dv/dy = v_0/d$, $\tau = \mu(v_0/d)$.

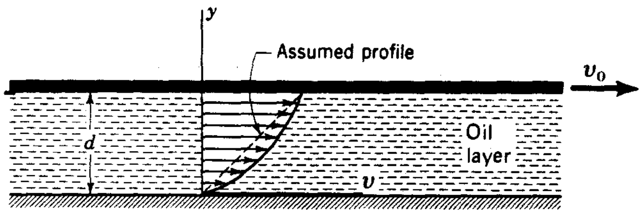



**Fig. 1-5**

**1.71** A square block weighing 1.1 kN and 250 mm on an edge slides down an incline on a film of oil 6.0 μm thick (see Fig. 1-6). Assuming a linear velocity profile in the oil, what is the terminal speed of the block? The viscosity of the oil is 7 mPa · s.

▌ $\tau = \mu\,(dv/dy) = (7 \times 10^{-3})[v_T/(6.0 \times 10^{-6})] = 1167v_T \qquad F_f = \tau A = (1167v_T)(0.250)^2 = 72.9v_T$

At the terminal condition, equilibrium occurs. Hence, $1100 \sin 20° = 72.9v_T$, $v_T = 5.16$ m/s.

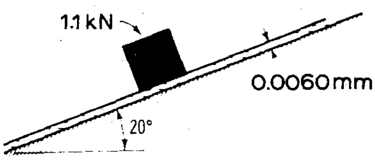



**Fig. 1-6(a)**

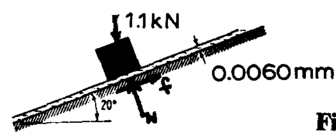



**Fig. 1-6(b)**

**1.72** A piston of weight 21 lb slides in a lubricated pipe, as shown in Fig. 1-7. The clearance between piston and pipe is 0.001 in. If the piston decelerates at 2.1 ft/s² when the speed is 21 ft/s, what is the viscosity of the oil?

▌

$$\tau = \mu\,(dv/dy) = \mu[v/(0.001/12)] = 12\,000\mu v$$

$$F_f = \tau A = 12\,000\mu v[(\pi)(\tfrac{6}{12})(\tfrac{5}{12})] = 7854\mu v$$

$$\Sigma F = ma \qquad 21 - (7854)(\mu)(21) = (21/32.2)(-2.1) \qquad \mu = 1.36 \times 10^{-4}\ \text{lb} \cdot \text{s/ft}^2$$

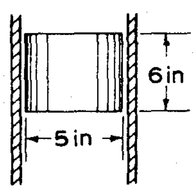



**Fig. 1-7**

**1.73** A piston is moving through a cylinder at a speed of 19 ft/s, as shown in Fig. 1-8. The film of oil separating the piston from the cylinder has a viscosity of 0.020 lb · s/ft². What is the force required to maintain this motion?

▌ Assume a cylindrically symmetric, linear velocity profile for the flow of oil in the film. To find the frictional resistance, compute the shear stress at the piston surface.

$$\tau = \mu\frac{dv}{dr} = 0.020\left[\frac{19}{(5.000 - 4.990)/2}\right](12) = 912\ \text{lb/ft}^2 \qquad F_f = \tau A = 912\left[\pi\left(\frac{4.990}{12}\right)\left(\frac{3}{12}\right)\right] = 298\ \text{lb}$$

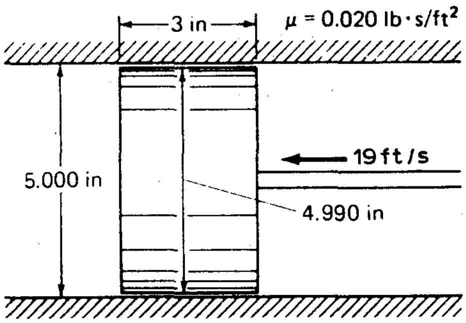



Fig. 1-8(a)

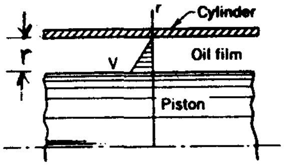



Fig. 1-8(b)

**1.74** To damp oscillations, the pointer of a galvanometer is fixed to a circular disk which turns in a container of oil (see Fig. 1-9). What is the damping torque for $\omega = 0.3$ rad/s if the oil has a viscosity of $8 \times 10^{-3}$ Pa · s? Neglect edge effects.

▌ Assume at any point that the velocity profile of the oil is linear $dv/dn = r\omega/(0.5/1000) = (r)(0.3)/(0.5/1000) = 600r$; $\tau = \mu\,(dv/dn) = \mu(600r) = (8 \times 10^{-3})(600r) = 4.80r$. The force $dF_f$ on $dA$ on the upper face of the disc is then $dF_f = \tau\, dA = (4.80r)(r\, d\theta\, dr) = 4.80r^2\, d\theta\, dr$. The torque $dT$ for $dA$ on the upper face is then $dT = r\, dF_f = r(4.80r^2\, d\theta\, dr) = 4.80r^3\, d\theta\, dr$. The total resisting torque on both faces is

$$T = 2\left[\int_0^{0.075/2}\int_0^{2\pi} 4.80r^3\, d\theta\, dr\right] = (9.60)(2\pi)\left[\frac{r^4}{4}\right]_0^{0.075/2} = 2.98 \times 10^{-5}\,\mathrm{N \cdot m}$$

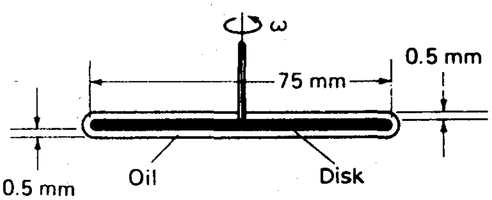



Fig. 1-9(a)

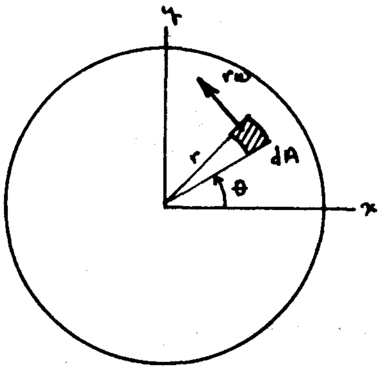



Fig. 1-9(b)

**1.75** For angular velocity 0.3 rad/s of the mechanism of Prob. 1.74, express the damping torque (in N · m) as a function of displacement $x$ (in mm) of the disk from its center position (Fig. 1-10).

▌ Assume at any point that the velocity profile of the oil is linear; $\tau = \mu\,(dv/dn)$. For the upper face, $dv/dn = r\omega/[(0.5 - x)/1000] = (r)(0.3)/[(0.5 - x)/1000]$; $\tau = (8 \times 10^{-3})\{(r)(0.3)/[(0.5 - x)/1000]\} = 2.40r/(0.5 - x)$. The force $dF_f$ on $dA$ on the upper face of the disc is then $dF_f = \tau\, dA = [2.40r/(0.5 - x)](r\, d\theta\, dr) = [2.40r^2/(0.5 - x)]\,(d\theta\, dr)$. The torque $dT$ for $dA$ on the upper face is then $dT = r\, dF_f = r[2.40r^2/(0.5 - x)]\,(d\theta\, dr) = [2.40r^3/(0.5 - x)]\,(d\theta\, dr)$. For the lower face, $dv/dn = r\omega/[(0.5 + x)/1000] = r(0.3)/[(0.5 + x)/1000]$; $\tau = (8 \times 10^{-3})\{r(0.3)/[(0.5 + x)/1000]\} = 2.40r/(0.5 + x)$. The force $dF_f$ on $dA$ on the lower face of the disc is then $dF_f = \tau\, dA = [2.40r/(0.5 + x)](r\, d\theta\, dr) = [2.40r^2/(0.5 + x)]\,(d\theta\, dr)$. The torque $dT$ for $dA$ on the lower face is then $dT = r\, dF_f = r[2.40r^2/(0.5 + x)]\,(d\theta\, dr) = [2.40r^3/(0.5 + x)]\,(d\theta\, dr)$. The total resisting torque on both faces is

$$\begin{aligned} T &= \int_0^{0.075/2}\int_0^{2\pi} \frac{2.40r^3}{0.5 - x}\, d\theta\, dr + \int_0^{0.075/2}\int_0^{2\pi} \frac{2.40r^3}{0.5 + x}\, d\theta\, dr \\ &= \left(\frac{1}{0.5 - x} + \frac{1}{0.5 + x}\right)(2.40)(2\pi)\left[\frac{r^4}{4}\right]_0^{0.075/2} = \left(\frac{0.5 + x + 0.5 - x}{0.25 - x^2}\right)(7.46 \times 10^{-6}) \\ &= \frac{7.46 \times 10^{-6}}{0.25 - x^2} \end{aligned}$$

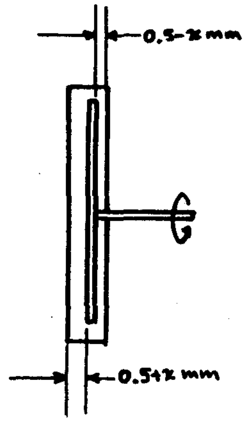



Fig. 1-10

**1.76** A conical body turns in a container, as shown in Fig. 1-11, at constant speed 11 rad/s. A uniform 0.01-in film of oil with viscosity $3.125 \times 10^{-7}$ lb · s/in² separates the cone from the container. What torque is required to maintain this motion, if the cone has a 2-in radius at its base and is 4 in tall?

▌ Consider the conical surface first ($r/2 = z/4$, $r = z/2$). The stress on this element is $\tau = \mu\,(dv/dx) = \mu(r\omega/0.01) = (3.125 \times 10^{-7})[(z/2)(11)/0.01] = 1.719 \times 10^{-4}z$. The area of the strip shown is $dA = 2\pi r\,ds = (2\pi z/2)[dz/(4/\sqrt{20})] = 3.512z\,dz$. The torque on the strip is $dT = \tau\,(dA)(r) = (1.719 \times 10^{-4}z)(3.512z\,dz)(z/2) = 3.019 \times 10^{-4}z^3\,dz$.

$$T_1 = \int_0^4 3.019 \times 10^{-4}z^3\,dz = 3.019 \times 10^{-4}\left[\frac{z^4}{4}\right]_0^4 = 0.01932 \text{ in} \cdot \text{lb}$$

Next consider the base: $dF_f = \tau\,dA$, $\tau = \mu(r\omega/0.01) = (3.125 \times 10^{-7})[(r)(11)/0.01] = 3.438 \times 10^{-4}r$, $dF_f = (3.438 \times 10^{-4}r)(r\,d\theta\,dr) = 3.438 \times 10^{-4}r^2\,d\theta\,dr$, $dT_2 = (3.438 \times 10^{-4}r^2\,d\theta\,dr)(r) = 3.438 \times 10^{-4}r^3\,d\theta\,dr$.

$$T_2 = \int_0^2\int_0^{2\pi} 3.438 \times 10^{-4}r^3\,d\theta dr = (3.438 \times 10^{-4})(2\pi)\left[\frac{r^4}{4}\right]_0^2 = 0.00864 \text{ in} \cdot \text{lb}$$

$$T_{tot} = 0.01932 + 0.00864 = 0.0280 \text{ in} \cdot \text{lb}$$

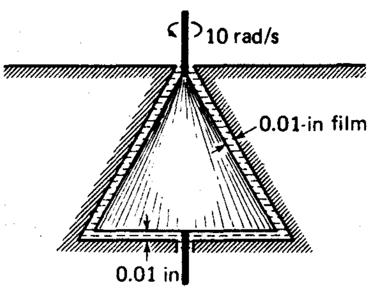



Fig. 1-11(*a*)

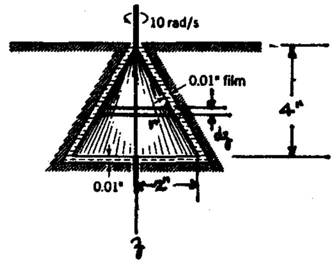



Fig. 1-11(*b*)

**1.77** In Fig. 1-12, if the fluid is SAE 30 oil at 20 °C and $D = 7$ mm, what shear stress is required to move the upper plate at 3.5 m/s? Compute the Reynolds number based on $D$.

▌
$$\tau = \mu\,(dv/dh) = (0.440)[3.5/(\tfrac{7}{1000})] = 220 \text{ Pa}$$
$$N_R = \rho Dv/\mu = (888)(\tfrac{7}{1000})(3.5)/0.440 = 49.4$$

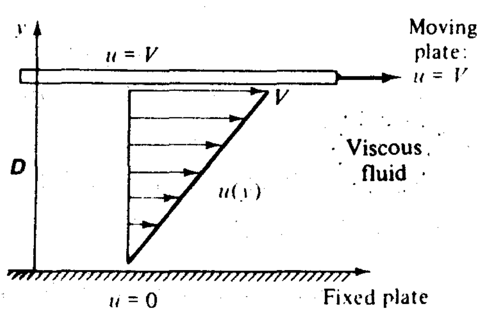



Fig. 1-12

**1.78** Benzene at 20 °C has a viscosity of 0.000651 Pa · s. What shear stress is required to deform this fluid at a strain rate of 4900 $s^{-1}$?

$$\tau = \mu\,(dv/dx) = (0.000651)(4900) = 3.19 \text{ Pa}$$

**1.79** SAE 30 oil at 20 °C is sheared between two parallel plates 0.005 in apart with the lower plate fixed and the upper plate moving at 13 ft/s. Compute the shear stress in the oil.

$$\tau = \mu\,(dv/dh) = (9.20 \times 10^{-3})[13/(0.005/12)] = 287 \text{ lb/ft}^2$$

**1.80** An 18-kg slab slides down a 15° inclined plane on a 3-mm-thick film of SAE 10 oil at 20 °C; the contact area is 0.3 $m^2$. Find the terminal velocity of the slab.

See Fig. 1-13.

$$\Sigma F_x = 0 \qquad W \sin\theta - \tau A_{\text{bottom}} = 0$$

$$\tau = \mu\,(dv/dy) = (8.14 \times 10^{-2})(v_T/0.003) = 27.1 v_T$$

$$[(18)(9.81)](\sin 15°) - (27.1 v_T)(0.3) = 0 \qquad v_T = 5.62 \text{ m/s}$$

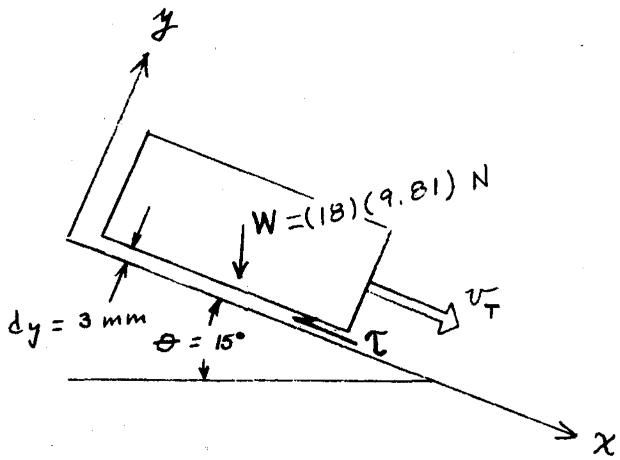



**Fig. 1-13**

**1.81** A shaft 70.0 mm in diameter is being pushed at a speed of 400 mm/s through a bearing sleeve 70.2 mm in diameter and 250 mm long. The clearance, assumed uniform, is filled with oil at 20 °C with $\nu = 0.005$ $m^2$/s and s.g. = 0.9. Find the force exerted by the oil on the shaft.

$$F = \tau A \qquad \tau = \mu\,(dv/dr) \qquad \mu = \rho\nu = [(0.9)(998)](0.005) = 4.49 \text{ kg/(m} \cdot \text{s)}$$

$$dr = (0.0702 - 0.0700)/2 = 0.0001 \text{ m} \qquad \tau = (4.49)(0.4/0.0001) = 17\,960 \text{ N/m}^2$$

$$A = (\pi)(7.00/100)(25/100) = 0.05498 \text{ m}^2 \qquad F = (17\,960)(0.05498) = 987 \text{ N}$$

**1.82** If the shaft in Prob. 1.81 is fixed axially and rotated inside the sleeve at 2000 rpm, determine the resisting torque exerted by the oil and the power required to rotate the shaft.

$$T = \tau A r \qquad \tau = \mu\,(dv/dr)$$

$$v = r\omega = [(7.00/2)/100][(2000)(2\pi/60)] = 7.330 \text{ m/s} \qquad dr = 0.0001 \text{ m}$$

$$\tau = (4.49)(7.330/0.0001) = 329.1 \times 10^3 \text{ N/m}^2 \qquad A = (\pi)(7.00/100)(\tfrac{25}{100}) = 0.05498 \text{ m}^2$$

$$T = (329.1 \times 10^3)(0.05498)[(7.00/2)/100] = 633 \text{ N} \cdot \text{m}$$

$$P = \omega T = [(2000)(2\pi/60)](633) = 132.6 \times 10^3 \text{ W} \quad \text{or} \quad 132.6 \text{ kW}$$

**1.83** A steel (7850-kg/$m^3$) shaft 40.0 mm in diameter and 350 mm long falls of its own weight inside a vertical open tube 40.2 mm in diameter. The clearance, assumed uniform, is a film of SAE 30 oil at 20 °C. What speed will the cylinder ultimately reach?

$$W_{shaft} = \tau A = [(7850)(9.81)][(0.350)(\pi)(0.0400)^2/4] = 33.87 \text{ N}$$
$$dr = (0.0402 - 0.0400)/2 = 0.0001 \text{ m}$$
$$\tau = \mu\,(dy/dr) = (0.440)(v_T/0.0001) = 4400v_T$$
$$A = (\pi)(4.00/100)(\tfrac{35}{100}) = 0.04398 \text{ m}^2 \qquad 33.87 = (4400v_T)(0.04398) \qquad v_T = 0.1750 \text{ m/s}$$

**1.84** Air at 20 °C forms a boundary layer near a solid wall, in which the velocity profile is sinusoidal (see Fig. 1-14). The boundary-layer thickness is 7 mm and the peak velocity is 9 m/s. Compute the shear stress in the boundary layer at $y$ equal to (*a*) 0, (*b*) 3.5 mm, and (*c*) 7 mm.

$$\tau = \mu\,(dv/dy) \qquad v = v_{max} \sin[\pi y/(2\delta)]$$
$$dv/dy = [\pi v_{max}/(2\delta)]\cos[\pi y/(2\delta)] = \{(\pi)(9)/[(2)(0.007)]\}\cos\{\pi y/[(2)(0.007)]\} = 2020\cos(224.4y)$$

**Note:** "224.4$y$" in the above equation is in radians.

$$\tau = (1.81 \times 10^{-5})[2020\cos(224.4y)] = 0.03656\cos(224.4y)$$

(*a*) At $y = 0$, $\tau = 0.03656\cos[(224.4)(0)] = 0.0366$ Pa. (*b*) At $y = 0.0035$ m, $\tau = 0.03656\cos[(224.4)(0.0035)] = 0.0259$ Pa. (*c*) At $y = 0.007$ m, $\tau = 0.03656\cos[(224.4)(0.007)] = 0$.

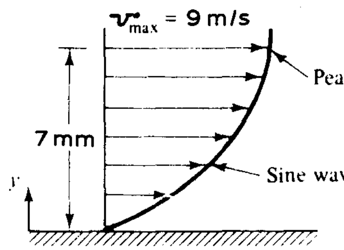



**Fig. 1-14**

**1.85** A disk of radius $r_0$ rotates at angular velocity $\omega$ inside an oil bath of viscosity $\mu$, as shown in Fig. 1-15. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.

$$\tau = \mu\,(dv/dy) = \mu(r\omega/h) \qquad \text{(on both sides)}$$
$$dT = (2)(r\tau\,dA) = (2)\{(r)[\mu(r\omega/h)](2\pi r\,dr)\} = (4\mu\omega\pi/h)(r^3\,dr)$$
$$T = \int_0^{r_0} \frac{4\mu\omega\pi}{h}(r^3\,dr) = \frac{4\mu\omega\pi}{h}\left[\frac{r^4}{4}\right]_0^{r_0} = \frac{\pi\mu\omega r_0^4}{h}$$

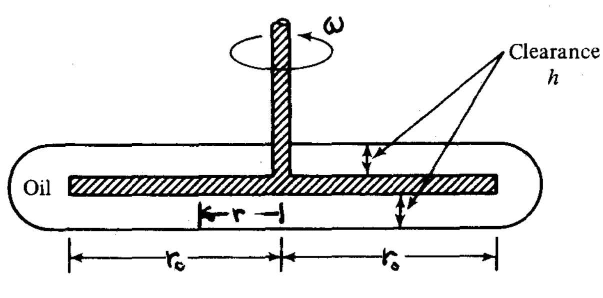



**Fig. 1-15**

**1.86** A 35-cm-by-55-cm block slides on oil ($\mu = 0.81$ Pa · s) over a large plane surface. What force is required to drag the block at 3 m/s, if the separating oil film is 0.6 mm thick?

$$\tau = \mu\,(dv/dx) = (0.81)[3/(0.6/1000)] = 4050 \text{ N/m}^2 \qquad F = \tau A = (4050)[(\tfrac{35}{100})(\tfrac{55}{100})] = 780 \text{ N}$$

**1.87** The 1.5-in (0.125-ft) gap between two large plane surfaces is filled with SAE 30 oil at 80 °F ($\mu = 0.0063$ lb · s/ft²). What force is required to drag a very thin plate of 5-ft² area between the surfaces at a speed of 0.5 ft/s if this plate is equally spaced between the two surfaces?

$$\tau = \mu\,(dv/dx) = (0.0063)[0.5/(0.125/2)] = 0.0504 \text{ lb/ft}^2 \qquad F = \tau A = (0.0504)(5) = 0.252 \text{ lb}$$

Since there are two sides, $F_{required} = (2)(0.252)$, or 0.504 lb.

**1.88** Rework Prob. 1.87 if the plate is at a distance of 0.50 in (0.0417 ft) from one surface.

▮

$$\tau = \mu\,(dv/dx) \qquad \tau_1 = (0.0063)(0.5/0.0417) = 0.0755\ \text{lb/ft}^2$$

$$F = \tau A \qquad F_1 = (0.0755)(5) = 0.3775\ \text{lb} \qquad \tau_2 = (0.0063)[0.5/(0.125 - 0.0417)] = 0.0378\ \text{lb/ft}^2$$

$$F_2 = (0.0378)(5) = 0.1890\ \text{lb} \qquad F_{\text{required}} = F_1 + F_2 = 0.3775 + 0.1890 = 0.566\ \text{lb}$$

**1.89** A 10.000-in-diameter plunger slides in a 10.006-in-diameter cylinder, the annular space being filled with oil having a kinematic viscosity of 0.004 $\text{ft}^2/\text{s}$ and specific gravity of 0.85. If the plunger moves at 0.6 ft/s, find the frictional resistance when 9 ft is engaged in the cylinder.

▮

$$\tau = \mu\,(dv/dx) \qquad \rho = \gamma/g = [(0.85)(62.4)]/32.2 = 1.647\ \text{slugs/ft}^3$$

$$\mu = \rho\nu = (1.647)(0.004) = 0.006588\ \text{lb}\cdot\text{s/ft}^2 \qquad dx = [(10.006 - 10.000)/2]/12 = 0.000250\ \text{ft}$$

$$\tau = (0.006588)(0.6/0.000250) = 15.81\ \text{lb/ft}^2 \qquad F_f = \tau A = (15.81)[(9)(\pi)(\tfrac{10}{12})] = 373\ \text{lb}$$

**1.90** A 6.00-in shaft rides in a 6.01-in sleeve 8 in long, the clearance space (assumed to be uniform) being filled with lubricating oil at 100 °F ($\mu = 0.0018\ \text{lb}\cdot\text{s/ft}^2$). Calculate the rate at which heat is generated when the shaft turns at 90 rpm.

▮

$$dv = \omega(\text{circumference}) = \tfrac{90}{60}[\pi(6.00/12)] = 2.356\ \text{ft/s}$$

$$dx = [(6.01 - 6.00)/2]/12 = 0.0004167\ \text{ft}$$

$$\tau = \mu\,(dv/dx) = (0.0018)(2.356/0.0004167) = 10.18\ \text{lb/ft}^2$$

$$F_f = \tau A = 10.18[\pi(8.00/12)(\tfrac{6}{12})] = 10.66\ \text{lb}$$

$$\text{Rate of energy loss} = F_f v = (10.66)(2.356) = 25.11\ \text{ft}\cdot\text{lb/s}$$

$$\text{Rate of heat generation} = (25.11)(3600)/778 = 116\ \text{Btu/h}$$

**1.91** A 10.00-cm shaft rides in an 10.03-cm sleeve 12 cm long, the clearance space (assumed to be uniform) being filled with lubricating oil at 40 °C ($\mu = 0.11\ \text{Pa}\cdot\text{s}$). Calculate the rate at which heat is generated when the shaft turns at 100 rpm.

▮

$$dv = \omega(\text{circumference}) = \tfrac{100}{60}[\pi(0.10)] = 0.5236\ \text{m/s} \qquad dx = (0.1003 - 0.1000)/2 = 0.00015\ \text{m}$$

$$\tau = \mu\,(dv/dx) = (0.11)(0.5236)/0.00015) = 384.0\ \text{N/m}^2$$

$$F_f = \tau A = 384.0[\pi(0.12)(0.10)] = 14.48\ \text{N}$$

$$\text{Rate of energy loss} = F_f v = (14.48)(0.5236) = 7.582\ \text{N}\cdot\text{m/s} = 7.582\ \text{W}$$

**1.92** In using a rotating-cylinder viscometer, a bottom correction must be applied to account for the drag on the flat bottom of the cylinder. Calculate the theoretical amount of this torque correction, neglecting centrifugal effects, for a cylinder of diameter $d$, rotated at a constant angular velocity $\omega$, in a liquid of viscosity $\mu$, with a clearance $\Delta h$ between the bottom of the inner cylinder and the floor of the outer one.

▮ Let $r$ = variable radius. $T = \int r\tau\,dA$, $\tau = \mu\,(dv/dx) = \mu(r\omega/\Delta h)$, $dA = 2\pi r\,dr$.

$$T = \int_0^{d/2} r\left[\mu\left(\frac{r\omega}{\Delta h}\right)\right](2\pi r\,dr) = \frac{2\pi\mu\omega}{\Delta h}\int_0^{d/2} r^3\,dr = \frac{2\pi\mu\omega}{\Delta h}\left[\frac{r^4}{4}\right]_0^{d/2} = \frac{\pi\mu\omega d^4}{32\,\Delta h}$$

**1.93** Assuming a boundary-layer velocity distribution as shown in Fig. 1-16, which is a parabola having its vertex 3 in from the wall, calculate the shear stresses for $y = 0$, 1 in, 2 in, and 3 in. Use $\mu = 0.00835\ \text{lb}\cdot\text{s/ft}^2$.

▮ $\tau = \mu\,(dv/dy)$. At $y = 0$, $v = 0$ and at $y = 3$ in, $v = 6$ ft/s, or 72 in/s. The equation of the parabola is $v = 72 - (8)(3 - y)^2$ ($y$ in inches gives $v$ in inches per second); $dv/dy = (16)(3 - y)$; $\tau = (0.00835)[(16)(3 - y)] = 0.4008 - 0.1336y$. At $y = 0$, $\tau = 0.4008 - (0.1336)(0) = 0.401\ \text{lb/ft}^2$. At $y = 1$ in, $\tau = 0.4008 - (0.1336)(1) = 0.267\ \text{lb/ft}^2$. At $y = 2$ in, $\tau = 0.4008 - (0.1336)(2) = 0.134\ \text{lb/ft}^2$. At $y = 3$ in, $\tau = 0.4008 - (0.1336)(3) = 0$.

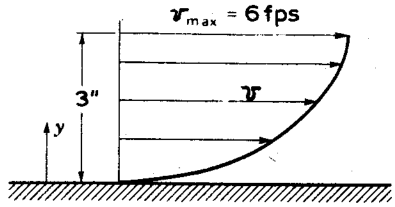



**Fig. 1-16**

**1.94** In Fig. 1-17*a*, oil of viscosity $\mu$ fills the small gap of thickness $Y$. Determine an expression for the torque $T$ required to rotate the truncated cone at constant speed $\omega$. Neglect fluid stress exerted on the circular bottom.

▌ See Fig. 1-17*b*. $\tau = \mu\,(dv/dy)$, $v = r\omega = (y \tan\alpha)(\omega)$, $dv/dy = (y\tan\alpha)(\omega)/Y$.

$$\tau = \mu\left[\frac{(y\tan\alpha)(\omega)}{Y}\right] = \frac{\mu y\omega\tan\alpha}{Y}$$

$$dA = 2\pi r\,ds = 2\pi(y\tan\alpha)(dy/\cos\alpha) = 2\pi y(\tan\alpha/\cos\alpha)(dy)$$

$$dF = \tau\,dA = \left(\frac{\mu y\omega\tan\alpha}{Y}\right)\left[2\pi y\left(\frac{\tan\alpha}{\cos\alpha}\right)(dy)\right] = \left(\frac{2\pi\mu\omega\tan^2\alpha}{Y\cos\alpha}\right)y^2\,dy$$

$$dT = r\,dF = (y\tan\alpha)\left(\frac{2\pi\mu\omega\tan^2\alpha}{Y\cos\alpha}\right)y^2\,dy = \left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)y^3\,dy$$

$$T = \int_a^{a+b}\left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)y^3\,dy = \left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)\left[\frac{y^4}{4}\right]_a^{a+b} = \left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)\left[\frac{(a+b)^4}{4} - \frac{a^4}{4}\right]$$

$$= \left(\frac{\pi\mu\omega\tan^3\alpha}{2Y\cos\alpha}\right)[(a+b)^4 - a^4]$$

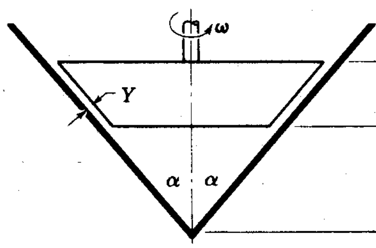



**Fig. 1-17(*a*)**

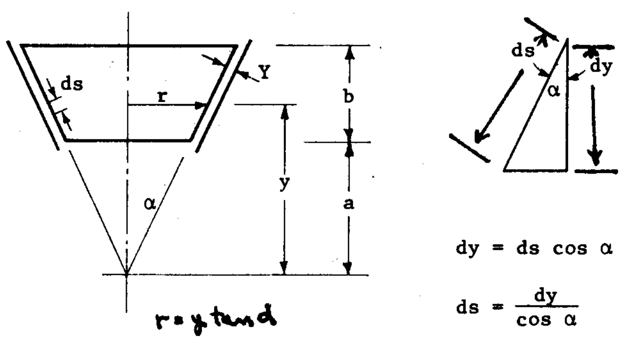



**Fig. 1-17(*b*)**

**1.95** A Newtonian fluid fills the gap between a shaft and a concentric sleeve. When a force of 788 N is applied to the sleeve parallel to the shaft, the sleeve attains a speed of 2 m/s. If a 1400-N force is applied, what speed will the sleeve attain? The temperature of the sleeve remains constant.

▌ $\tau = F/A = \mu\,(dv/dx)$; $F/dv = \mu A/dx$ = constant. Therefore, $F_1/dv_1 = F_2/dv_2$, $\frac{788}{2} = 1400/dv_2$, $dv_2 = 3.55$ m/s.

**1.96** A plate separated by 0.5 mm from a fixed plate moves at 0.50 m/s under a force per unit area of 4.0 N/m². Determine the viscosity of the fluid between the plates.

▌ $\tau = \mu\,(dv/dx) \qquad 4.0 = \mu[0.50/(0.0005)] \qquad \mu = 0.00400\ \mathrm{N\cdot s/m^2} = 4.00\ \mathrm{mPa\cdot s}$

**1.97** Determine the viscosity of fluid between shaft and sleeve in Fig. 1-18.

▌ $\tau = F/A = \mu\,(dv/dx) \qquad 25/[(\pi)(\frac{4}{12})(\frac{9}{12})] = \mu[0.5/(0.004/12)] \qquad \mu = 0.0212\ \mathrm{lb\cdot s/ft^2}$

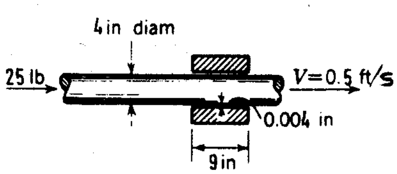



**Fig. 1-18**

**1.98** A 1-in-diameter steel cylinder 10 in long falls at 0.6 ft/s inside a tube of slightly larger diameter. A castor-oil film of constant thickness is between the cylinder and the tube. Determine the clearance between the cylinder and the tube, if the temperature is 100 °F, s.g. = 7.85 for steel, and $\mu = 6 \times 10^{-3}$ lb · s/ft$^2$ for castor oil.

▮
$$\tau = F/A = \mu\,(dv/dx)$$
$$F = W = \gamma V = [(7.85)(62.4)][(\tfrac{10}{12})(\pi)(\tfrac{1}{12})^2/4] = 2.226 \text{ lb} \quad 2.226/[(\tfrac{10}{12})(\pi)(\tfrac{1}{12})] = (6 \times 10^{-3})(0.6/dx)$$
$$dx = 0.0003528 \text{ ft} \quad \text{or} \quad 0.00423 \text{ in}$$

**1.99** A piston of diameter 70.00 mm moves inside a cylinder of diameter 70.10 mm. Determine the percent decrease in force necessary to move the piston when the lubricant warms from 0 to 120 °C. Values of $\mu$ for the lubricant are 0.01820 Pa · s at 0 °C and 0.00206 Pa · s at 120 °C.

▮ $\tau = F/A = \mu\,(dv/dx)$; $F/\mu = A\,(dv/dx) =$ constant. Therefore, $\Delta F/F_{0\,^\circ\mathrm{C}} = \Delta\mu/\mu_{0\,^\circ\mathrm{C}} = (0.01820 - 0.00206)/0.01820 = 0.887$, or 88.7%.

**1.100** A body weighing 100 lb with a flat surface area of 3 ft$^2$ slides down a lubricated inclined plane making a 35° angle with the horizontal. For viscosity of 0.002089 lb · s/ft$^2$ and a body speed of 3.5 ft/s, determine the lubricant film thickness.

▮
$$F = \text{weight of body along inclined plane} = 100 \sin 35° = 57.4 \text{ lb}$$
$$\tau = F/A = \mu\,(dv/dx) \qquad 57.4/3 = (0.002089)(3.5/dx) \qquad dx = 0.0003821 \text{ ft} \quad \text{or} \quad 0.00459 \text{ in}$$

**1.101** A small drop of water at 80 °F is in contact with the air and has a diameter of 0.0200 in. If the pressure within the droplet is 0.082 psi greater than the atmosphere, what is the value of the surface tension?

▮
$$p(\pi d^2/4) = (\pi d)(\sigma) \qquad \sigma = pd/4 = [(0.082)(144)](0.0200/12)/4 = 0.00492 \text{ lb/ft}$$

**1.102** Estimate the height to which water at 70 °F will rise in a capillary tube of diameter 0.120 in.

▮ $h = 4\sigma \cos\theta/(\gamma d)$. From Table A-1, $\sigma = 0.00500$ lb/ft and $\gamma = 62.3$ lb/ft$^3$ at 70 °F. Assume $\theta = 0°$ for a clean tube. $h = (4)(0.00500)(\cos 0°)/[(62.3)(0.120/12)] = 0.0321$ ft, or 0.385 in.

**1.103** The shape of a hanging drop of liquid is expressible by the following formulation developed from photographic studies of the drop: $\sigma = (\gamma - \gamma_0)(d_e)^2/H$, where $\sigma$ = surface tension, i.e., force per unit length, $\gamma$ = specific weight of liquid drop, $\gamma_0$ = specific weight of vapor around it, $d_e$ = diameter of drop at its equator, and $H$ = a function determined by experiment. For this equation to be dimensionally homogeneous, what dimensions must $H$ possess?

▮ Dimensionally, $(F/L) = (F/L^3)(L^2)/\{H\}$, $\{H\} = (1)$. Therefore, $H$ is dimensionless.

**1.104** Two clean, parallel glass plates, separated by a distance $d = 1.5$ mm, are dipped in a bath of water. How far does the water rise due to capillary action, if $\sigma = 0.0730$ N/m?

▮ Because the plates are clean, the angle of contact between water and glass is taken as zero. Consider the free-body diagram of a unit width of the raised water (Fig. 1-19). Summing forces in the vertical direction gives $(2)[(\sigma)(0.0015)] - (0.0015)^2(h)(\gamma) = 0$, $(2)[(0.0730)(0.0015)] - (0.0015)^2(h)(9790) = 0$, $h = 0.00994$ m, or 9.94 mm.

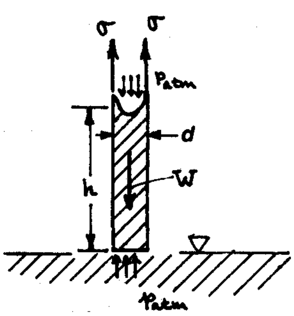



**Fig. 1-19(c)**

**1.105** A glass tube is inserted in mercury (Fig. 1-20); the common temperature is 20 °C. What is the upward force on the glass as a result of surface effects?

$F = (\sigma)(\pi d_o)(\cos 50°) + (\alpha)(\pi d_i)(\cos 50°) = (0.514)[(\pi)(0.035)](\cos 50°) + (0.514)[(\pi)(0.025)](\cos 50°) = 0.0623$ N

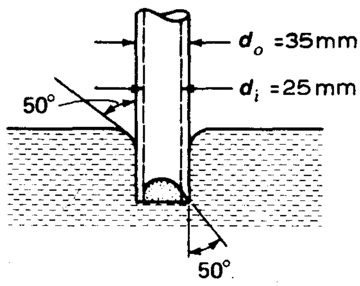



Fig. 1-20

**1.106** In Fig. 1-21*a* estimate the depression $h$ for mercury in the glass capillary tube. Angle $\theta$ is 40°.

Consider the meniscus of the mercury as a free body (see Fig. 1-21*b*) of negligible weight. Summing forces in the vertical direction gives $-(\sigma)(\pi d)(\cos\theta) + (p)(\pi d^2/4) = 0$, $-(0.514)[(\pi)(0.002)](\cos 40°) + [(13.6)(9790)(h)][(\pi)(0.002)^2/4] = 0$, $h = 0.00591$ m, or 5.91 mm. Actual $h$ must be larger because the weight of the meniscus was neglected.

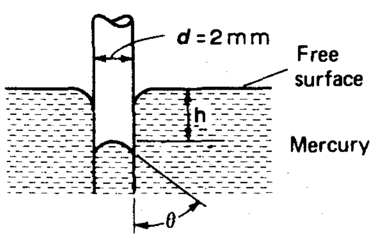



Fig. 1-21(*a*)

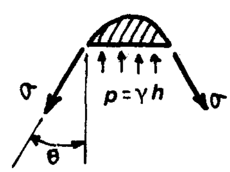



Fig. 1-21(*b*)

**1.107** A narrow trough (Fig. 1-22) is filled with water at 20 °C to the maximum extent. If the gage measures a gage pressure of 2.8458 kPa, what is the radius of curvature of the water surface (away from the ends)?

$$p = \sigma/r = p_{gage} - \gamma d = 2845.8 - (9790)(0.290) = 6.70 \text{ Pa gage}$$

$$6.70 = 0.0728/r \qquad r = 0.01087 \text{ m} \quad \text{or} \quad 10.87 \text{ mm}$$

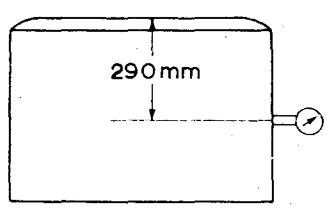



Fig. 1-22

**1.108** Water at 10 °C is poured into a region between concentric cylinders until water appears above the top of the open end (see Fig. 1-23). If the pressure measured by a gage 42 cm below the open end is 4147.38 Pa gage, what is the curvature of the water at the top?

$$p = \sigma/r = p_{gage} - \gamma d = 4147.38 - (9810)(0.42) = 27.18 \text{ Pa gage}$$

$$27.18 = 0.0742/r \qquad r = 0.00273 \text{ m} \quad \text{or} \quad 2.73 \text{ mm}$$

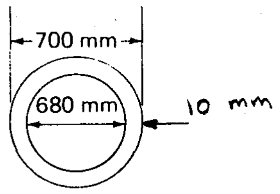



**Fig. 1-23**

**1.109** The rate of twist $\alpha$ of a shaft of any shape may be found by using *Prandtl's soap-film analogy*. A soap film is attached to a sharp edge having the shape of the outside boundary of the shaft cross section (a rectangle here, as shown in Fig. 1-24). Air pressure is increased under the film so that it forms an elevated curved surface above the boundary. Then

$$\alpha = \frac{M_x \Delta p}{4\sigma G V} \qquad \text{(radians per unit length)}$$

where $\Delta p$ = gage air pressure under the soap film, $M_x$ = torque transmitted by actual shaft, $G$ = shear modulus of actual shaft, and $V$ = volume of air under the soap film and above the cross section formed by the sharp edge. For the case at hand, $\Delta p = 0.4\ \text{lb/ft}^2$ gage and $V = 0.5\ \text{in}^3$. The angle $\theta$ along the long edge of the cross section is measured optically to be 30°. For a torque of 600 lb · ft on a shaft having $G = 10 \times 10^6\ \text{lb/in}^2$, what angle of twist does this analogy predict?

▮

$$\alpha = \frac{M_x \Delta p}{4\sigma G V}$$

To get $\sigma$, consider a unit length of the long side of the shaft cross section away from the ends (see Fig. 1-24*c*). For equilibrium of the film in the vertical direction (remembering there are two surfaces on each side) $(-4)[(\sigma)(L)(\cos\theta)] + pA = 0$, $(-4)[(\sigma)(\frac{1}{12})(\cos 30°)] + (0.4)[(0.5)(1)/144] = 0$, $\sigma = 0.00481$ lb/ft;

$$\alpha = \frac{(600)(0.4)}{(4)(0.00481)[(10 \times 10^6)(144)](0.5/1728)} = 0.0299\ \text{rad/ft}$$

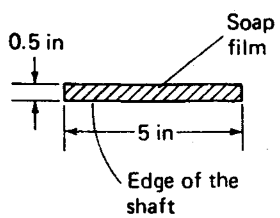



**Fig. 1-24(*a*)**

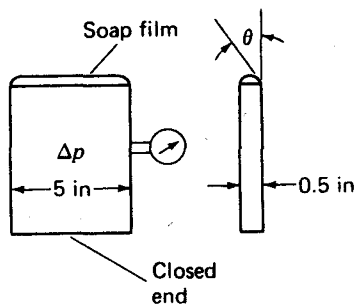



**Fig. 1-24(*b*)**

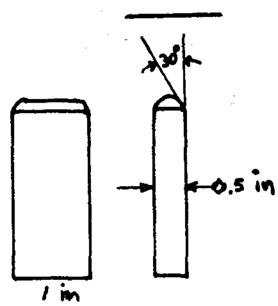



**Fig. 1-24(*c*)**

**1.110** In using Prandtl's soap-film analogy (see Prob. 1.109), we wish to check the mechanism for measuring the pressure $\Delta p$ under the soap film. Accordingly, we use a circular cross section (Fig. 1-25) for which we have an accurate theory for determining the rate of twist $\alpha$. The surface tension for the soap film is 0.1460 N/m and volume $V$ under the film is measured to be $0.001120\ \text{m}^3$. Compute $\Delta p$ from consideration of the soap film and from solid mechanics using the equation given in Prob. 1.109 and the well-known formula from strength of materials

$$\alpha = \frac{M_x}{GJ}$$

where $J$, the polar moment of inertia, is $\pi r^4/2$. Compare the results.

▌ From consideration of the film (see Fig. 1-25), $-2\sigma\pi\, d\cos 45° + (\Delta p)(\pi d^2)/4 = 0$, $-(2)(0.1460)(\pi)(\frac{200}{1000})(\cos 45°) + (\Delta p)[(\pi)(\frac{200}{1000})^2/4] = 0$, $\Delta p = 4.13$ Pa gage. From strength of materials, equate $\alpha$'s for the equations given in this problem and in Prob. 1.109.

$$\frac{M_x \Delta p}{4\sigma GV} = \frac{M_x}{GJ} \qquad J = \frac{\pi[(\frac{200}{2})/1000]^4}{2} = 0.0001571\ \text{m}^4 \qquad \frac{\Delta p}{(4)(0.1460)(0.001120)} = \frac{1}{0.0001571} \qquad \Delta p = 4.16\ \text{Pa gage}$$

The pressure measurement is quite close to what is expected from theory.

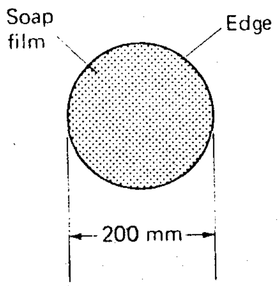



**Fig. 1-25(*a*)**

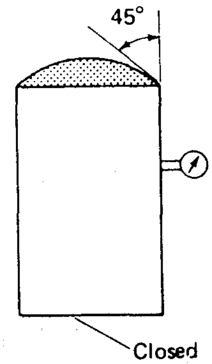



**Fig. 1-25(*b*)**

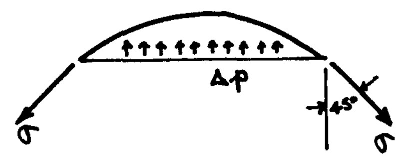



**Fig. 1-25(*c*)**

**1.111** Find the capillary rise in the tube shown in Fig. 1-26 for a water–air–glass interface ($\theta = 0°$) if the tube radius is 1 mm and the temperature is 20 °C.

▌
$$h = \frac{2\sigma\cos\theta}{\rho g r} = \frac{(2)(0.0728)(\cos 0°)}{(1000)(9.81)(\frac{1}{1000})} = 0.0148\ \text{m} \quad \text{or} \quad 14.8\ \text{mm}$$

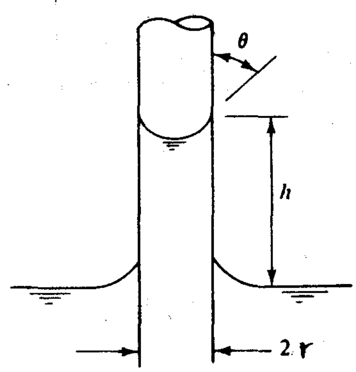



**Fig. 1-26**

**1.112** Find the capillary rise in the tube shown in Fig. 1-26 for a mercury–air–glass interface with $\theta = 130°$ if the tube radius is 1 mm and the temperature is 20 °C.

▌
$$h = \frac{2\sigma\cos\theta}{\rho g r} = \frac{(2)(0.514)(\cos 130°)}{(13\,570)(9.81)(\frac{1}{1000})} = -0.0050\ \text{m} \quad \text{or} \quad -5.0\ \text{mm}$$

**1.113** If a bubble is equivalent to an air–water interface with $\sigma = 0.005$ lb/ft, what is the pressure difference between the inside and outside of a bubble of diameter 0.003 in?

▌
$$p = 2\sigma/r = (2)(0.005)/[(0.003/2)/12] = 80.0\ \text{lb/ft}^2$$

**1.114** A small circular jet of mercury 200 $\mu$m in diameter issues from an opening. What is the pressure difference between the inside and outside of the jet at 20 °C?

▌ See Fig. 1-27. Equating the force due to surface tension ($2\sigma L$) and the force due to pressure ($pDL$), $2\sigma L = pDL$, $p = 2\sigma/D = (2)(0.514)/(200 \times 10^{-6}) = 5140$ Pa.

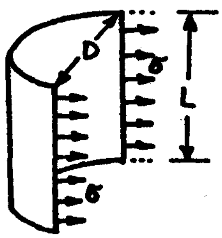



Fig. 1-27

**1.115** The surface tensions of mercury and water at 60 °C are 0.47 N/m and 0.0662 N/m, respectively. What capillary-height changes will occur in these two fluids when they are in contact with air in a glass tube of radius 0.30 mm? Use $\theta = 130°$ for mercury, and 0° for water; $\gamma = 132.3\ \text{kN/m}^3$ for mercury, and $9.650\ \text{kN/m}^3$ for water.

▮
$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

For mercury:

$$h = \frac{(2)(0.47)(\cos 130°)}{(132\,300)(0.30/1000)} = -0.0152\ \text{m} \quad \text{or} \quad -15.2\ \text{mm}$$

For water:

$$h = \frac{(2)(0.0662)(\cos 0°)}{(9650)(0.30/1000)} = 0.0457\ \text{m} \quad \text{or} \quad 45.7\ \text{mm}$$

**1.116** At 30 °C what diameter glass tube is necessary to keep the capillary-height change of water less than 2 mm?

▮
$$h = \frac{2\sigma\cos\theta}{\rho g r} \qquad \frac{2}{1000} = \frac{(2)(0.0712)(\cos 0°)}{(996)(9.81)(r)}$$

$$r = 0.00729\ \text{m} \quad \text{or} \quad 7.29\ \text{mm} \qquad d = (2)(7.29) = 14.6\ \text{mm (or greater)}$$

**1.117** A 1-in-diameter soap bubble has an internal pressure 0.0045 lb/in² greater than that of the outside atmosphere. Compute the surface tension of the soap–air interface. Note that a soap bubble has two interfaces with air, an inner and outer surface of nearly the same radius.

▮
$$p = 4\sigma/r \qquad (0.0045)(144) = (4)(\sigma)/[(\tfrac{1}{2})/12] \qquad \sigma = 0.00675\ \text{lb/ft}$$

**1.118** What force is required to lift a thin wire ring 6 cm in diameter from a water surface at 20 °C?

▮ Neglecting the weight of the wire, $F = \sigma L$. Since there is resistance on the inside and outside of the ring, $F = (2)(\sigma)(\pi d) = (2)(0.0728)[(\pi)(0.06)] = 0.0274\ \text{N}$.

**1.119** The glass tube in Fig. 1-28 is used to measure pressure $p_1$ in the water tank. The tube diameter is 1 mm and the water is at 30 °C. After correcting for surface tension, what is the true water height in the tube?

▮
$$h = \frac{2\sigma\cos\theta}{\rho g r} = \frac{(2)(0.0712)(\cos 0°)}{(996)(9.81)[(\tfrac{1}{2})/1000]} = 0.029\ \text{m} \quad \text{or} \quad 2.9\ \text{cm}$$

True water height in the tube = 17 − 2.9 = 14.1 cm.

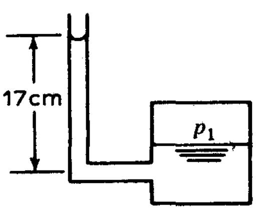



Fig. 1-28

**1.120** An atomizer forms water droplets 45 $\mu$m in diameter. Find the excess pressure within these droplets for water at 30 °C?

$$p = 2\sigma/r = (2)(0.0712)/[(45 \times 10^{-6})/2] = 6329 \text{ Pa}$$

**1.121** Rework Prob. 1.120 for droplets of 0.0018 in diameter and at 68 °F.

$$p = 2\sigma/r = (2)(0.005)/[(0.0018/2)/12] = 133 \text{ lb/ft}^2 \quad \text{or} \quad 0.93 \text{ lb/in}^2$$

**1.122** What is the pressure difference between the inside and outside of a cylindrical water jet when the diameter is 2.2 mm and the temperature is 10 °C? (See Fig. 1-27.)

$$p = \sigma/r = 0.0742/0.0011 = 67.5 \text{ Pa}$$

**1.123** Find the angle the surface tension film leaves the glass for a vertical tube immersed in water if the diameter is 0.25 in and the capillary rise is 0.08 in. Use $\sigma = 0.005$ lb/ft.

$$h = \frac{2\sigma\cos\theta}{\rho g r} \qquad \frac{0.08}{12} = \frac{(2)(0.005)(\cos\theta)}{(1.94)(32.2)[(0.25/2)/12]} \qquad \cos\theta = 0.433806 \qquad \theta = 64.3°$$

**1.124** Develop a formula for capillary rise between two concentric glass tubes of radii $r_o$ and $r_i$ and contact angle $\theta$.

See Fig. 1-29. Equating the force due to pressure and the force due to surface tension,

$$(h)(\gamma)(\pi r_o^2 - \pi r_i^2) = \sigma(2\pi r_i + 2\pi r_o)(\cos\theta)$$

$$h = \frac{(2)(\sigma)(r_i + r_o)(\cos\theta)}{\gamma(r_o^2 - r_i^2)} = \frac{2\sigma\cos\theta}{\gamma(r_o - r_i)}$$

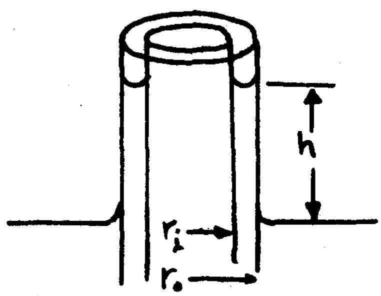



Fig. 1-29

**1.125** Distilled water at 10 °C stands in a glass tube of 9.0-mm diameter at a height of 24.0 mm. What is the true static height?

$$h = \frac{2\sigma\cos\theta}{\rho g r} = \frac{(2)(0.0742)(\cos 0°)}{(1000)(9.81)[(9.0/2)/1000]} = 0.0034 \text{ m} \quad \text{or} \quad 3.4 \text{ mm}$$

True static height = 24.0 − 3.4 = 20.6 mm.

**1.126** What capillary depression of mercury ($\theta = 140°$) may be expected in a 0.08-in-diameter tube at 68 °F?

$$h = \frac{2\sigma\cos\theta}{\rho g r} = \frac{(2)(0.0352)(\cos 140°)}{(26.34)(32.2)[(0.08/2)/12]} = -0.01908 \text{ ft} \quad \text{or} \quad -0.23 \text{ in}$$

**1.127** At the top of Mount Olympus water boils at 85 °C. Approximately how high is the mountain?

From Table A-2, water boiling at 85 °C corresponds to a vapor pressure of 58.8 kPa. From Table A-8, this corresponds to a standard atmosphere elevation of approximately 4200 m.

**1.128** At approximately what temperature will water boil at an elevation of 12 500 ft?

▮ From Table A-7, the pressure of the standard atmosphere at 12 500-ft evevation is 9.205 psia, or 1326 lb/ft$^2$ abs. From Table A-1, the saturation pressure of water is 1326 lb/ft$^2$ abs at about 189 °F. Hence, the water will boil at 193 °F; this explains why it takes longer to cook at high altitudes.

**1.129** At approximately what temperature will water boil in Denver (elevation 5280 ft)?

▮ From Table A-7, the pressure of the standard atmosphere at 5280-ft elevation is 12.12 psia, or 1745 lb/ft$^2$ abs. From Table A-1, the saturation pressure of water is 1745 lb/ft$^2$ abs at about 202 °F. Hence, the water will boil at 198 °F.

**1.130** Water at 105 °F is placed in a beaker within an airtight container. Air is gradually pumped out of the container. What reduction below standard atmospheric pressure of 14.7 psia must be achieved before the water boils?

▮ From Table A-1, $p_v = 162$ lb/ft$^2$ abs, or 1.12 psia at 105 °F. Hence, pressure must be reduced by 14.7 − 1.12, or 13.58 psi.

**1.131** At what pressure will 50 °C water boil?

▮ From Table A-2, $p_v = 12.3$ kPa at 50 °C. Hence, water will boil at 12.3 kPa.

**1.132** At what pressure will cavitation occur at the inlet of a pump that is drawing water at 25 °C?

▮ Cavitation occurs when the internal pressure drops to the vapor pressure. From Table A-2, the vapor pressure of water at 25 °C is 3.29 kPa.

**1.133** For low-speed (laminar) flow through a circular pipe, as shown in Fig. 1-30, the velocity distribution takes the form $v = (B/\mu)(r_0^2 - r^2)$, where $\mu$ is the fluid viscosity. What are the units of the constant $B$?

▮ Dimensionally, $(L/T) = [\{B\}/(M/LT)](L^2)$, $\{B\} = ML^{-2}T^{-2}$. In SI units, $B$ could be kg/(m$^2 \cdot$ s$^2$), or Pa/m.

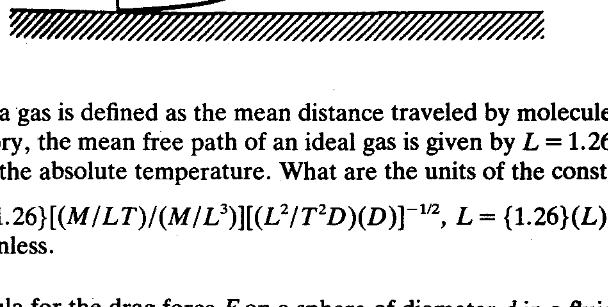



Fig. 1-30

**1.134** The mean free path $L$ of a gas is defined as the mean distance traveled by molecules between collisions. According to kinetic theory, the mean free path of an ideal gas is given by $L = 1.26(\mu/\rho)(RT)^{-1/2}$, where $R$ is the gas constant and $T$ is the absolute temperature. What are the units of the constant 1.26?

▮ Dimensionally, $L = \{1.26\}[(M/LT)/(M/L^3)][(L^2/T^2D)(D)]^{-1/2}$, $L = \{1.26\}(L)$, $\{1.26\} = 1$. Therefore, the constant 1.26 is dimensionless.

**1.135** The Stokes–Oseen formula for the drag force $F$ on a sphere of diameter $d$ in a fluid stream of low velocity $v$ is $F = 3\pi\mu dv + (9\pi/16)(\rho v^2 d^2)$. Is this formula dimensionally consistent?

▮ Dimensionally, $(F) = (1)(M/LT)(L)(L/T) + (1)(M/L^3)(L/T)^2(L)^2 = (ML/T^2) + (ML/T^2) = (F) + (F)$. Therefore, the formula is dimensionally consistent.

**1.136** The speed of propagation $C$ of waves traveling at the interface between two fluids is given by $C = (\pi\sigma/\rho_a\lambda)^{1/2}$, where $\lambda$ is the wavelength and $\rho_a$ is the average density of the two fluids. If the formula is dimensionally consistent, what are the units of $\sigma$? What might it represent?

▮ Dimensionally, $(L/T) = [(1)\{\sigma\}/(M/L^3)(L)]^{1/2} = [\{\sigma\}(L^2/M)]^{1/2}$, $\{\sigma\} = M/T^2 = F/L$. In SI units, $\sigma$ could be N/m. (In this formula, $\sigma$ is actually the surface tension.)

**1.137** Is the following equation dimensionally homogeneous? $a = 2d/t^2 - 2v_0/t$, where $a$ = acceleration, $d$ = distance, $v_0$ = velocity, and $t$ = time.

▌ $L/T^2 = (L)/(T^2) - (L/T)/(T) = (L/T^2) - (L/T^2)$. Therefore, the equation is homogeneous.

**1.138** A popular formula in the hydraulics literature is the Hazen-Williams formula for volume flow rate $Q$ in a pipe of diameter $D$ and pressure gradient $dp/dx$: $Q = 61.9D^{2.63}(dp/dx)^{0.54}$. What are the dimensions of the constant 61.9?

▌ $$\frac{L^3}{T} = \{61.9\}(L)^{2.63}\left(\frac{M}{L^2T^2}\right)^{0.54} \qquad \{61.9\} = L^{1.45}T^{0.08}M^{-0.54}$$

# CHAPTER 2
# Fluid Statics

**2.1** For the dam shown in Fig. 2-1, find the horizontal pressure acting at the face of the dam at 20-ft depth.

$$p = \gamma h = (62.4)(20) = 1248\ \text{lb/ft}^2$$

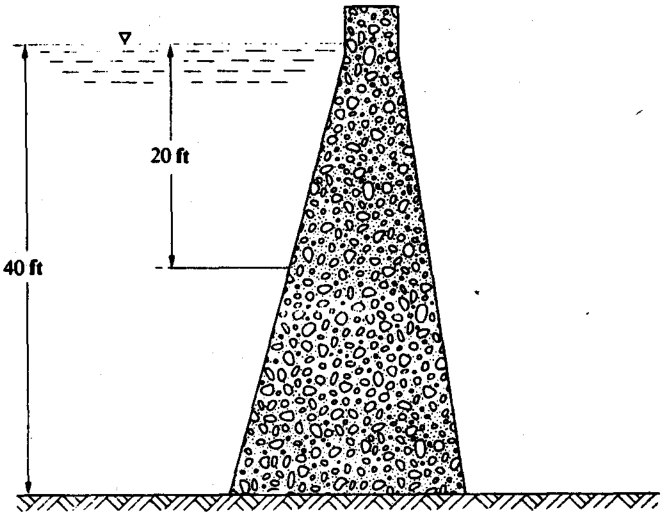



**Fig. 2-1.** Dam.

**2.2** For the vessel containing glycerin under pressure as shown in Fig. 2-2, find the pressure at the bottom of the tank.

$$p = 50 + \gamma h = 50 + (12.34)(2.0) = 74.68\ \text{kN/m}^2 \quad \text{or} \quad 74.68\ \text{kPa}$$

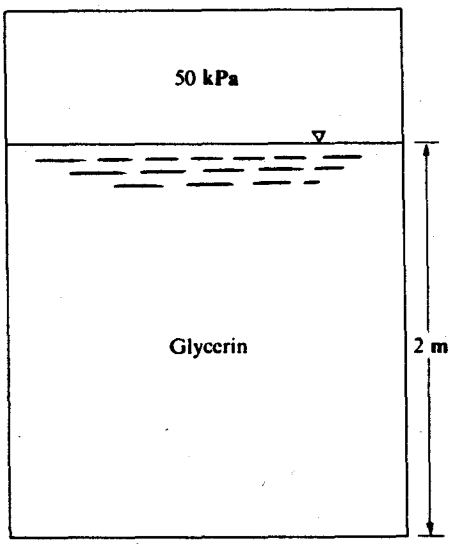



**Fig. 2-2**

**2.3** If the pressure in a tank is 50 psi, find the equivalent pressure head of (*a*) water, (*b*) mercury, and (*c*) heavy fuel oil with a specific gravity of 0.92.

$$h = p/\gamma$$

(*a*) $h = [(50)(144)]/62.4 = 115.38\ \text{ft}$

(*b*) $h = [(50)(144)]/847.3 = 8.50\ \text{ft}$

(*c*) $h = [(50)(144)]/[(0.92)(62.4)] = 125.42\ \text{ft}$

**2.4** A weather report indicates the barometric pressure is 29.75 in of mercury. What is the atmospheric pressure in pounds per square inch?

■ $$p = \gamma h = [(13.6)(62.4)][(29.75/12)]/144 = 14.61\ \text{lb/in}^2 \quad \text{or} \quad 14.61\ \text{psi}$$

**2.5** Find the atmospheric pressure in kilopascals if a mercury barometer reads 742 mm.

■ $$p = \gamma h = (133.1)(\tfrac{742}{1000}) = 98.8\ \text{kN/m}^2 \quad \text{or} \quad 98.8\ \text{kPa}$$

**2.6** A pressure gage 7.0 m above the bottom of a tank containing a liquid reads 64.94 kPa; another gage at height 4.0 m reads 87.53 kPa. Compute the specific weight and mass density of the fluid.

■ $$\gamma = \Delta p/\Delta h = (87.53 - 64.94)/(7.0 - 4.0) = 7.53\ \text{kN/m}^3 \quad \text{or} \quad 7530\ \text{N/m}^3$$
$$\rho = \gamma/g = 7530/9.81 = 786\ \text{kg/m}^3$$

**2.7** A pressure gage 19.0 ft above the bottom of a tank containing a liquid reads 13.19 psi; another gage at height 14.0 ft reads 15.12 psi. Compute the specific weight, mass density, and specific gravity of the liquid.

■ $$\Delta p = \gamma(\Delta h) \qquad (15.12 - 13.19)(144) = (\gamma)(19.0 - 14.0) \qquad \gamma = 55.6\ \text{lb/ft}^3$$
$$\rho = \gamma/g = 55.6/32.2 = 1.73\ \text{slug/ft}^3 \qquad \text{s.g.} = 55.6/62.4 = 0.891$$

**2.8** An open tank contains 5.7 m of water covered with 2.8 m of kerosene ($\gamma = 8.0\ \text{kN/m}^3$). Find the pressure at the interface and at the bottom of the tank.

■ $$p_{\text{int}} = \gamma h = (8.0)(2.8) = 22.4\ \text{kPa}$$
$$p_{\text{bot}} = 22.4 + (9.79)(5.7) = 78.2\ \text{kPa}$$

**2.9** An open tank contains 9.4 ft of water beneath 1.8 ft of oil (s.g. = 0.85). Find the pressure at the interface and at the bottom of the tank.

■ $$p_{\text{int}} = \gamma h = [(0.85)(62.4)](1.8)/144 = 0.663\ \text{psi}$$
$$p_{\text{bot}} = 0.663 + (62.4)(9.4)/144 = 4.74\ \text{psi}$$

**2.10** If air had a constant specific weight of $0.076\ \text{lb/ft}^3$ and were incompressible, what would be the height of the atmosphere if sea-level pressure were 14.92 psia?

■ $$h = p/\gamma = (14.92)(144)/0.076 = 28\,270\ \text{ft}$$

**2.11** If the weight density of mud is given by $\gamma = 65.0 + 0.2h$, where $\gamma$ is in $\text{lb/ft}^3$ and depth $h$ is in ft, determine the pressure, in psi, at a depth of 17 ft.

■ $dp = \gamma\, dh = (65.0 + 0.2h)\, dh$. Integrating both sides: $p = 65.0h + 0.1h^2$. For $h = 17$ ft: $p = (65.0)(17)/144 + (0.1)(17)^2/144 = 7.87$ psi.

**2.12** If the absolute pressure in a gas is 40.0 psia and the atmospheric pressure is 846 mbar abs, find the gage pressure in (*a*) $\text{lb/in}^2$; (*b*) kPa; (*c*) bar.

■ (*a*) $p_{\text{atm}} = (846)(0.0145) = 12.3\ \text{lb/in}^2 \qquad p_{\text{gage}} = 40.0 - 12.3 = 27.7\ \text{lb/in}^2$

(*b*) $p_{\text{abs}} = (40.0)(6.894) = 276\ \text{kPa} \qquad p_{\text{atm}} = (846)(0.100) = 85\ \text{kPa} \qquad p_{\text{gage}} = 276 - 85 = 191\ \text{kPa}$

(*c*) $p_{\text{abs}} = 40.0/14.5 = 2.759\ \text{bar} \qquad p_{\text{gage}} = 2.759 - 0.846 = 1.913\ \text{bar}$

**2.13** If the atmospheric pressure is 0.900 bar abs and a gage attached to a tank reads 390 mmHg vacuum, what is the absolute pressure within the tank?

■ $$p = \gamma h \qquad p_{\text{atm}} = 0.900 \times 100 = 90.0\ \text{kPa}$$
$$p_{\text{gage}} = [(13.6)(9.79)](\tfrac{390}{1000}) = 51.9\ \text{kPa vacuum} \quad \text{or} \quad -51.9\ \text{kPa}$$
$$p_{\text{abs}} = 90.0 + (-51.9) = 38.1\ \text{kPa}$$

**2.14** If atmospheric pressure is 13.99 psia and a gage attached to a tank reads 7.4 inHg vacuum, find the absolute pressure within the tank.

$$p = \gamma h \qquad p_{\text{gage}} = [(13.6)(62.4)][(7.4/12)/144] = 3.63 \text{ psi vacuum} \quad \text{or} \quad -3.63 \text{ psi}$$

$$p_{\text{abs}} = 13.99 + (-3.63) = 10.36 \text{ psia}$$

**2.15** The closed tank in Fig. 2-3 is at 20 °C. If the pressure at point $A$ is 98 kPa abs, what is the absolute pressure at point $B$? What percent error results from neglecting the specific weight of the air?

$p_A + \gamma_{\text{air}} h_{AC} - \gamma_{H_2O} h_{DC} - \gamma_{\text{air}} h_{DB} = p_B$, $98 + (0.0118)(5) - (9.790)(5-3) - (0.0118)(3) = p_B = 78.444$ kPa. Neglecting air, $p_B = 98 - (9.790)(5-3) = 78.420$ kPa; error $= (78.444 - 78.420)/78.444 = 0.00031$, or 0.031%.

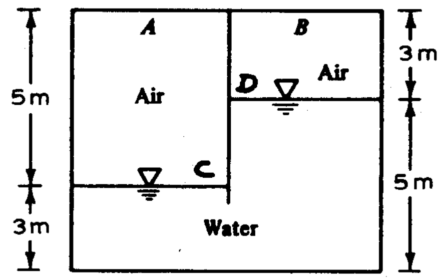



Fig. 2-3

**2.16** The system in Fig. 2-4 is at 70 °F. If the pressure at point $A$ is 2900 lb/ft², determine the pressures at points $B$, $C$, and $D$.

$$p_B = 2900 - (62.4)(4-3) = 2838 \text{ lb/ft}^2 \qquad p_D = 2900 + (62.4)(6) = 3274 \text{ lb/ft}^2$$

$$p_C = 2900 + (62.4)(6-2) - (0.075)(5+3) = 3149 \text{ lb/ft}^2$$

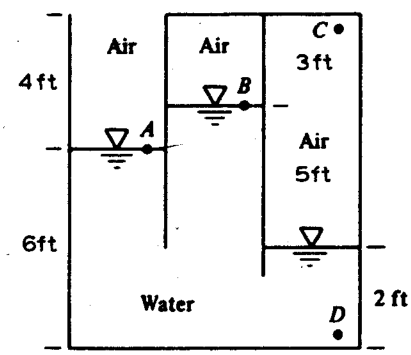



Fig. 2-4

**2.17** The system in Fig. 2-5 is at 20 °C. If atmospheric pressure is 101.03 kPa and the absolute pressure at the bottom of the tank is 231.3 kPa, what is the specific gravity of olive oil?

$101.03 + (0.89)(9.79)(1.5) + (9.79)(2.5) + (\text{s.g.})(9.79)(2.9) + (13.6)(9.79)(0.4) = 231.3 \qquad \text{s.g.} = 1.39$

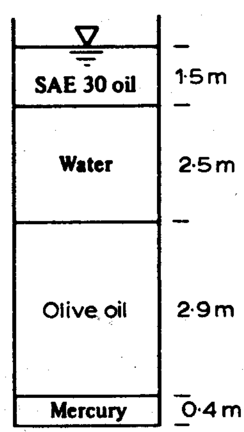



Fig. 2-5

**2.18** Find the pressures at $A$, $B$, $C$, and $D$ in Fig. 2-6.

▌ $p_A = (62.4)(4+2) = 374\ \text{lb/ft}^2$, $p_B = -(62.4)(2) = -125\ \text{lb/ft}^2$. Neglecting air, $p_C = p_B = -125\ \text{lb/ft}^2$; $p_D = -125 - (62.4)(4+2+2) = -624\ \text{lb/ft}^2$.

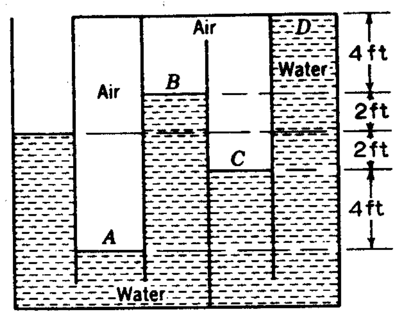



**Fig. 2-6**

**2.19** The tube shown in Fig. 2-7 is filled with oil. Determine the pressure heads at $A$ and $B$ in meters of water.

▌ $(h_{H_2O})(\gamma_{H_2O}) = (h_{oil})(\gamma_{oil}) = (h_{oil})[(\text{s.g.}_{oil})(\gamma_{H_2O})]$; therefore, $h_{H_2O} = (h_{oil})(\text{s.g.}_{oil})$. Thus, $h_A = -(2.2+0.6)(0.85) = -2.38\ \text{m } H_2O$ and $h_B = (-0.6)(0.85) = -0.51\ \text{m } H_2O$.

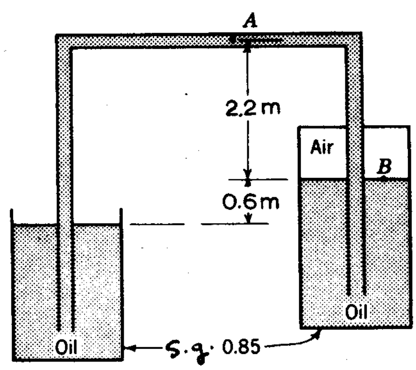



**Fig. 2-7**

**2.20** Calculate the pressure, in kPa, at $A$, $B$, $C$, and $D$ in Fig. 2-8.

▌ $p_A = -(0.4+0.4)(9.790) = -7.832\ \text{kPa}$; $p_B = (0.5)(9.790) = 4.895\ \text{kPa}$. Neglecting air, $p_C = p_B = 4.895\ \text{kPa}$; $p_D = 4.895 + (0.9)(9.790)(1+0.5+0.4) = 21.636\ \text{kPa}$.

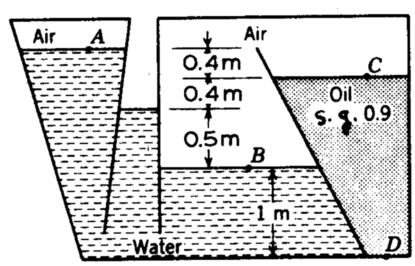



**Fig. 2-8**

**2.21** Convert 9 psi to **(a)** inches of mercury, **(b)** feet of water, **(c)** feet of ichor (s.g. = 2.94).

▌ **(a)** $h = p/\gamma = [(9)(144)]/[(13.6)(62.4)] = 1.527$ ft, or 18.33 inHg
**(b)** $h = [(9)(144)]/62.4 = 20.77$ ft of water
**(c)** $h = [(9)(144)]/[(2.94)(62.4)] = 7.06$ ft ichor

**2.22** Express an absolute pressure of 5 atm in meters of water gage when the barometer reads 760 mmHg.

▌ $p_{abs} = (5)(101.3)/9.79 = 51.74$ m of water $\qquad p_{atm} = (0.760)(13.6) = 10.34$ m of water

$$p_{gage} = 51.74 - 10.34 = 41.40 \text{ m of water}$$

**2.23** Figure 2-9 shows one pressurized tank inside another. If the sum of the readings of Bourdan gages $A$ and $B$ is 34.1 psi, and an aneroid barometer reads 29.90 inHg, what is the absolute pressure at $A$, in inHg?

■ $$h = p/\gamma \qquad h_A + h_B = 34.1/[(13.6)(62.4)/(12)^3] = 69.44 \text{ inHg}$$

$$(h_A)_{abs} = 29.90 + 69.44 = 99.34 \text{ inHg}$$

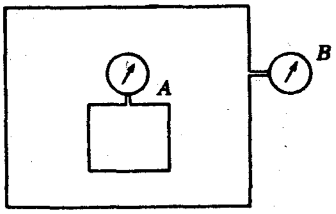



**Fig. 2-9**

**2.24** Determine the heights of columns of water, kerosene (ker), and nectar (s.g. = 2.94) equivalent to 277 mmHg.

■ $$(h_{Hg})(\gamma_{Hg}) = (H_{H_2O})(\gamma_{H_2O}) = (h_{ker})(\gamma_{ker}) = (h_{nectar})(\gamma_{nectar})$$

$$0.277[(13.6)(9.79)] = (h_{H_2O})(9.79) \qquad h_{H_2O} = 3.77 \text{ m}$$

$$0.277[(13.6)(9.79)] = (h_{ker})[(0.82)(9.79)] \qquad h_{ker} = 4.59 \text{ m}$$

$$0.277[(13.6)(9.79)] = (h_{nectar})[(2.94)(9.79)] \qquad h_{nectar} = 1.28 \text{ m}$$

**2.25** In Fig. 2-10, if $h = 25.5$ in, determine the pressure at $A$. The liquid has a specific gravity of 1.85.

■ $$p = \gamma h = [(1.85)(62.4)][25.5/12] = 245.3 \text{ lb/ft}^2 \quad \text{or} \quad 1.70 \text{ psi}$$

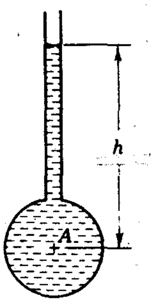



**Fig. 2-10**

**2.26** For the pressure vessel containing glycerin, with piezometer attached, as shown in Fig. 2-11, what is the pressure at point $A$?

■ $$p = \gamma h = [(1.26)(62.4)](40.8/12) = 267 \text{ lb/ft}^2$$

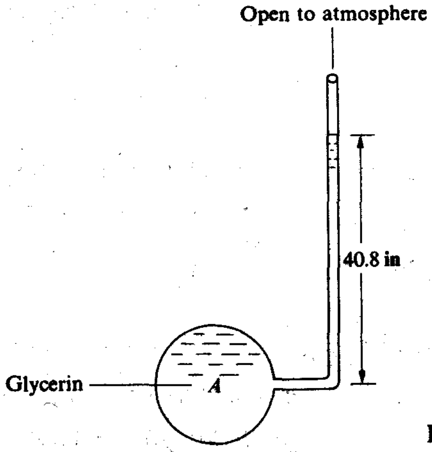



**Fig. 2-11**

**2.27** For the open tank, with piezometers attached on the side, containing two different immiscible liquids, as shown in Fig. 2-12, find the **(*a*)** elevation of the liquid surface in piezometer $A$, **(*b*)** elevation of the liquid surface in piezometer $B$, and **(*c*)** total pressure at the bottom of the tank.

**I** **(*a*)** Liquid $A$ will simply rise in piezometer $A$ to the same elevation as liquid $A$ in the tank (i.e., to elevation 2 m). **(*b*)** Liquid $B$ will rise in piezometer $B$ to elevation 0.3 m (as a result of the pressure exerted by liquid $B$) plus an additional amount as a result of the overlying pressure of liquid $A$. The overlying pressure can be determined by $p = \gamma h = [(0.72)(9.79)](2 - 0.3) = 11.98\ \text{kN/m}^2$. The height liquid $B$ will rise in piezometer $B$ as a result of the overlying pressure of liquid $A$ can be determined by $h = p/\gamma = 11.98/[(2.36)(9.79)] = 0.519\ \text{m}$. Hence, liquid $B$ will rise in piezometer $B$ to an elevation of 0.3 m + 0.519 m, or 0.819 m.
**(*c*)** $p_{\text{bottom}} = [(0.72)(9.79)](2 - 0.3) + [(2.36)(9.79)](0.3) = 18.9\ \text{kPa}$.

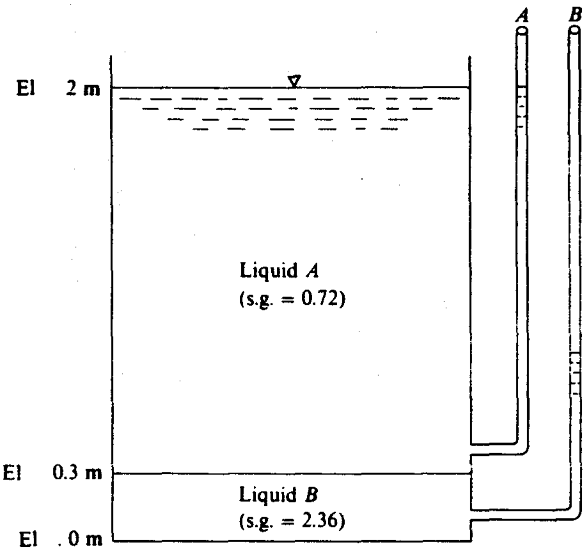



**Fig. 2-12**

**2.28** The air–oil–water system shown in Fig. 2-13 is at 70 °F. If gage $A$ reads 16.1 lb/in² abs and gage $B$ reads 2.00 lb/in² less than gage $C$, compute **(*a*)** the specific weight of the oil and **(*b*)** the reading of gage $C$.

**I** **(*a*)** $(16.1)(144) + (0.0750)(3) + (\gamma_{\text{oil}})(2) = p_B$, $p_B + (\gamma_{\text{oil}})(2) + (62.4)(3) = p_C$. Since $p_C - p_B = 2.00$, $(\gamma_{\text{oil}})(2) + (62.4)(3) = (2.00)(144)$, $\gamma_{\text{oil}} = 50.4\ \text{lb/ft}^3$. **(*b*)** $(16.1)(144) + (0.0750)(3) + (50.4)(2) = p_B$, $p_B = 2419\ \text{lb/ft}^2$; $p_C = 2419 + (2.00)(144) = 2707\ \text{lb/ft}^2$, or $18.80\ \text{lb/in}^2$.

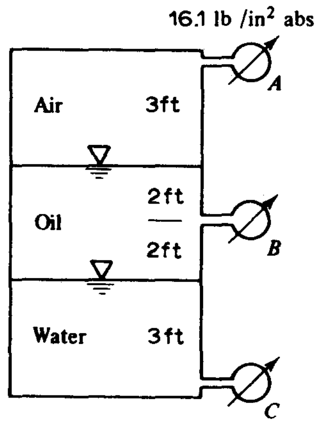



**Fig. 2-13**

**2.29** For a gage reading at $A$ of −2.50 psi, determine the **(*a*)** elevations of the liquids in the open piezometer columns $E$, $F$, and $G$ and **(*b*)** deflection of the mercury in the U-tube gage in Fig. 2-14. Neglect the weight of the air.

**I** **(*a*)** The liquid between the air and the water would rise to elevation 49.00 ft in piezometer column $E$ as a result of its weight. The actual liquid level in the piezometer will be lower, however, because of the vacuum in the air above the liquid. The amount the liquid level will be lowered ($h$ in Fig. 2-14) can be determined by

$(-2.50)(144) + [(0.700)(62.4)](h) = 0$, $h = 8.24$ ft. Elevation at $L = 49.00 - 8.24 = 40.76$ ft; $(-2.50)(144) + [(0.700)(62.4)][49.00 - 38.00) = p_M$, $p_M = 120.5$ lb/ft$^2$. Hence, pressure head at $M = 120.5/62.4 = 1.93$ ft of water. Elevation at $N = 38.00 + 1.93 = 39.93$ ft; $120.5 + (62.4)(38.00 - 26.00) = p_O$, $p_O = 869.3$ lb/ft$^2$. Hence, pressure head at $O = 869.3/[(1.600)(62.4)] = 8.71$ ft (of the liquid with s.g. = 1.600). Elevation at $Q = 26.00 + 8.71 = 34.71$ ft. **(b)** $869.3 + (62.4)(26.00 - 14.00) - [(13.6)(62.4)](h_1) = 0$, $h_1 = 1.91$ ft.

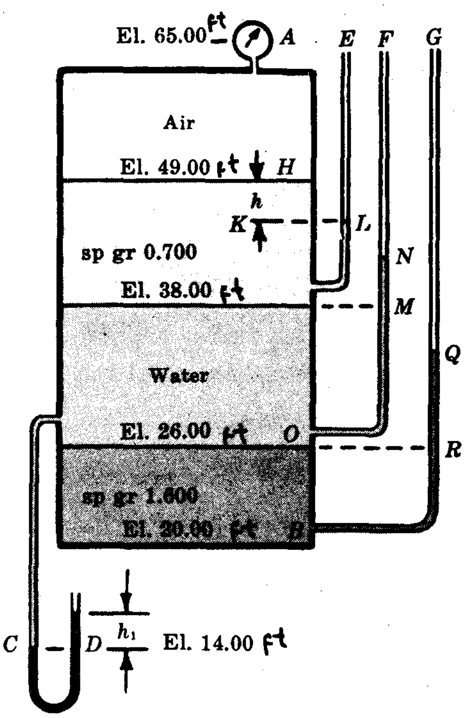



**Fig. 2-14**

**2.30** A vessel containing oil under pressure is shown in Fig. 2-15. Find the elevation of the oil surface in the attached piezometer.

**I** Elevation of oil surface in piezometer $= 2 + 35/[(0.83)(9.79)] = 6.31$ m

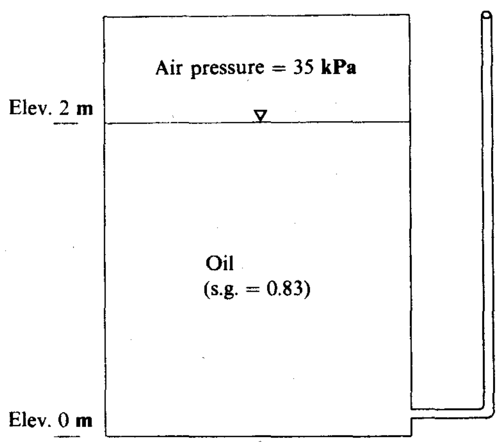



**Fig. 2-15**

**2.31** The reading of an automobile fuel gage is proportional to the gage pressure at the bottom of the tank (Fig. 2-16). If the tank is 32 cm deep and is contaminated with 3 cm of water, how many centimeters of air remains at the top when the gage indicates "full"? Use $\gamma_{gasoline} = 6670$ N/m$^3$ and $\gamma_{air} = 11.8$ N/m$^3$.

**I** When full of gasoline, $p_{gage} = (6670)(0.32) = 2134$ Pa. With water added, $2134 = (9790)(0.03) + (6670)[(0.32 - 0.03) - h] + (11.8)(h)$, $h = 0.0141$ m, or 1.41 cm.

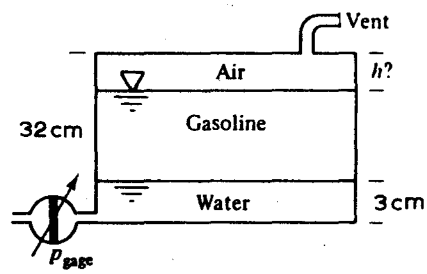



Fig. 2-16

2.32 The hydraulic jack shown in Fig. 2-17 is filled with oil at 55 lb/ft$^3$. Neglecting the weight of the two pistons, what force $F$ on the handle is required to support the 2200-lb weight?

The pressure against the large and the small piston is the same. $p = W/A_{\text{large}} = 2200/[\pi(\frac{3}{12})^2/4] =$ 44 818 lb/ft$^2$. Let $P$ be the force from the small piston onto the handle. $P = pA_{\text{small}} = (44\,818)[\pi(\frac{1}{12})^2/4] = 244$ lb. For the handle, $\Sigma M_A = 0 = (16+1)(F) - (1)(244)$, $F = 14.4$ lb.

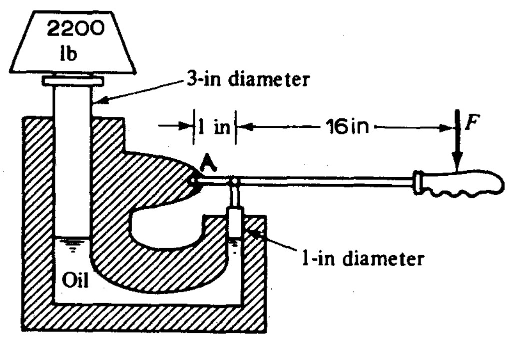



Fig. 2-17

2.33 Figure 2-18 shows a setup with a vessel containing a plunger and a cylinder. What force $F$ is required to balance the weight of the cylinder if the weight of the plunger is negligible?

$$10\,000/500 - [(0.78)(62.4)](15)/144 = F/5 \qquad F = 74.6\text{ lb}$$

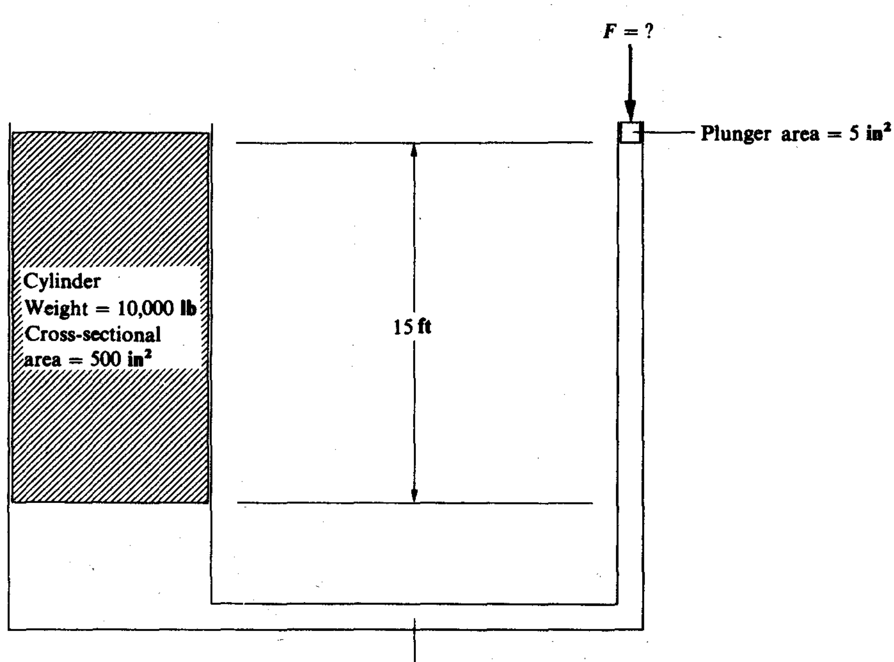



Fig. 2-18

2.34 For the vertical pipe with manometer attached, as shown in Fig. 2-19, find the pressure in the oil at point $A$.

$$p_A + [(0.91)(62.4)](7.22) - [(13.6)(62.4)](1.00) = 0 \qquad p_A = 438.7\text{ lb/ft}^2 \quad\text{or}\quad 3.05\text{ lb/in}^2$$

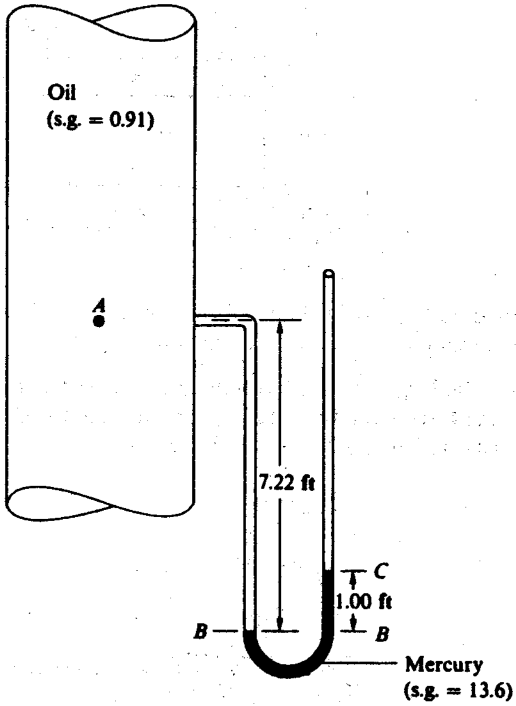



**Fig. 2-19**

**2.35** A monometer is attached to a tank containing three different fluids, as shown in Fig. 2-20. What will be the difference in elevation of the mercury column in the manometer (i.e., $y$ in Fig. 2-20)?

$$30+[(0.82)(9.79)](5-2)+(9.79)(2-0)+(9.79)(1.00)-[(13.6)(9.79)]y=0 \qquad y=0.627\text{ m}$$

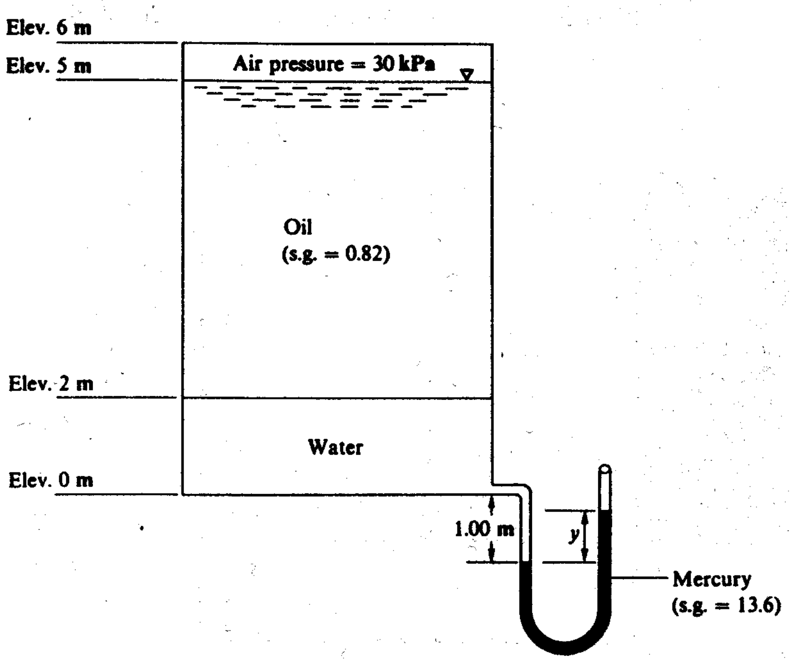



**Fig. 2-20**

**2.36** Oil of specific gravity 0.750 flows through the nozzle shown in Fig. 2-21 and deflects the mercury in the U-tube gage. Determine the value of $h$ if the pressure at $A$ is 20.0 psi.

$$20.0+[(0.750)(62.4)](2.75+h)/144-[(13.6)(62.4)](h)/144=0 \qquad h=3.75\text{ ft}$$

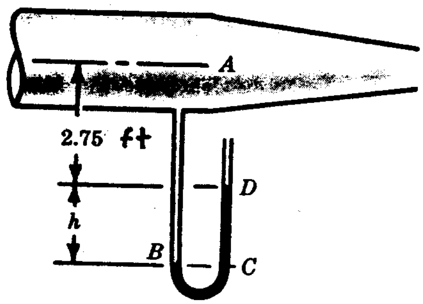



Fig. 2-21

**2.37** Determine the reading $h$ in Fig. 2-22 for $p_A = 39$ kPa vacuum if the liquid is kerosene (s.g. = 0.83).

$$-39 + [(0.83)(9.79)]h = 0 \qquad h = 4.800 \text{ m}$$

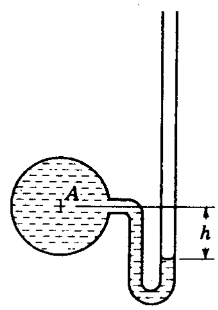



Fig. 2-22

**2.38** In Fig. 2-22, the liquid is water. If $h = 9$ in and the barometer reading is 29.8 inHg, find $p_A$ in feet of water absolute.

$$p_A + \tfrac{9}{12} = (13.6)(29.8/12) \qquad p_A = 33.0 \text{ ft of water absolute}$$

**2.39** In Fig. 2-23, s.g.$_1 = 0.84$, s.g.$_2 = 1.0$, $h_2 = 96$ mm, and $h_1 = 159$ mm. Find $p_A$ in mmHg gage. If the barometer reading is 729 mmHg, what is $p_A$ in $\text{mmH}_2\text{O}$ absolute?

$$p_A + (0.84)(96) - (1.0)(159) = 0$$

$$p_A = 78.4 \text{ mmH}_2\text{O gage} = 78.4/13.6 = 5.76 \text{ mmHg gage}$$
$$= 78.4 + (13.6)(729) = 9993 \text{ mmH}_2\text{O absolute}$$

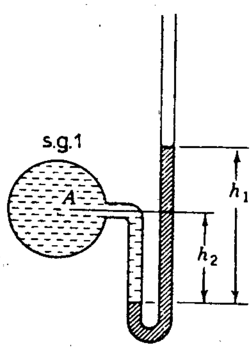



Fig. 2-23

**2.40** At 20 °C, gage $A$ in Fig. 2-24 reads 290 kPa abs. What is the height $h$ of water? What does gage $B$ read?

$$290 - [(13.6)(9.79)](\tfrac{70}{100}) - 9.79h = 175 \qquad h = 2.227 \text{ m}$$
$$p_B - (9.79)(\tfrac{70}{100} + 2.227) = 175 \qquad p_B = 204 \text{ kPa}$$

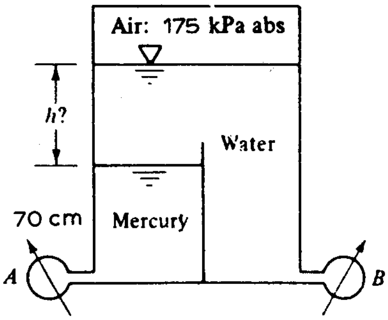



Fig. 2-24

**2.41** The U-tube shown in Fig. 2-25*a* is 10 mm in diameter and contains mercury. If 12.0 mL of water is poured into the right-hand leg, what are the ultimate heights in the two legs?

▌ After the water is poured, the orientation of the liquids will be as shown in Fig. 2-25*b*; $h = (12.0 \times 10^3\ \text{mm}^3)/\pi(5\ \text{mm})^2 = 152.8\ \text{mm}$, $(13.6)(240 - L) = 13.6L + 152.8$, $L = 114.4\ \text{mm}$. Left leg height above bottom of U-tube $= 240 - 114.4 = 125.6\ \text{mm}$; right leg height above bottom of U-tube $= 114.4 + 152.8 = 267.2\ \text{mm}$.

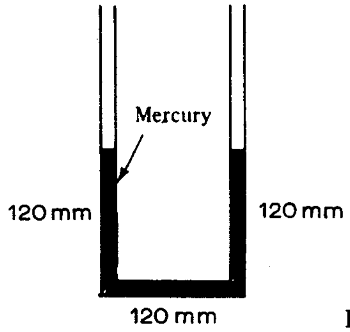



Fig. 2-25(*a*)

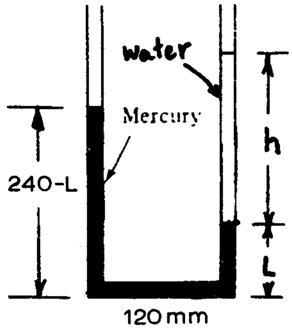



Fig. 2-25(*b*)

**2.42** Assuming sea water to have a constant specific weight of 10.05 kN/m³, what is the absolute pressure at a depth of 10 km?

▌
$$p = 1 + (10.05)(10\,000)/101.3 = 993\ \text{atm}$$

**2.43** In Fig. 2-26, fluid 2 is carbon tetrachloride and fluid 1 is benzene. If $p_{atm}$ is 101.5 kPa, determine the absolute pressure at point $A$.

▌
$$101.5 + (15.57)(0.35) - (8.62)(0.12) = p_A \qquad p_A = 105.9\ \text{kPa}$$

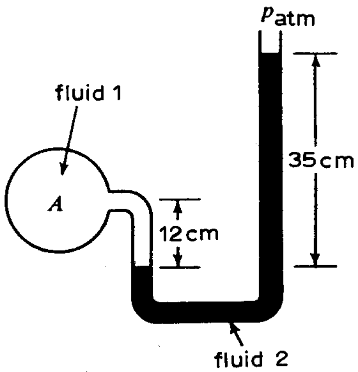



Fig. 2-26

**2.44** In Fig. 2-27*a*, the manometer reads 4 in when atmospheric pressure is 14.7 psia. If the absolute pressure at $A$ is doubled, what is the new manometer reading?

▌ $p_A + (62.4)(3.5) - [(13.6)(62.4)](\frac{4}{12}) = (14.7)(144)$, $p_A = 2181\ \text{lb/ft}^2$. If $p_A$ is doubled to $4362\ \text{lb/ft}^2$, the mercury level will fall $x$ inches on the left side of the manometer and will rise by that amount on the right side of the manometer (see Fig. 2-27*b*). Hence, $4362 + (62.4)(3.5 + x/12) - [(13.6)(62.4)][(4 + 2x)/12] = (14.7)(144)$, $x = 16.0$ in. New manometer reading $= 4 + (2)(16.0) = 36.0$ in.

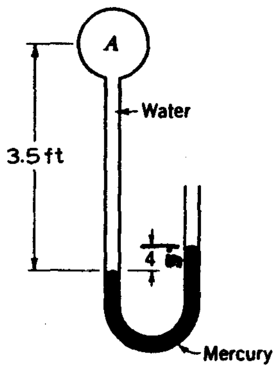



Fig. 2-27(*a*)

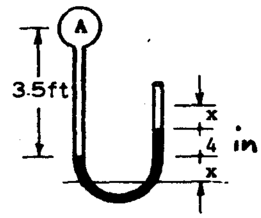



Fig. 2-27(*b*)

**2.45** In Fig. 2-28*a*, $A$ contains water, and the manometer fluid has density $2900\ \mathrm{kg/m^3}$. When the left meniscus is at zero on the scale, $p_A = 100$ mm of water. Find the reading of the right meniscus for $p_A = 10$ kPa with no adjustment of the U-tube or scale.

First, determine the reading of the right meniscus for $p_A = 100$ mm of water (see Fig. 2-28*b*): $100 + 500 - 2.90h = 0$, $h = 206.9$ mm. When $p_A = 10$ kPa, the mercury level will fall some amount, $d$, on the left side of the manometer and will rise by that amount on the right side of the manometer (see Fig. 2-28*b*). Hence, $10/9.79 + (500 + d)/1000 - [(206.9 + 2d)/1000](2.90) = 0$, $d = 192.0$ mm. Scale reading for $p_A = 10$ kPa is $206.9 + 192.0$, or 398.9 mm.

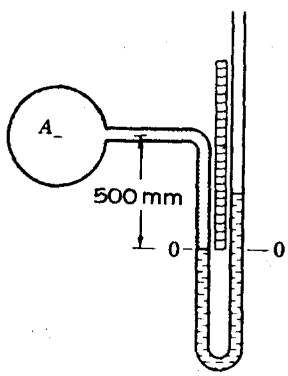



Fig. 2-28(*a*)

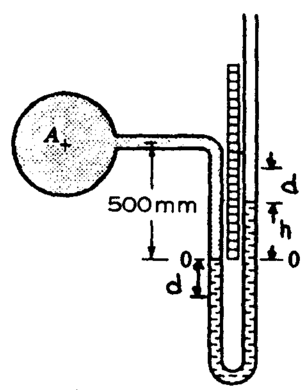



Fig. 2-28(*b*)

**2.46** A manometer is attached to a conduit, as shown in Fig. 2-29. Calculate the pressure at point $A$.

$$p_A + (62.4)[(5 + 15)/12] - [(13.6)(62.4)](\tfrac{15}{12}) = 0 \qquad p_A = 957\ \mathrm{lb/ft^2}$$

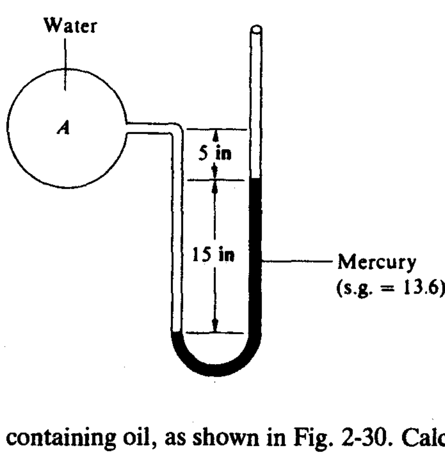



Fig. 2-29

**2.47** A manometer is attached to a pipe containing oil, as shown in Fig. 2-30. Calculate the pressure at point $A$.

$$p_A + [(0.85)(9.79)](0.2) - (9.79)(1.5) = 0 \qquad p_A = 13.02\ \mathrm{kN/m^2}$$

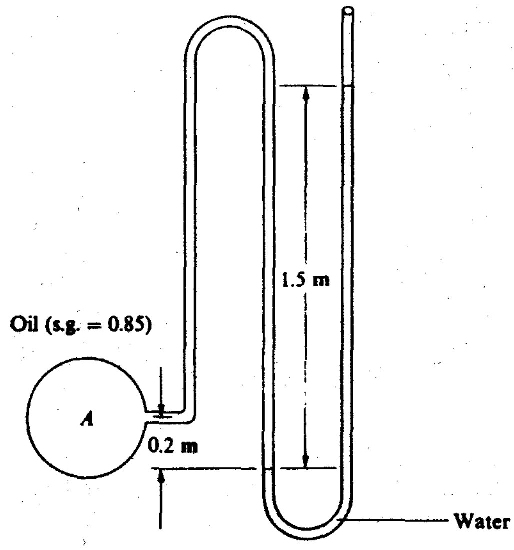


Fig. 2-30

**2.48** A monometer is attached to a pipe to measure pressure, as shown in Fig. 2-31. Calculate the pressure at point $A$.

▌ $$p_A + (62.4)(\tfrac{18}{12}) - [(13.6)(62.4)](\tfrac{6}{12}) = 0 \qquad p_A = 331 \text{ lb/ft}^2$$

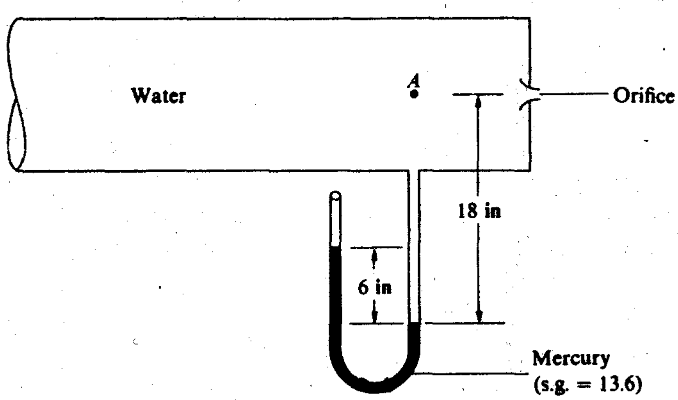


Fig. 2-31

**2.49** A glass U-tube open to the atmosphere at both ends is shown in Fig. 2-32. if the U-tube contains oil and water as shown, determine the specific gravity of the oil.

▌ $$[(\text{s.g.}_{\text{oil}})(9.79)](0.35) - (9.79)(0.30) = 0 \qquad \text{s.g.}_{\text{oil}} = 0.86$$

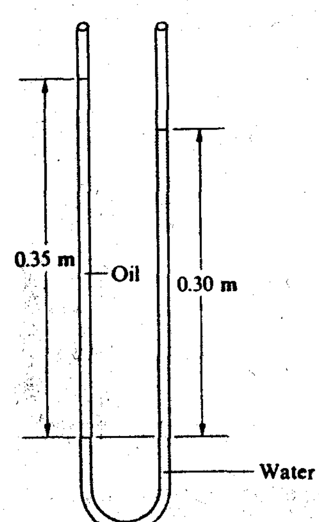


Fig. 2-32

**2.50** A differential manometer is shown in Fig. 2-33. Calculate the pressure difference between points $A$ and $B$.

$$p_A + [(0.92)(62.4)][(x+12)/12] - [(13.6)(62.4)](\tfrac{12}{12}) - [(0.92)(62.4)][(x+24)/12] = p_B$$

$$p_A - p_B = 906 \text{ lb/ft}^2$$

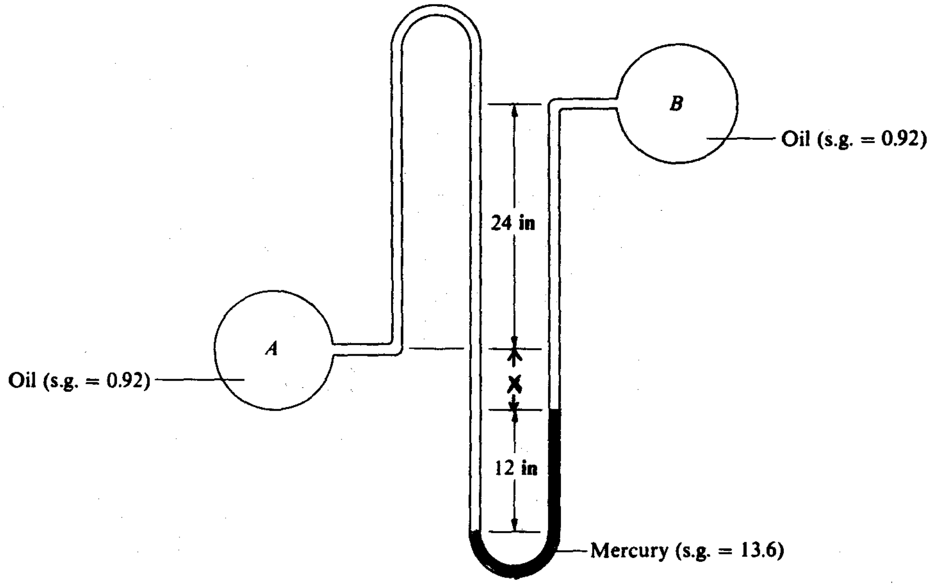



**Fig. 2-33**

**2.51** A differential manometer is attached to a pipe, as shown in Fig. 2.34. Calculate the pressure difference between points $A$ and $B$.

$$p_A + [(0.91)(62.4)](y/12) - [(13.6)(62.4)](\tfrac{4}{12}) - [(0.91)(62.4)][(y-4)/12] = p_B$$

$$p_A - p_B = 264 \text{ lb/ft}^2$$

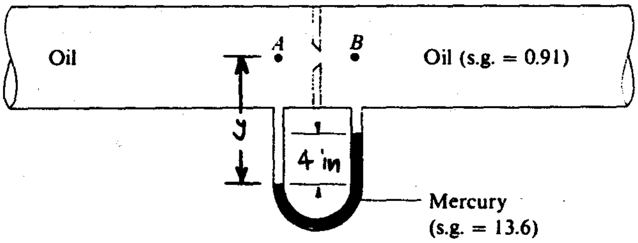



**Fig. 2-34**

**2.52** A differential manometer is attached to a pipe, as shown in Fig. 2-35. Calculate the pressure difference between points $A$ and B.

$$p_A - [(0.91)(62.4)](y/12) - [(13.6)(62.4)](\tfrac{4}{12}) + [(0.91)(62.4)][(y+4)/12] = p_B$$

$$p_A - p_B = 264 \text{ lb/ft}^2$$

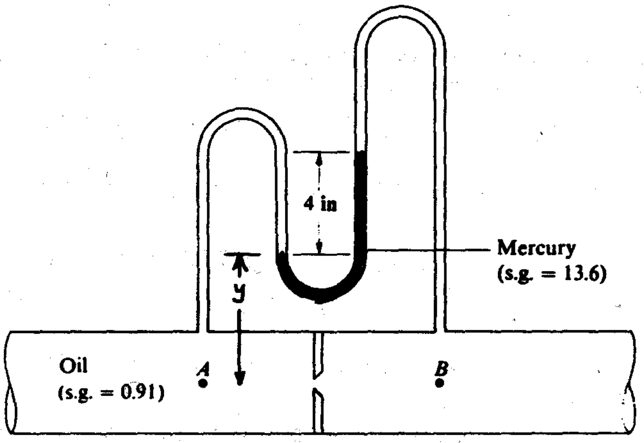



**Fig. 2-35**

**2.53** For the configuration shown in Fig. 2-36, calculate the weight of the piston if the gage pressure reading is 70.0 kPa.

▮ Let $W$ = weight of the piston. $W/[(\pi)(1)^2/4] - [(0.86)(9.79)](1) = 70.0$, $W = 61.6$ kN.

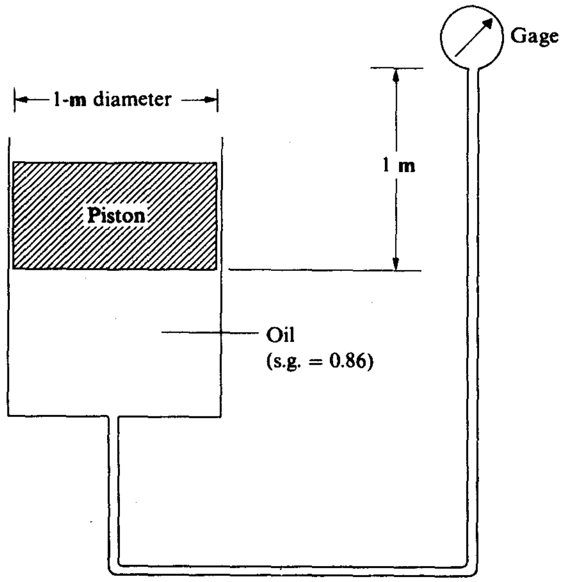



**Fig. 2-36**

**2.54** A manometer is attached to a horizontal oil pipe, as shown in Fig. 2-37. If the pressure at point $A$ is 10 psi, find the distance between the two mercury surfaces in the manometer (i.e., determine the distance $y$ in Fig. 2-37).

▮ $$(10)(144) + [(0.90)(62.4)](3 + y) - [(13.6)(62.4)]y = 0 \qquad y = 2.03 \text{ ft} \quad \text{or} \quad 24.4 \text{ in}$$

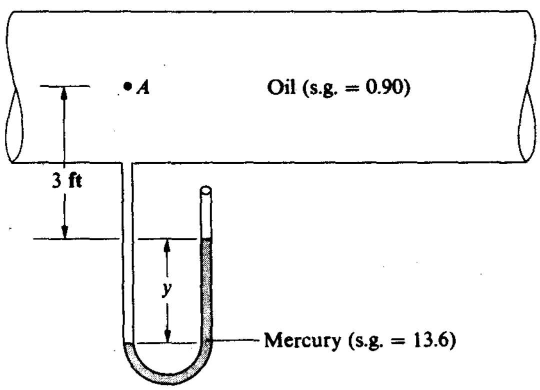



**Fig. 2-37**

**2.55** A vertical pipe with attached gage and manometer is shown in Fig. 2-38. What will be the gage reading in pounds per square inch if there is no flow in the pipe?

▮ $$\text{Gage reading} + [(0.85)(62.4)](2 + 8)/144 - [(13.6)(62.4)](\tfrac{18}{12})/144 = 0 \qquad \text{Gage reading} = 5.16 \text{ psi}$$

**2.56** A monometer is attached to a vertical pipe, as shown in Fig. 2-39. Calculate the pressure difference between points $A$ and $B$.

▮ $$p_A - (62.4)(5 + 1) - [(13.6)(62.4)](2) + (62.4)(2 + 1) = p_B$$

$$p_A - p_B = 1884 \text{ lb/ft}^2 \quad \text{or} \quad 13.1 \text{ lb/in}^2$$

**2.57** A manometer is attached to a water tank, as shown in Fig. 2-40. Find the height of the free water surface above the bottom of the tank.

▮ $$(9.79)(H - 0.15) - [(13.6)(9.79)](0.20) = 0 \qquad H = 2.87 \text{ m}$$

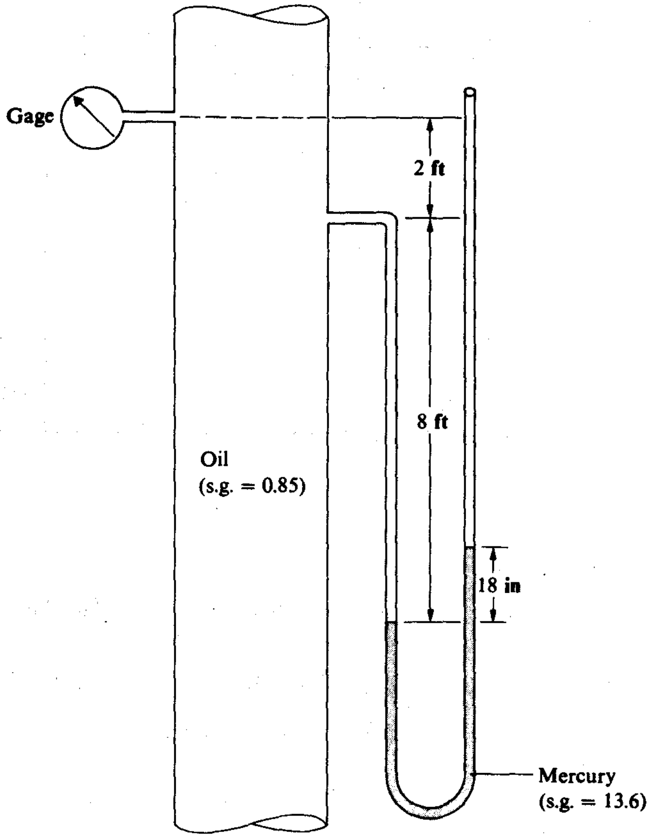



**Fig. 2-38**

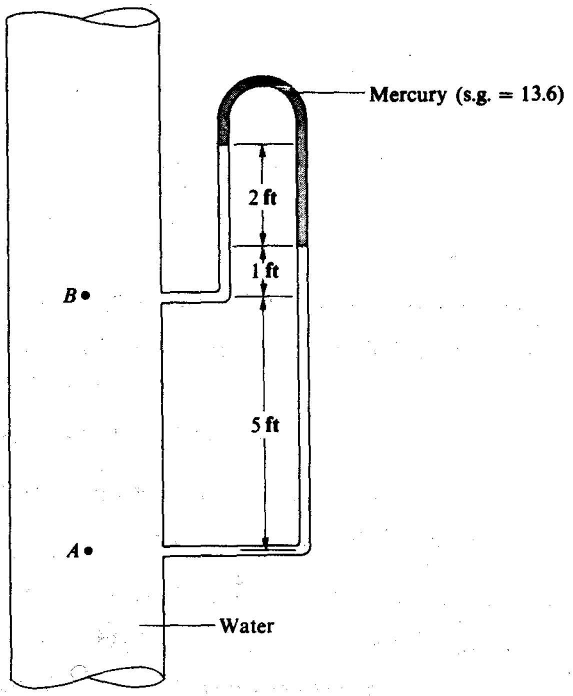



**Fig. 2-39**

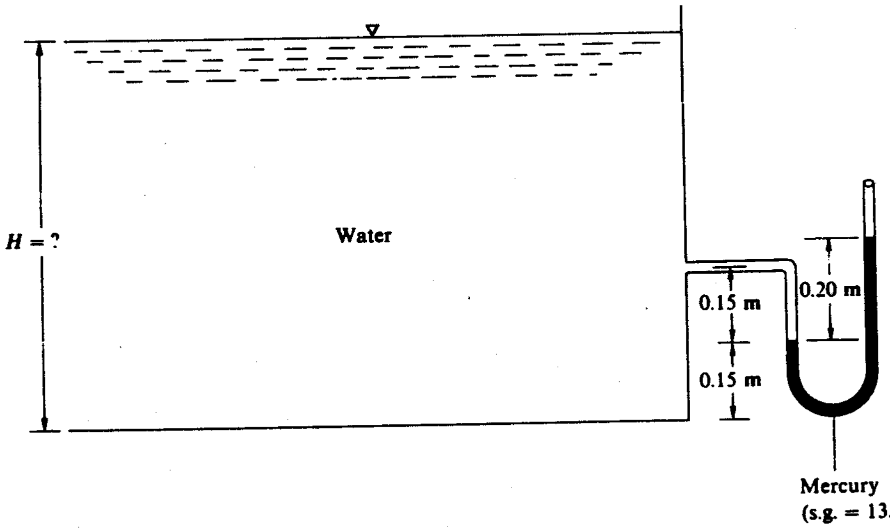



**Fig. 2-40**

**2.58** A differential manometer is attached to two tanks, as shown in Fig. 2-41. Calculate the pressure difference between chambers $A$ and $B$.

▮

$$p_A + [(0.89)(9.79)](1.1) + [(13.6)(9.79)](0.3) - [(1.59)(9.79)](0.8) = p_B$$

$$p_A - p_B = -37.1 \text{ kN/m}^2 \qquad (\text{i.e., } p_B > p_A)$$

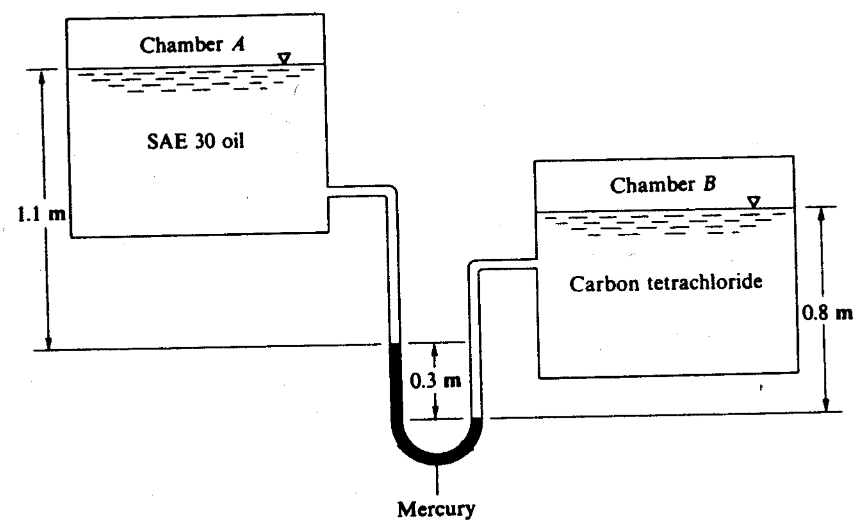



**Fig. 2-41**

**2.59** Calculate the pressure difference between $A$ and $B$ for the setup shown in Fig. 2-42.

▮ $$p_A + (62.4)(66.6/12) - [(13.6)(62.4)](40.3/12) + (62.4)(22.2/12)$$
$$- [(13.6)(62.4)](30.0/12) - (62.4)(10.0/12) = p_B$$

$$p_A - p_B = 4562 \text{ lb/ft}^2 \quad \text{or} \quad 31.7 \text{ lb/in}^2$$

**2.60** Calculate the pressure difference between $A$ and $B$ for the setup shown in Fig. 2-43.

▮ $$p_A - (9.79)x - [(0.8)(9.79)](0.70) + (9.79)(x - 0.80) = p_B \qquad p_A - p_B = 13.3 \text{ kN/m}^2$$

**2.61** Calculate the pressure difference between $A$ and $B$ for the setup shown in Fig. 2-44.

▮ $$p_A + (62.4)(x + 4) - [(13.6)(62.4)](4) + (62.4)(7 - x) = p_B$$

$$p_A - p_B = 2708 \text{ lb/ft}^2 \quad \text{or} \quad 18.8 \text{ lb/in}^2$$

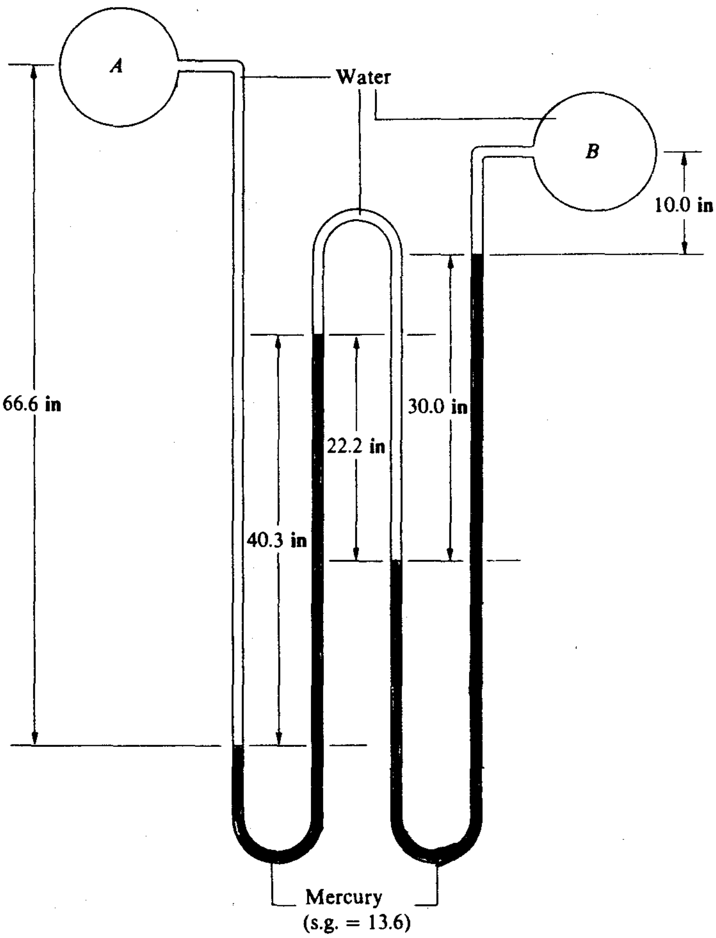



Fig. 2-42

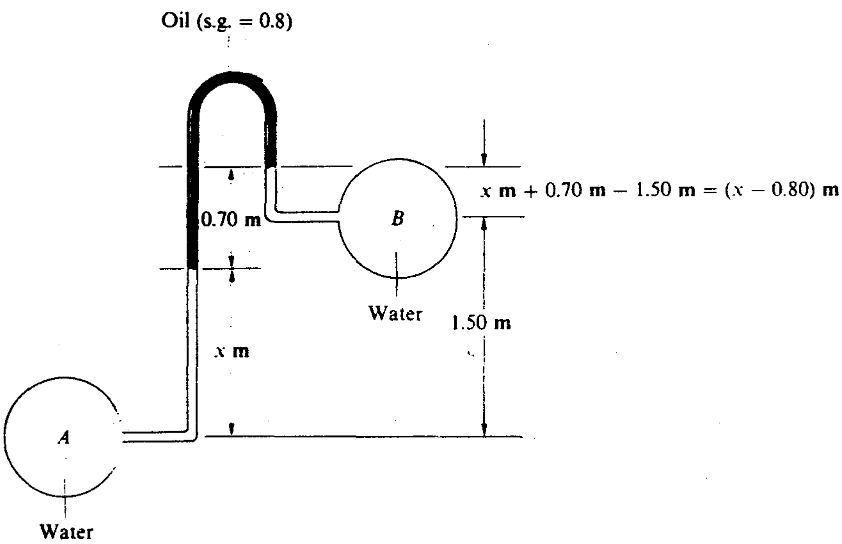



Fig. 2-43

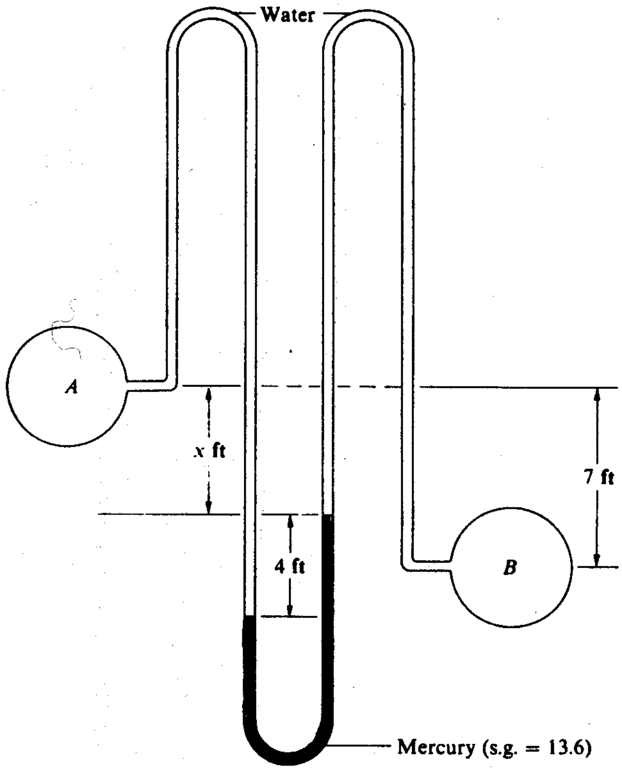



**Fig. 2-44**

**2.62** Vessels $A$ and $B$ in Fig. 2-45 contain water under pressures of 40.0 psi and 20.0 psi, respectively. What is the deflection of the mercury in the differential gage?

▮ $(40.0)(144) + (62.4)(x + h) - [(13.6)(62.4)]h + 62.4y = (20.0)(144)$. Since $x + y = 16.00 - 10.00$, or 6.00 ft, $h = 4.14$ ft.

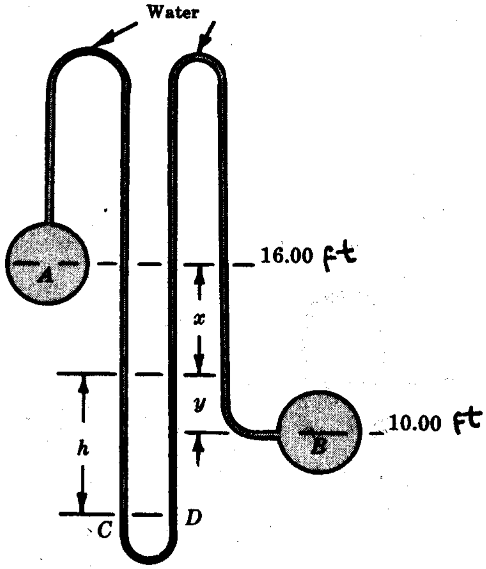



**Fig. 2-45**

**2.63** For a gage pressure at $A$ in Fig. 2-46 of $-1.58$ psi, find the specific gravity of gage liquid $B$.

▮ $(-1.58)(144) + [(1.60)(62.4)](10.50 - 9.00) - (0.0750)(11.25 - 9.00) + [(\text{s.g.}_{\text{liq. }B})(62.4)](11.25 - 10.00) = 0$

$$\text{s.g.}_{\text{liq. }B} = 1.00$$

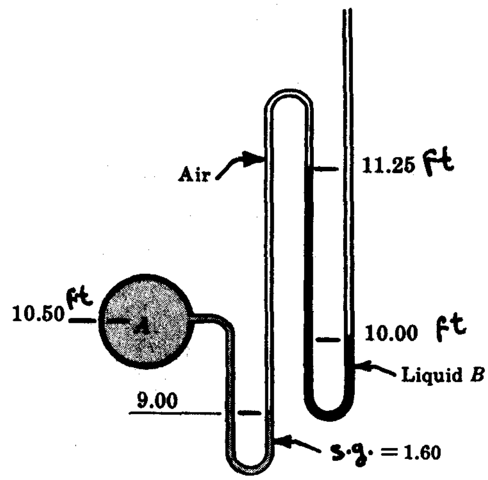



Fig. 2-46

**2.64** In Fig. 2-47, liquid $A$ weighs 53.5 lb/ft$^3$ and liquid $B$ weighs 78.8 lb/ft$^3$. Manometer liquid $M$ is mercury. If the pressure at $B$ is 30 psi, find the pressure at $A$.

$$p_A - (53.5)(6.5 + 1.3) + [(13.6)(62.4)](1.3) + (78.8)(6.5 + 10.0) = (30)(144)$$

$$p_A = 2334 \text{ lb/ft}^2 \quad \text{or} \quad 16.2 \text{ lb/in}^2$$

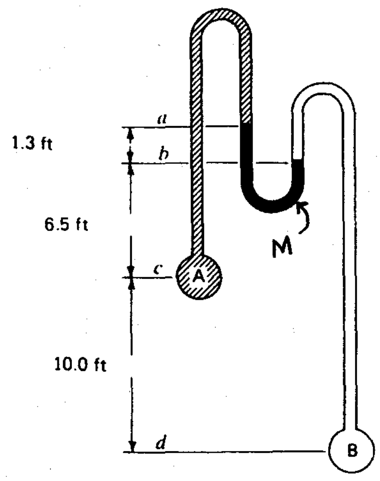



Fig. 2-47

**2.65** What would be the manometer reading in Fig. 2-47 if $p_B - p_A$ is 165 kPa?

Converting to lb/ft$^2$, $p_B - p_A = 3446$ lb/ft$^2$. The mercury level will rise some amount, $x$, on the left side of the manometer and will fall by that amount on the right side of the manometer (see Fig. 2.48). Hence, taking weight densities from Prob. 2.64, $p_A - (53.5)(6.5 + 1.3 + x) + [(13.6)(62.4)](1.3 + 2x) + (78.8)(6.5 + 10.0 - x) = p_B$, $1644x + 1986 = p_B - p_A = 3446$, $x = 0.89$ ft; manometer reading $= 1.3 + (2)(0.89) = 3.08$ ft.

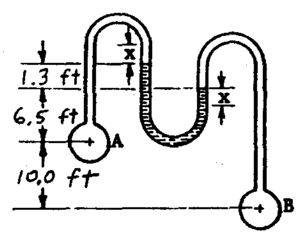



Fig. 2-48

**2.66** In Fig. 2-49, water is contained in $A$ and rises in the tube to a level 85 in above $A$; glycerin is contained in $B$. The inverted U-tube is filled with air at 23 psi and 70 °F. Atmospheric pressure is 14.6 psia. Determine the difference in pressure (psi) between $A$ and $B$ if $y$ is 16 in. What is the absolute pressure in $B$ in inches of mercury and in feet of glycerin?

$$p_A - (62.4)(\tfrac{85}{12}) = (23)(144) \qquad p_A = 3754.0\ \text{lb/ft}^2$$

$$p_B - [(1.26)(62.4)][(85-16)/12] = (23)(144) \qquad p_B = 3764.1\ \text{lb/ft}^2$$

$$p_A - p_B = 3754.0 - 3764.1 = -10.1\ \text{lb/ft}^2 \quad \text{or} \quad -0.070\ \text{lb/in}^2$$

$$(p_{abs})_B = (3764.1/144 + 14.6)/[(13.6)(62.4)/(12)^3] = 83.0\ \text{inHg}$$

$$(p_{abs})_B = (3764.1/144 + 14.6)/[(1.26)(62.4)/(12)^3] = 895.4\ \text{in} \quad \text{or} \quad 74.6\ \text{ft of glycerin}$$

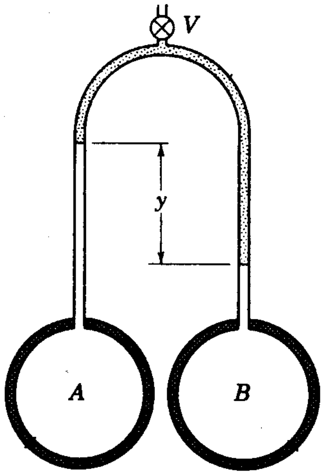



Fig. 2-49

2.67 Gas confined in a rigid container exerts a gage pressure of 150 kPa when its temperature is 7 °C. What pressure would the gas exert at 67 °C? Barometric pressure remains constant at 719 mmHg.

$$p_{atm} = [(13.6)(9.79)](0.719) = 95.7\ \text{kPa} \qquad p_{abs} = 95.7 + 150 = 245.7\ \text{kPa}$$

$$p_1V_1/T_1 = p_2V_2/T_2 \qquad (245.7)(V)/(273+7) = (p_2)(V)/(273+67) \qquad [V \text{ (volume) is constant}]$$

$$p_2 = 298.4\ \text{kPa (absolute)} = 298.4 - 95.7 = 202.7\ \text{kPa (gage)}$$

2.68 In Fig. 2-50, atmospheric pressure is 14.6 psia, the gage reading at $A$ is 6.1 psi, and the vapor pressure of the alcohol is 1.7 psia. Compute $x$ and $y$.

Working in terms of absolute pressure heads, $[(6.1+14.6)(144)]/[(0.90)(62.4)] - x = (1.7)(144)/[(0.90)(62.4)]$, $x = 48.72$ ft; $[(6.1+14.6)(144)]/[(0.90)(62.4)] + (y+4.2) - (4.2)(13.6/0.90) = 0$, $y = 6.19$ ft.

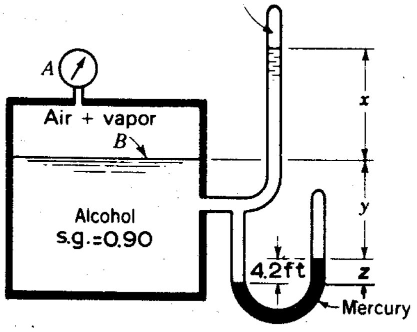



Fig. 2-50

2.69 In Fig. 2-50, assume the following: atmospheric pressure = 858 mbar abs, vapor pressure of the alcohol = 160 mbar abs, $x = 2.90$ m, $y = 2.10$ m. Compute the reading on the pressure gage ($p_A$) and on the manometer ($z$).

▌ Working in terms of absolute pressure heads, $[(p_A)_{gage} + 858](0.100)/[(0.90)(9.79)] - 2.90 = (160)(0.100)/[(0.90)(9.79)]$, $(p_A)_{gage} = -442$ mbar; $[(-442 + 858)(0.100)]/[(0.90)(9.79)] + (2.10 + z) - (z)(13.6/0.90) = 0$, $z = 0.483$ m.

**2.70** A pipeline contains an incompressible gas ($\gamma = 0.05\ \text{lb/ft}^3$) at rest; at point $A$ the pressure is 4.69 in of water. What is the pressure, in inches of water, at point $B$, 492 ft higher than $A$?

▌ The change in pressure in the atmosphere must be considered; assume, however, that $\gamma_{air} = 0.076\ \text{lb/ft}^3$ is constant.

$$(p_A/\gamma)_{abs} = (p_A/\gamma)_{atm} + 4.69/12 \text{ ft of water} \tag{1}$$

$$(p_B/\gamma)_{abs} = (p_B/\gamma)_{atm} + x/12 \text{ ft of water} \tag{2}$$

Subtracting Eq. (2) from Eq. (1),

$$(p_A/\gamma)_{abs} - (p_B/\gamma)_{abs} = (p_A/\gamma)_{atm} - (p_B/\gamma)_{atm} + 4.69/12 - x/12 \tag{3}$$

$$(p_A/\gamma)_{atm} - (p_B/\gamma)_{atm} = 492 \text{ ft of air} = (492)(0.076/62.4) = 0.599 \text{ ft of water}$$

$$(p_A/\gamma)_{abs} - (p_B/\gamma)_{abs} = 492 \text{ ft of gas} = (492)(0.05/62.4) = 0.394 \text{ ft of water}$$

Substituting these relationships into Eq. (3), $0.394 = 0.599 + 4.69/12 - x/12$, $x = 7.15$ in of water.

**2.71** Determine the pressure difference between points $A$ and $B$ in Fig. 2-51.

▌ $$p_A + [(0.88)(9.79)](0.21) - [(13.6)(9.79)](0.09) - [(0.82)(9.79)](0.41 - 0.09) + (9.79)(0.41 - 0.15) - (0.0118)(0.10) = p_B$$

$$p_A - p_B = 10.2 \text{ kPa}$$

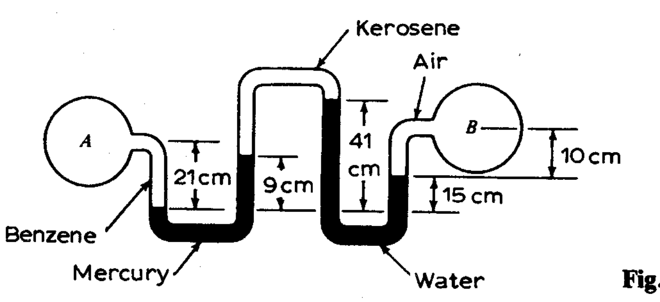



**Fig. 2-51**

**2.72** In Fig. 2-52, if $p_B - p_A = 97.4$ kPa, calculate $H$.

▌ $$p_A - (9.79)(H/100) - [(0.827)(9.79)](\tfrac{17}{100}) + [(13.6)(9.79)][(34 + H + 17)/100] = p_B$$

$$1.234H + 66.53 = p_B - p_A = 97.4 \qquad H = 25.0 \text{ cm}$$

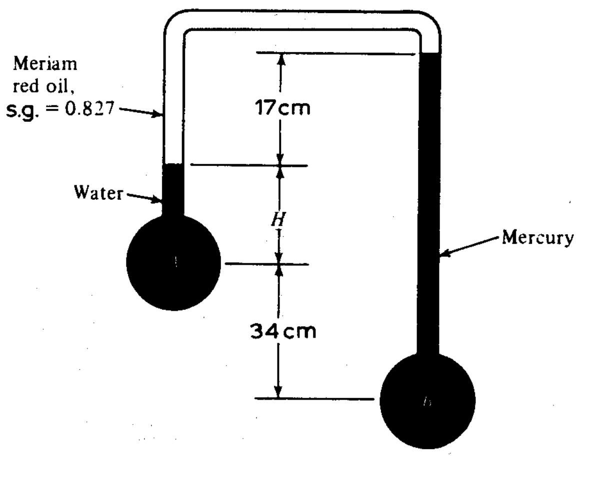



**Fig. 2-52**

**2.73** For Fig. 2-53, if fluid 1 is water and fluid 2 is mercury, and $z_A = 0$ and $z_1 = -11$ cm, what is level $z_2$ at which $p_A = p_{atm}$?

$$0 + (9.79)[0 - (-11)]/100 - [(13.6)(9.79)][z_2 - (-11)]/100 = 0 \qquad z_2 = -10.19 \text{ cm}$$

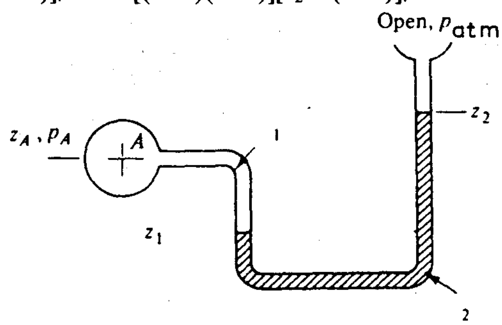



**Fig. 2-53**

**2.74** The inclined manometer in Fig. 2-54*a* contains Meriam red manometer oil (s.g. = 0.827). Assume the reservoir is very large. What should the angle $\theta$ be if each inch along the scale is to represent a change of 0.8 lb/ft² in gage pressure $p_A$?

From Fig. 2-54*b*, $\Delta p = \gamma \Delta z$, or

$$0.8 \text{ lb/ft}^2 = [(0.827)(62.4 \text{ lb/ft}^3)](\tfrac{1}{12} \text{ ft})(\sin\theta)$$

from which $\theta = 10.72°$.

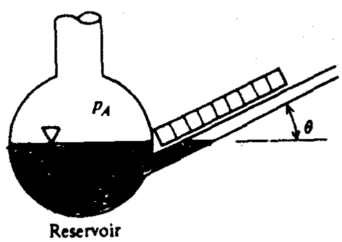



**Fig. 2-54(*a*)**

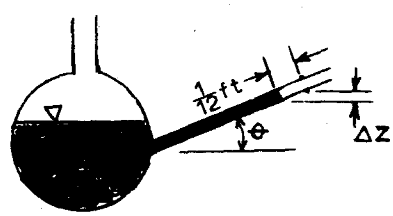



**Fig. 2-54(*b*)**

**2.75** The system in Fig. 2-55 is at 20 °C. Compute the absolute pressure at point $A$.

$$p_A + [(0.85)(62.4)](\tfrac{7}{12}) - [(13.6)(62.4)](\tfrac{9}{12}) + (62.4)(\tfrac{6}{12}) = (14.7)(144) \qquad p_A = 2691 \text{ lb/ft}^2 \text{ abs}$$

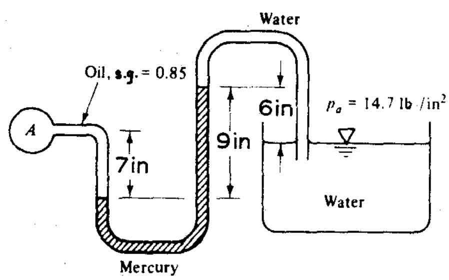



**Fig. 2-55**

**2.76** Very small pressure differences $p_A - p_B$ can be measured accurately by the two-fluid differential manometer shown in Fig. 2-56. Density $\rho_2$ is only slightly larger than the upper fluid $\rho_1$. Derive an expression for the proportionality between $h$ and $p_A - p_B$ if the reservoirs are very large.

$p_A + \rho_1 g h_1 - \rho_2 g h - \rho_1 g(h_1 - h) = p_B$, $p_A - p_B = (\rho_2 - \rho_1)gh$. If $(\rho_2 - \rho_1)$ is small, $h$ will be large (sensitive).

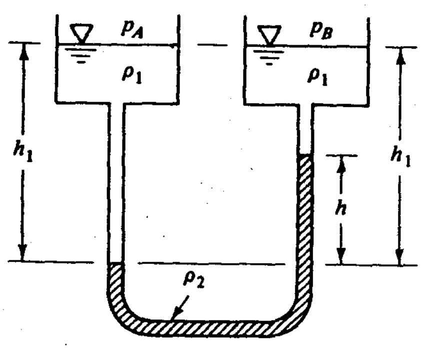



**Fig. 2-56**

**2.77** Water flows downward in a pipe at 35°, as shown in Fig. 2-57. The pressure drop $p_1 - p_2$ is partly due to gravity and partly due to friction. The mercury manometer reads a 5-in height difference. What is the total pressure drop $p_1 - p_2$? What is the pressure drop due to friction only between 1 and 2? Does the manometer reading correspond only to friction drop?

$$p_1 + (62.4)(6 \sin 35° + x/12 + \tfrac{5}{12}) - [(13.6)(62.4)](\tfrac{5}{12}) - (62.4)(x/12) = p_2$$

$$p_1 - p_2 = 112.9 \text{ lb/ft}^2 \qquad \text{(total pressure drop)}$$

$$\text{Pressure drop due to friction only} = [(13.6)(62.4) - 62.4](\tfrac{5}{12}) = 327.6 \text{ lb/ft}^2$$

Manometer reads only the friction loss.

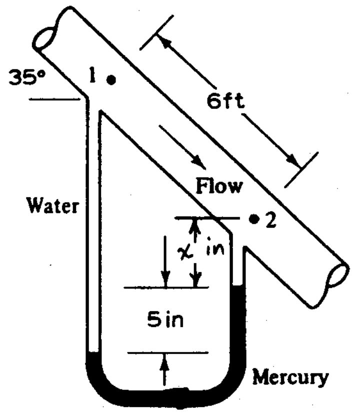



Fig. 2-57

**2.78** Determine the gage pressure at point $A$ in Fig. 2-58.

$$p_A - (9.79)(0.50) + (0.0118)(0.33) + [(13.6)(9.79)](0.17) - [(0.83)(9.79)](0.44) = 0 \qquad p_A = -14.17 \text{ kPa}$$

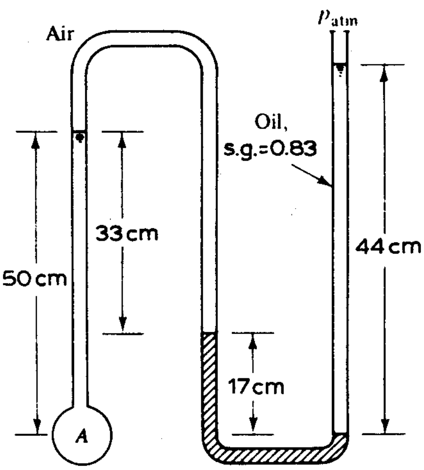



Fig. 2-58

**2.79** In Fig. 2-59, calculate level $h$ of the oil in the right-hand tube. Both tubes are open to the atmosphere.

$$0 + (9.79)(0.110 + 0.240) - [(0.83)(9.79)](0.240 + h) = 0 \qquad h = 0.1817 \text{ m} = 181.7 \text{ mm}$$

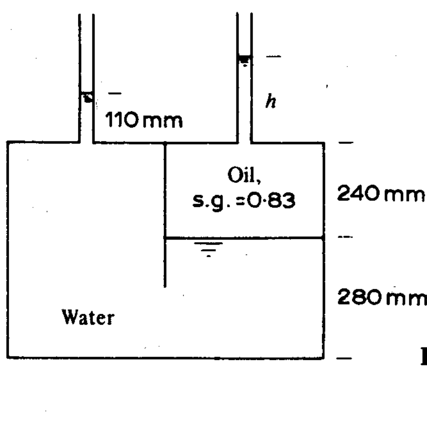



Fig. 2-59

**2.80** In Fig. 2-60*a* the inclined manometer measures the excess pressure at $A$ over that at $B$. The reservoir diameter is 2.5 in and that of the inclined tube is $\frac{1}{4}$ in. For $\theta = 32°$ and gage fluid with s.g. = 0.832, calibrate the scale in psi per ft.

$$p_A = \gamma(\Delta h + \Delta y) + p_B \qquad \text{(see Fig. 2-60}b\text{)} \qquad p_A - p_B = \gamma(\Delta h + \Delta y)$$

From Fig. 2-60*b*, $(A_A)(\Delta y) = (A_B)(R)$ or $\Delta y = A_B R/A_A$, $\Delta h = R \sin\theta$, $p_A - p_B = \gamma(R\sin\theta + A_B R/A_A) = \gamma R(\sin\theta + A_B/A_A)$, $A_B/A_A = [\pi(\frac{1}{4})^2/4]/[\pi(2.5)^2/4] = \frac{1}{100}$; $p_A - p_B = [(0.832)(62.4)](R)(\sin 32° + \frac{1}{100})/144 = 0.1947R$. The scale factor is thus 0.1947 psi/ft.

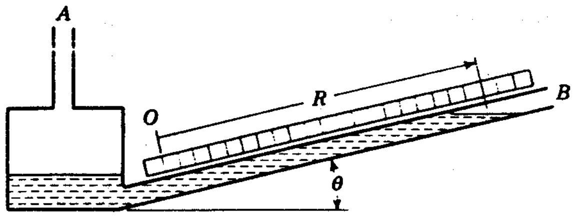



Fig. 2-60(*a*)

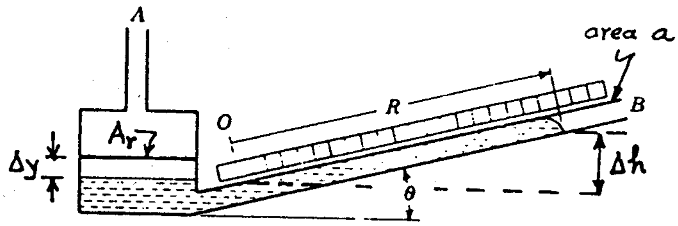



Fig. 2-60(*b*)

**2.81** Determine the weight $W$ that can be equilibrated by the force acting on the piston of Fig. 2-61.

$$p_1 = p_2 = F_1/A_1 = F_2/A_2 \qquad 1.25/[\pi(35)^2/4] = W/[\pi(250)^2/4] \qquad W = 63.8\text{ kN}$$

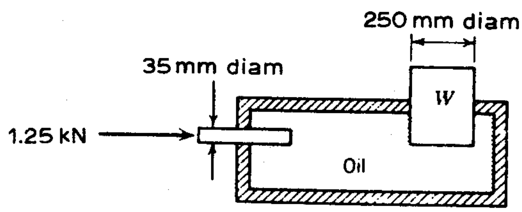



Fig. 2-61

**2.82** Neglecting the container's weight in Fig. 2-62, find the force tending to lift the circular top $CD$.

$$p_{CD} - [(0.8)(62.4)](4) = 0 \qquad p_{CD} = 199.7\text{ lb/ft}^2 \qquad F = pA = (199.7)[\pi(2.5)^2/4] = 980\text{ lb}$$

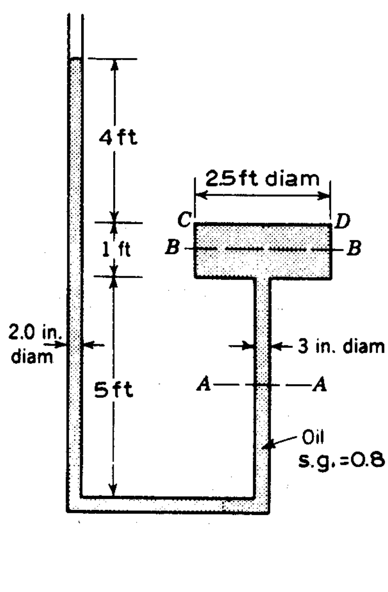



Fig. 2-62

**2.83** Find the force of oil on the top surface $CD$ of Fig. 2-62 if the liquid level in the open pipe is reduced by 1.3 m.

▌ $p_{CD} - [(0.8)(62.4)][4 - (1.3)(3.281)] = 0 \qquad p_{CD} = -13.24\ \text{lb/ft}^2$ (i.e., a downward pressure by $CD$)

$$F = pA = (-13.24)[\pi(2.5)^2/4] = -65.0\ \text{lb}$$

**2.84** A drum 2.25 ft in diameter filled with water has a vertical pipe of 0.70-in diameter attached to the top. How many pounds of water must be poured into the pipe to exert a force of 1500 lb on the top of the drum?

▌ $$p = F/A = 1500/[\pi(2.25)^2/4] = 377.3\ \text{lb/ft}^2 \qquad h = p/\gamma = 377.3/62.4 = 6.05\ \text{ft}$$

$$W_{H_2O} = (6.05)[\pi(0.70/12)^2/4](62.4) = 1.01\ \text{lb}$$

**2.85** In Fig. 2-63, the liquid at $A$ and $B$ is water and the manometer liquid is oil with s.g. $= 0.80$, $h_1 = 300$ mm, $h_2 = 200$ mm, and $h_3 = 600$ mm. **(a)** Determine $p_A - p_B$. **(b)** If $p_B = 50$ kPa and the barometer reading is 730 mmHg, find the absolute pressure at $A$ in meters of water.

▌ **(a)** $$p_A - (9.79)(\tfrac{300}{1000}) - [(0.80)(9.79)](\tfrac{200}{1000}) + (9.79)(\tfrac{600}{1000}) = p_B \qquad p_A - p_B = -1.37\ \text{kPa}$$

**(b)** $$p_A - (9.79)(\tfrac{300}{1000}) - [(0.80)(9.79)](\tfrac{200}{1000}) + (9.79)(\tfrac{600}{1000}) = 50$$

$$p_A = 48.63\ \text{kPa}\ \text{(gage)} = 48.63/9.79 + \tfrac{730}{1000}(13.6) = 14.90\ \text{m water (absolute)}$$

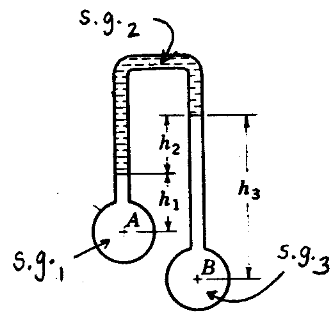



**Fig. 2-63**

**2.86** In Fig. 2-63, s.g.$_1 = 1.0$, s.g.$_2 = 0.96$, s.g.$_3 = 1.0$, $h_1 = h_2 = 269$ mm, and $h_3 = 1.2$ m. Compute $p_A - p_B$ in millimeters of water.

▌ $$p_A - (1.0)(269) - (0.96)(269) + (1.0)(1200) = p_B \qquad p_A - p_B = -673\ \text{mm of water}$$

**2.87** In Fig. 2-63, s.g.$_1 = 1.0$, s.g.$_2 = 0.94$, s.g.$_3 = 1.0$, $h_1 = 300$ mm, $h_3 = 1.1$ m, and $p_A - p_B = -360$ mm of water. Find the gage difference ($h_2$).

▌ $$p_A - (1.0)(300) - (0.94)(h_2) + (1.0)(1100) = p_B \qquad p_A - p_B = -360 = -800 + (0.94)(h_2) \qquad h_2 = 468\ \text{mm}$$

**2.88** What is the pressure difference, in pounds per square inch, of a 1000-ft water column?

▌ $$p = \gamma h = (62.4)(1000)/144 = 433\ \text{psi}$$

**2.89** Find the pressure at a point 9.5 m below the free surface in a fluid whose density varies with depth $h$ (in m) according to

$$\rho = (450\ \text{kg/m}^3) + (11\ \text{kg/m}^4)h$$

▌ $dp = \gamma\, dh = \rho g\, dh = (g)(450 + 11h)\, dh$. Integrating both sides: $p = (g)(450h + 11h^2/2)$. For $h = 9.5$ m: $p = (9.81)[(450)(9.5) + (11)(9.5)^2/2] = 46.807$ kPa.

**2.90** If atmospheric pressure is 29.72 inHg, what will be the height of water in a water barometer if the temperature of the water is **(a)** 50 °F, **(b)** 100 °F, and **(c)** 150 °F?

▌ $$p = \gamma h = [(13.6)(62.4)](29.72/12) = 2102\ \text{lb/ft}^2 \quad \text{or} \quad 14.60\ \text{lb/in}^2$$

**(a)** At 50 °F, $\gamma = 62.4\ \text{lb/ft}^3$ and $p_{\text{vapor}} = 25.7/144$, or $0.178\ \text{lb/in}^2$, $h_{H_2O} = (14.60 - 0.178)(144)/62.4 = 33.28$ ft.
**(b)** At 100 °F, $\gamma = 62.0\ \text{lb/ft}^3$ and $p_{\text{vapor}} = \tfrac{135}{144}$, or $0.938\ \text{lb/in}^2$, $h_{H_2O} = (14.60 - 0.938)(144)/62.0 = 31.73$ ft.
**(c)** At 150 °F, $\gamma = 61.2\ \text{lb/ft}^3$ and $p_{\text{vapor}} = \tfrac{545}{144}$, or $3.78\ \text{lb/in}^2$, $h_{H_2O} = (14.60 - 3.78)(144)/61.2 = 25.46$ ft.

**2.91** A bicycle tire is inflated at sea level (where atmospheric pressure is 14.6 psia and the temperature is 69 °F) to 65.0 psi. Assuming the tire does not expand, what is the gage pressure within the tire on the top of Everest (altitude 30 000 ft), where atmospheric pressure is 4.3 psia and the temperature is −38 °F?

▌ Let subscript 1 indicate sea level and subscript 2 indicate altitude 30 000 ft.

$$(p_1)_{abs} = 14.6 + 65.0 = 79.6 \text{ psia} \qquad p_1V_1/T_1 = p_2V_2/T_2$$
$$(79.6)(V)/(460+69) = (p_2)(V)/[460+(-38)] \qquad (V \text{ is constant})$$
$$(p_2)_{abs} = 63.5 \text{ psia} \qquad (p_2)_{gage} = 63.5 - 4.3 = 59.2 \text{ psi}$$

**2.92** Find the difference in pressure between tanks $A$ and $B$ in Fig. 2-64 if $d_1 = 330$ mm, $d_2 = 160$ mm, $d_3 = 480$ mm, and $d_4 = 230$ mm.

▌
$$p_A + (9.79)(0.330) - [(13.6)(9.79)](0.480 + 0.230 \sin 45°) = p_B \qquad p_A - p_B = 82.33 \text{ kPa}$$

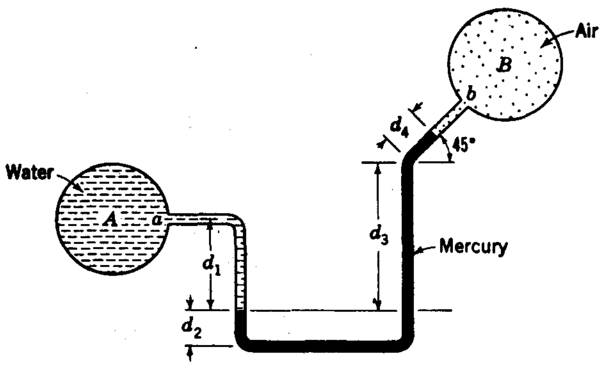



**Fig. 2-64**

**2.93** A cylindrical tank contains water at a height of 55 mm, as shown in Fig. 2-65. Inside is a smaller open cylindrical tank containing cleaning fluid (s.g. = 0.8) at height $h$. If $p_B = 13.40$ kPa gage and $p_C = 13.42$ kPa gage, what are gage pressure $p_A$ and height $h$ of cleaning fluid? Assume that the cleaning fluid is prevented from moving to the top of the tank.

▌
$$p_A + (9.79)(0.055) = 13.42 \qquad p_A = 12.88 \text{ kPa}$$
$$12.88 + (9.79)(0.055 - h) + [(0.8)(9.79)]h = 13.40 \qquad h = 0.00942 \text{ m} = 9.42 \text{ mm}$$

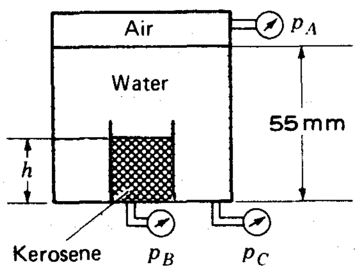



**Fig. 2-65**

**2.94** An open tube is attached to a tank, as shown in Fig. 2-66. If the water rises to a height of 800 mm in the tube, what are the pressures $p_A$ and $p_B$ of the air above the water? Neglect capillary effects in the tube.

▌
$$p_A - (9.79)[(800 - 300 - 100)/1000] = 0 \qquad p_A = 3.92 \text{ kPa}$$
$$p_B - (9.79)[(800 - 300)/1000] = 0 \qquad p_B = 4.90 \text{ kPa}$$

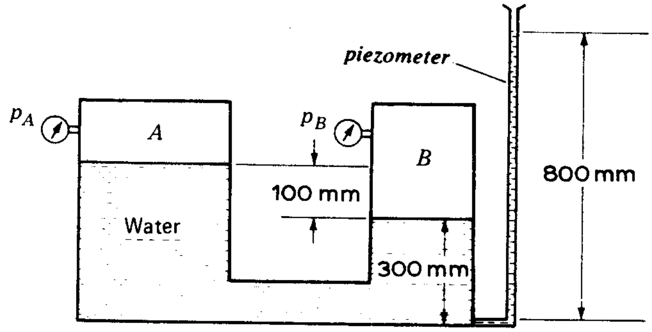



**Fig. 2-66**

**2.95** For the setup shown in Fig. 2-67, what is the pressure $p_A$ if the specific gravity of the oil is 0.82?

▌ $$p_A + [(0.82)(9.79)](3) + (9.79)(4-3) - [(13.6)(9.79)](0.320) = 0 \qquad p_A = 8.73 \text{ kPa}$$

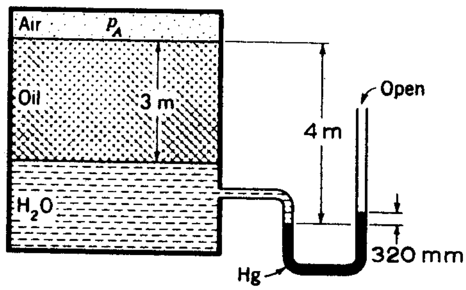



**Fig. 2-67**

**2.96** For the setup shown in Fig. 2-68, calculate the absolute pressure at $a$. Assume standard atmospheric pressure, 101.3 kPa.

▌ $$101.3 + (9.79)(0.600 - 0.200) - [(13.6)(9.79)](0.140) + [(0.83)(9.79)](0.140 + 0.090) = p_A$$

$$p_A = 88.44 \text{ kPa}$$

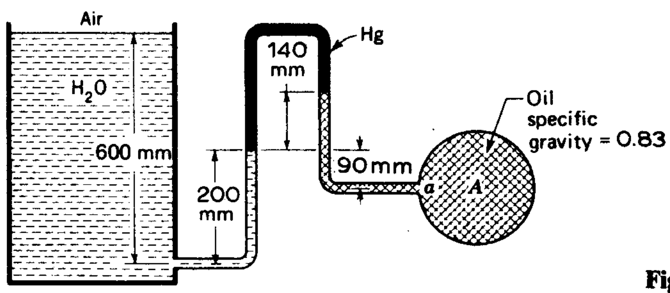



**Fig. 2-68**

**2.97** A force of 460 N is exerted on lever $AB$, as shown in Fig. 2-69. End $B$ is connected to a piston which fits into a cylinder having a diameter of 60 mm. What force $F_D$ acts on the larger piston, if the volume between $C$ and $D$ is filled with water?

▌ Let $F_C$ = force exerted on smaller piston at $C$: $F_C = (460)(\frac{220}{120}) = 843$ N. $F_C/A_C = F_D/A_D$, $(843)/[\pi(\frac{60}{1000})^2/4] = F_D/[\pi(\frac{260}{1000})^2/4]$, $F_D = 15\,830$ N, or 15.83 kN.

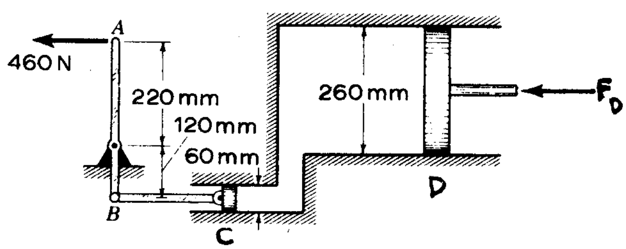



**Fig. 2-69**

# CHAPTER 3

# Forces on Submerged Plane Areas

**3.1** If a triangle of height $d$ and base $b$ is vertical and submerged in liquid with its vertex at the liquid surface (see Fig. 3-1), derive an expression for the depth to its center of pressure.

**I**

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \frac{2d}{3} + \frac{bd^3/36}{(2d/3)(bd/2)} = \frac{3d}{4}$$

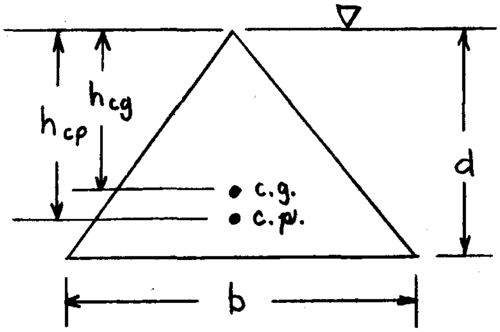



Fig. 3-1

**3.2** If a triangle of height $d$ and base $b$ is vertical and submerged in liquid with its vertex a distance $a$ below the liquid surface (see Fig. 3-2), derive an expression for the depth to its center of pressure.

**I**

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \left(a + \frac{2d}{3}\right) + \frac{bd^3/36}{(a + 2d/3)(bd/2)} = \left(a + \frac{2d}{3}\right) + \frac{d^2}{18(a + 2d/3)}$$

$$= \frac{18(a^2 + 4ad/3 + 4d^2/9) + d^2}{18(a + 2d/3)} = \frac{6a^2 + 8ad + 3d^2}{6(a + 2d/3)}$$

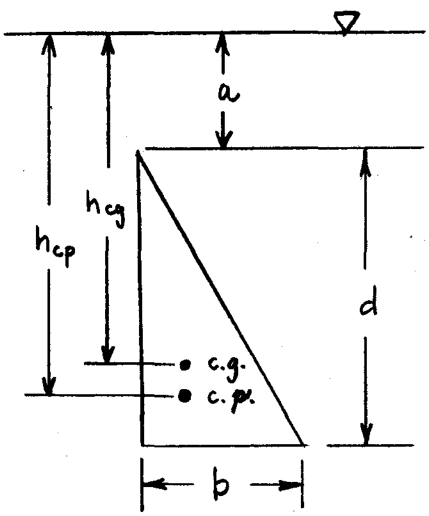



Fig. 3-2

**3.3** If a triangle of height $d$ and base $b$ is vertical and submerged in liquid with its base at the liquid surface (see Fig. 3-3), derive an expression for the depth to its center of pressure.

**I**

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \frac{d}{3} + \frac{bd^3/36}{(d/3)(bd/2)} = \frac{d}{3} + \frac{d}{6} = \frac{d}{2}$$

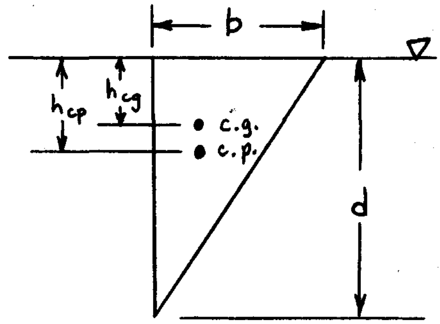



Fig. 3-3

**3.4** A circular area of diameter $d$ is vertical and submerged in a liquid. Its upper edge is coincident with the liquid surface (see Fig. 3-4). Derive an expression for the depth to its center of pressure.

▮
$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \frac{d}{2} + \frac{\pi d^4/64}{(d/2)(\pi d^2/4)} = \frac{d}{2} + \frac{d}{8} = \frac{5d}{8}$$

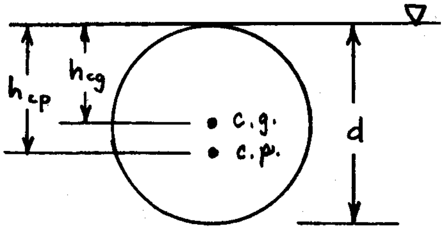



Fig. 3-4

**3.5** A vertical semicircular area of diameter $d$ and radius $r$ is submerged and has its diameter in a liquid surface (see Fig. 3-5). Derive an expression for the depth to its center of pressure.

▮
$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} \qquad h_{cg} = \frac{4r}{3\pi} \qquad I_x = \frac{1}{2}\left(\frac{\pi d^4}{64}\right) = \frac{1}{2}\left[\frac{\pi (2r)^4}{64}\right] = \frac{\pi r^4}{8}$$

$$I_{cg} = \frac{\pi r^4}{8} - \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)(r^4) \qquad h_{cp} = \frac{4r}{3\pi} + \frac{[\pi/8 - 8/(9\pi)](r^4)}{[4r/(3\pi)][(\pi r^2/2)]} = 0.589r$$

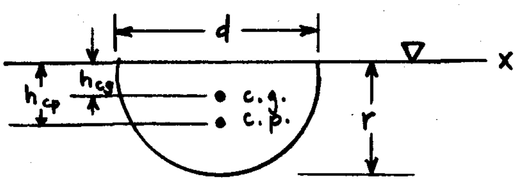



Fig. 3-5

**3.6** A dam 20 m long retains 7 m of water, as shown in Fig. 3-6. Find the total resultant force acting on the dam and the location of the center of pressure.

▮ $F = \gamma hA = (9.79)[(0 + 7)/2][(20)(7/\sin 60°)] = 5339$ kN. The center of pressure is located at two-thirds the total water depth of 7 m, or 4.667 m below the water surface (i.e., $h_{cp} = 4.667$ m in Fig. 3-6).

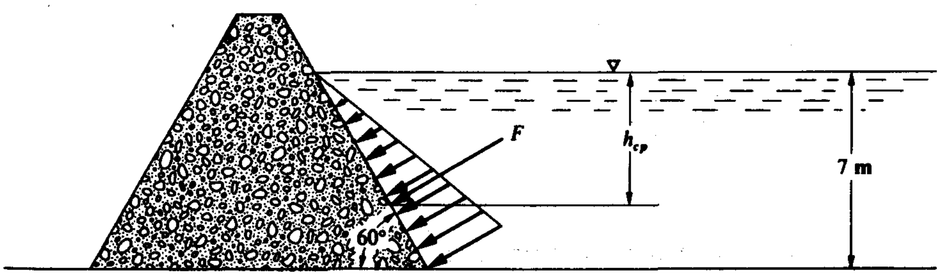



Fig. 3-6

**3.7** A vertical, rectangular gate with water on one side is shown in Fig. 3-7. Determine the total resultant force acting on the gate and the location of the center of pressure.

■

$$F = \gamma h_{cg} A = (9.79)(3 + 1.2/2)[(2)(1.2)] = 84.59 \text{ kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \left(3 + \frac{1.2}{2}\right) + \frac{(2)(1.2)^3/12}{(3 + 1.2/2)[(2)(1.2)]} = 3.633 \text{ m}$$

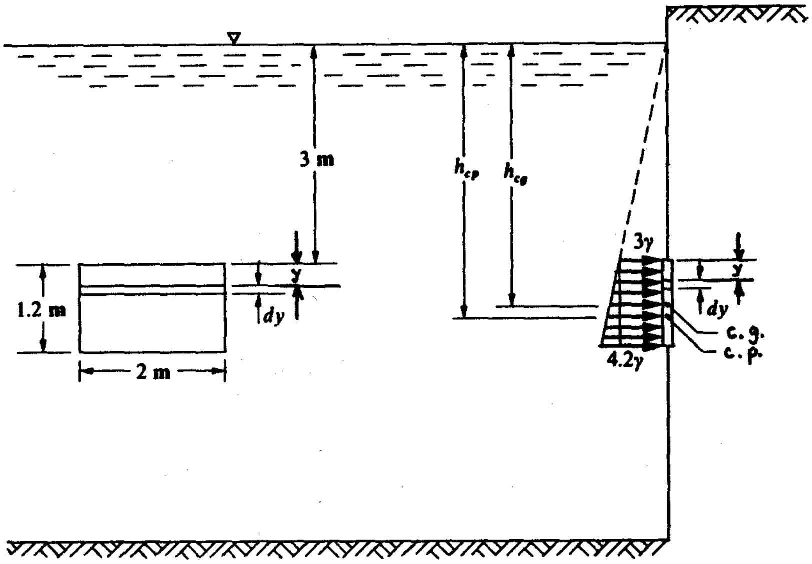



Fig. 3-7

**3.8** Solve Prob. 3.7 by the integration method.

■

$$F = \int \gamma h \, dA = \int_0^{1.2} (9.79)(3 + y)(2\,dy) = (19.58)\left[3y + \frac{y^2}{2}\right]_0^{1.2} = 84.59 \text{ kN}$$

$$h_{cp} = \frac{\int \gamma h^2 \, dA}{F} = \frac{\int_0^{1.2} (9.79)(3 + y)^2(2\,dy)}{84.59} = \frac{(19.58)[9y + 3y^2 + y^3/3]_0^{1.2}}{84.59} = 3.633 \text{ m}$$

**3.9** A vertical, triangular gate with water on one side is shown in Fig. 3-8. Determine the total resultant force acting on the gate and the location of the center of pressure.

■

$$F = \gamma h_{cg} A = (62.4)(6 + 3/3)[(2)(3)/2] = 1310 \text{ lb}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \left(6 + \frac{3}{3}\right) + \frac{(2)(3)^3/36}{(6 + 3/3)[(2)(3)/2]} = 7.07 \text{ ft}$$

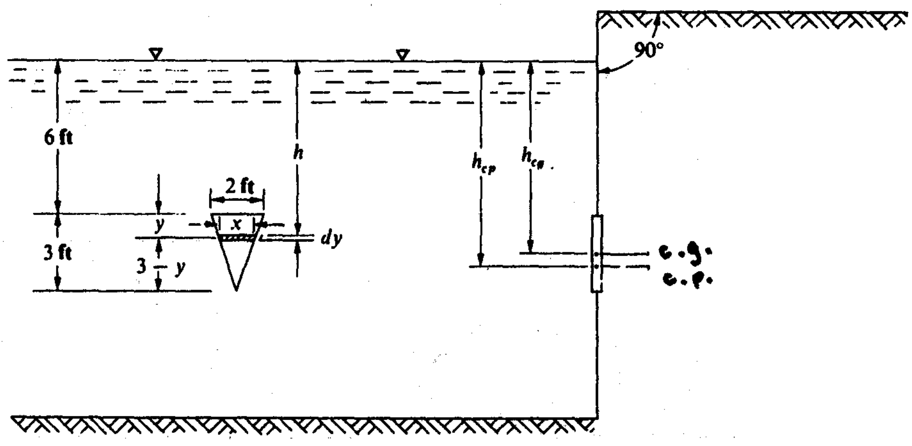



Fig. 3-8

**3.10** Solve Prob. 3.9 by the integration method.

$$F = \gamma h_{cg} A = [(0.82)(9.79)][4 + (1 + 1.2/2)(\sin 40°)][(0.8)(1.2)] = 38.75 \text{ kN}$$

$$F = \int_0^3 (62.4)(6+y)[(2 - 2y/3)\,dy] = \int_0^3 (62.4)(12 - 2y - 2y^2/3)\,dy = (62.4)[12y - y^2 - 2y^3/9]_0^3 = 1310 \text{ lb}$$

$$h_{cp} = \frac{\int \gamma h^2\,dA}{F} = \frac{\int_0^3 (62.4)(6+y)^2(2 - 2y/3)\,dy}{1310} = \frac{\int_0^3 (62.4)(72 - 6y^2 - 2y^3/3)\,dy}{1310}$$

$$= \frac{(62.4)[72y - 2y^3 - y^4/6]_0^3}{1310} = 7.07 \text{ ft}$$

**3.11** An inclined, rectangular gate with water on one side is shown in Fig. 3-9. Determine the total resultant force acting on the gate and the location of the center of pressure.

$$F = \gamma h_{cg} A = (62.4)[8 + \tfrac{1}{2}(4 \cos 60°)][(4)(5)] = 11\,230 \text{ lb}$$

$$z_{cp} = z_{cg} + \frac{I_{cg}}{z_{cg}A} = \left(\frac{8}{\cos 60°} + \frac{4}{2}\right) + \frac{(5)(4)^3/12}{(8/\cos 60° + \frac{4}{2})[(4)(5)]} = 18.07 \text{ ft}$$

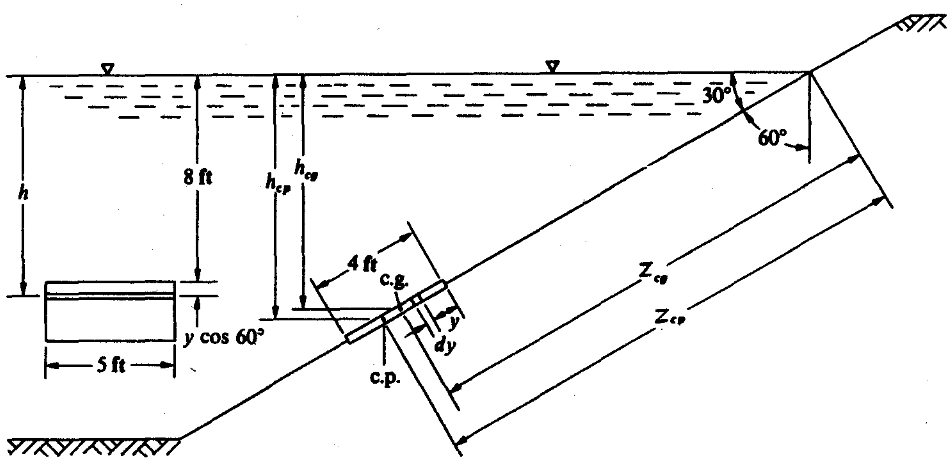



Fig. 3-9

**3.12** Solve Prob. 3.11 by the integration method.

$$F = \int \gamma h\,dA = \int_0^4 (62.4)(8 + y\cos 60°)(5\,dy) = (312)\left[8y + \frac{y^2}{4}\right]_0^4 = 11\,230 \text{ lb}$$

$$h_{cp} = \frac{\int \gamma h^2\,dA}{F} = \frac{\int_0^4 (62.4)(8 + y\cos 60°)^2(5\,dy)}{11\,230} = \frac{\int_0^4 (312)(64 + 8y + y^2/4)\,dy}{11\,230}$$

$$= \frac{(312)[64y + 4y^2 + y^3/12]_0^4}{11\,230} = 9.04 \text{ ft}$$

**Note:** $h_{cp}$ is the vertical distance from the water surface to the center of pressure. The distance from the water surface to the center of pressure as measured along the inclination of the gate ($z_{cp}$) would be 9.04/cos 60°, or 18.08 ft.

**3.13** An inclined, circular gate with water on one side is shown in Fig. 3-10. Determine the total resultant force acting on the gate and the location of the center of pressure.

$$F = \gamma h_{cg} A = (9.79)[1.5 + \tfrac{1}{2}(1.0 \sin 60°)][\pi(1.0)^2/4] = 14.86 \text{ kN}$$

$$z_{cp} = z_{cg} + \frac{I_{cg}}{z_{cg}A} = \left[\frac{1.5}{\sin 60°} + \frac{1}{2}(1.0)\right] + \frac{\pi(1.0)^4/64}{[1.5/\sin 60° + \frac{1}{2}(1.0)][\pi(1.0)^2/4]} = 2.260 \text{ m}$$

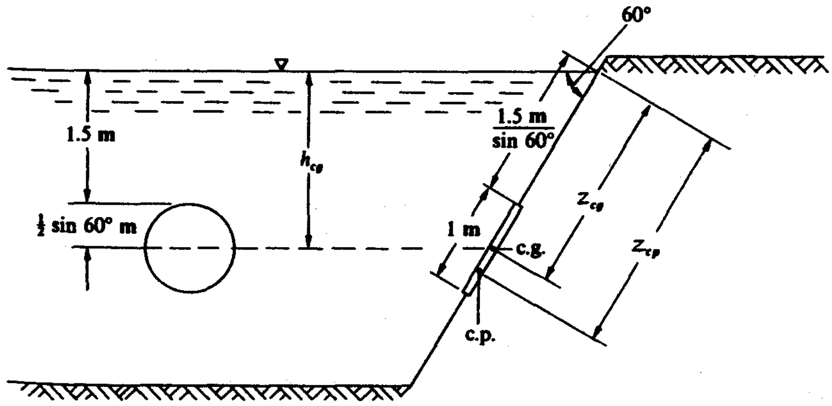



Fig. 3-10

**3.14** A vertical, triangular gate with water on one side is shown in Fig. 3-11. Determine the total resultant force acting on the gate and the location of the center of pressure.

$$F = \gamma h_{cg} A = (9.79)[3 + \tfrac{2}{3}(1)][(1.2)(1)/2] = 21.54 \text{ kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = [3 + (\tfrac{2}{3})(1)] + \frac{(1.2)(1)^3/36}{[3 + \tfrac{2}{3}(1)][(1.2)(1)/2]} = 3.68 \text{ m}$$

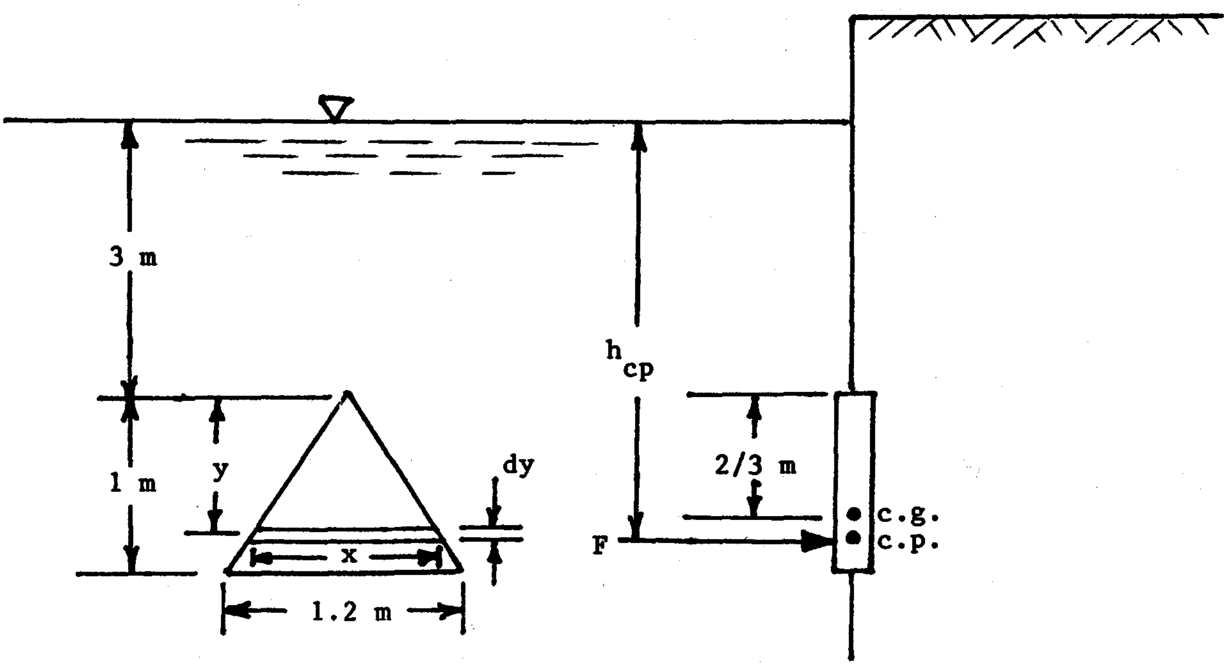



Fig. 3-11

**3.15** Solve Prob. 3.14 by the integration method.

$F = \int \gamma h \, dA$. From Fig. 3-11, $y/x = 1/1.2$. Therefore, $x = 1.2y$.

$$F = \int_0^1 (9.79)(3 + y)(1.2y \, dy) = \int_0^1 (11.75)(3y + y^2) \, dy = (11.75)\left[\frac{3y^2}{2} + \frac{y^3}{3}\right]_0^1 = 21.54 \text{ kN}$$

$$h_{cp} = \frac{\int \gamma h^2 \, dA}{F} = \frac{\int_0^1 (9.79)(3 + y)^2(1.2y \, dy)}{21.54} = \frac{\int_0^1 (11.75)(9y + 6y^2 + y^3) \, dy}{21.54}$$

$$= \frac{(11.75)[9y^2/2 + 2y^3 + y^4/4]_0^1}{21.54} = 3.68 \text{ m}$$

**3.16** A tank containing water is shown in Fig. 3-12. Calculate the total resultant force acting on side $ABCD$ of the container and the location of the center of pressure.

▮

$$F = \gamma h A = (62.4)[(0+6)/2][(20)(6)] = 22\,500 \text{ lb}$$

$$h_{cp} = (\tfrac{2}{3})(6) = 4.00 \text{ ft} \qquad \text{(vertically below the water surface)}$$

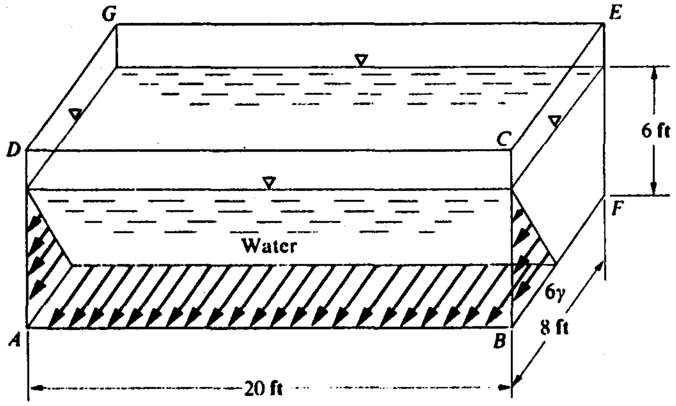



**Fig. 3-12**

**3.17** The gate in Fig. 3-13 is 4 ft wide, is hinged at point $B$, and rests against a smooth wall at $A$. Compute (*a*) the force on the gate due to seawater pressure, (*b*) the (horizontal) force $P$ exerted by the wall at point $A$, and (*c*) the reaction at hinge $B$.

▮ (*a*)

$$F = \gamma h_{cg} A = (64)(17 - \tfrac{7.2}{2})[(4)(12)] = 30\,106 \text{ lb}$$

(*b*)

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(4)(12)^3/12](\frac{7.2}{12})}{(17-\frac{7.2}{2})[(4)(12)]} = -0.537 \text{ ft}$$

$$\sum M_B = 0 \qquad (P)(7.2) - (30\,106)(12 - 6 - 0.537) = 0 \qquad P = 22\,843 \text{ lb}$$

(*c*)

$$\sum F_x = 0 \qquad B_x + (30\,106)(\tfrac{7.2}{12}) - 22\,843 = 0 \qquad B_x = 4779 \text{ lb}$$

$$\sum F_y = 0 \qquad B_y - (30\,106)(\tfrac{9.6}{12}) = 0 \qquad B_y = 24\,085 \text{ lb}$$

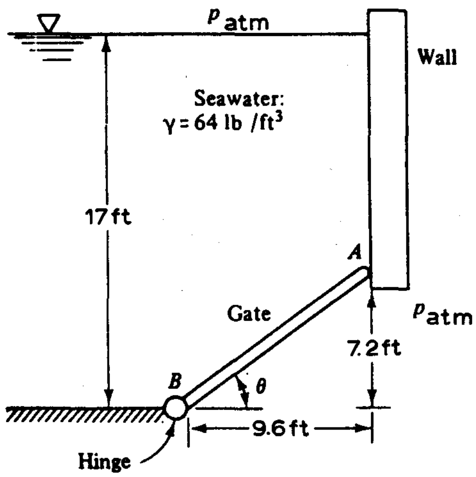



**Fig. 3-13(*a*)**

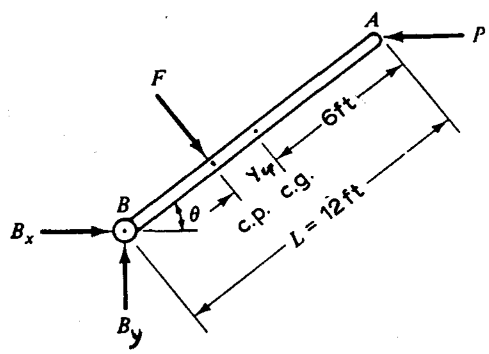



**Fig. 3-13(*b*)**

**3.18** Repeat Prob. 3.17, but instead let the hinge be at point $A$ and let point $B$ rest against a smooth bottom.

▮ (*a*) From Prob. 3.17, $F = 30\,106$ lb. (*b*) From Prob. 3.17, $y_{cp} = -0.537$ ft; $\sum M_A = 0$; $(B_y)(9.6) - (30\,106)(6 + 0.537) = 0$, $B_y = 20\,500$ lb.

(*c*)

$$\sum F_x = 0 \qquad (30\,106)(\tfrac{7.2}{12}) - A_x = 0 \qquad A_x = 18\,064 \text{ lb}$$

$$\sum F_y = 0 \qquad A_y - (30\,106)(\tfrac{9.6}{12}) + 20\,500 = 0 \qquad A_y = 3585 \text{ lb}$$

**3.19** A tank of dye has a right-triangular panel near the bottom as shown in Fig. 3-14*a*. Calculate the resultant force on the panel and locate its center of pressure.

▌

$$F = \gamma h_{cg} A = \rho g h_{cg} A = (820)(9.81)(6+8)[\tfrac{1}{2}(8+16)(8+4)] = 16.22\ \text{MN}$$

$$I_{xx} = \frac{bh^3}{36} = \frac{(4+8)(8+16)^3}{36} = 4608\ \text{m}^4 \qquad y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-(4608)(\sin 30°)}{(6+8)[\tfrac{1}{2}(8+16)(8+4)]} = -1.143\ \text{m}$$

$$I_{xy} = b(b-2s)(h)^2/72 = (4+8)[(4+8)-(2)(4+8)](8+16)^2/72 = -1152\ \text{m}^4$$

$$x_{cp} = \frac{-I_{xy}\sin\theta}{h_{cg}A} = \frac{-(-1152)(\sin 30°)}{(6+8)[\tfrac{1}{2}(8+16)(8+4)]} = +0.286\ \text{m}$$

(The resultant force acts at 1.143 m down and 0.286 m to the right of the centroid.)

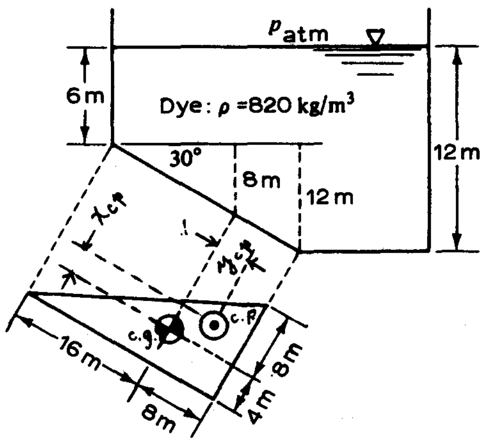



**Fig. 3-14(*a*)**

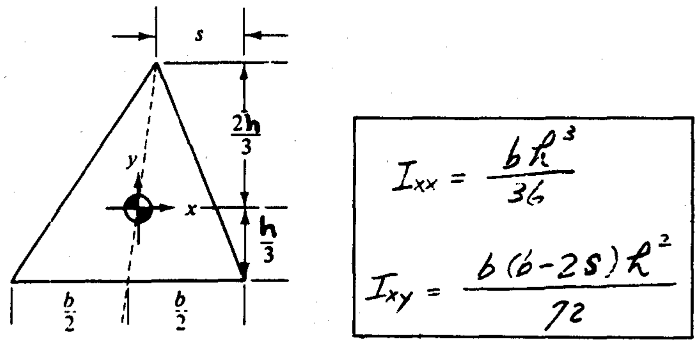



**Fig. 3-14(*b*)**

**3.20** Gate *AB* in Fig. 3-15 is 1.0 m long and 0.9 m wide. Calculate force *F* on the gate and the position *X* of its center of pressure.

▌

$$F = \gamma h_{cg} A = [(0.81)(9.79)][3 + (1 + 1.0/2)(\sin 50°)][(0.9)(1.0)] = 29.61\ \text{kN}$$

$$y_{cp} = \frac{-I_{XX}\sin\theta}{h_{cg}A} = \frac{-[(0.9)(1.0)^3/12](\sin 50°)}{[3 + (1 + 1.0/2)(\sin 50°)][(0.9)(1.0)]}$$

$$= -0.015\ \text{m from the centroid}$$

$$X = 1.0/2 + 0.015 = 0.515\ \text{m from point } A$$

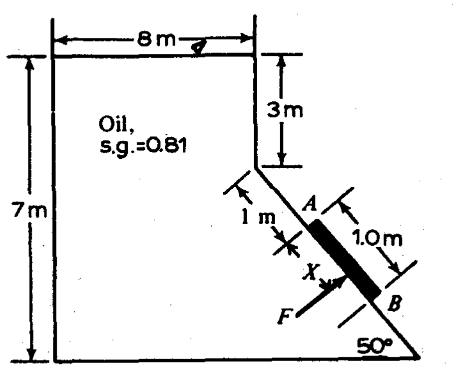



**Fig. 3-15**

**3.21** A fishpond gate 6 ft wide and 9 ft high is hinged at the top and held closed by water pressure as shown in Fig. 3-16. What horizontal force applied at the bottom of the gate is required to open it?

$$F = \gamma h_{cg} A = (62.4)(8+4.5)[(6)(9)] = 42\,120 \text{ lb}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = (8+4.5) + \frac{(6)(9)^3/12}{(8+4.5)[(6)(9)]} = 13.04 \text{ ft}$$

$$\sum M_A = 0 \qquad (P)(9) - (42\,120)(13.04-8) = 0 \qquad P = 23\,587 \text{ lb}$$

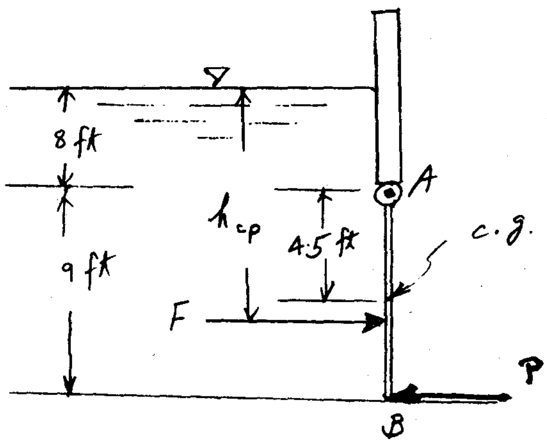



**Fig. 3-16**

**3.22** A vat holding paint (s.g. = 0.80) is 8 m long and 4 m deep and has a trapezoidal cross section 3 m wide at the bottom and 5 m wide at the top (see Fig. 3-17). Compute (*a*) the weight of the paint, (*b*) the force on the bottom of the vat, and (*c*) the force on the trapezoidal end panel.

(*a*)
$$W = \gamma V = [(0.80)(9.79)][(8)(4)(5+3)/2] = 1002 \text{ kN}$$

(*b*)
$$F = \gamma h_{cg} A \qquad F_{\text{bottom}} = [(0.80)(9.79)](4)[(3)(8)] = 752 \text{ kN}$$

(*c*)
$$F_{\text{end}} = F_{\text{square}} + 2F_{\text{triangle}} = [(0.80)(9.79)][(0+4)/2][(4)(3)] + (2)[(0.80)(9.79)](\tfrac{4}{3})[(4)(1)/2] = 230 \text{ kN}$$

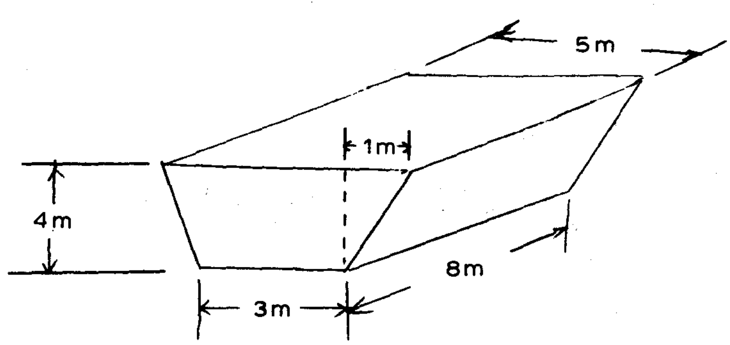



**Fig. 3-17**

**3.23** Gate $AB$ in Fig. 3-18 is 5 ft wide, hinged at point $A$, and restrained by a stop at point $B$. Compute the force on the stop and the components of the reaction at $A$ if water depth $h$ is 9 ft.

$$F = \gamma h_{cg} A = (62.4)(9 - \tfrac{3}{2})[(3)(5)] = 7020 \text{ lb}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(5)(3)^3/12](\sin 90°)}{(9-3/2)[(3)(5)]} = -0.100 \text{ ft}$$

$$\sum M_A = 0 \qquad (B_x)(3) - (7020)(1.5+0.100) = 0 \qquad B_x = 3744 \text{ lb}$$

$$\sum F_x = 0 \qquad 7020 - 3744 - A_x = 0 \qquad A_x = 3276 \text{ lb}$$

If gate weight is neglected, $A_y = 0$.

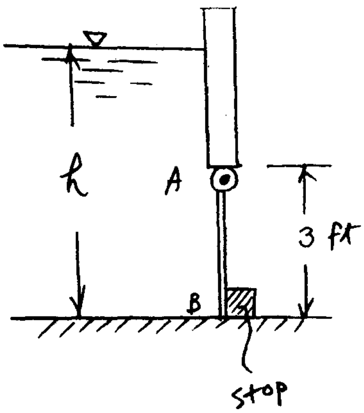



Fig. 3-18(a)

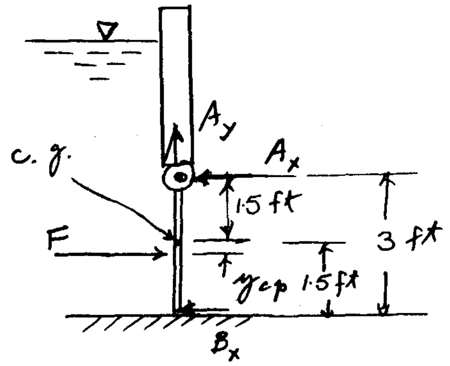



Fig. 3-18(b)

**3.24** In Fig. 3-18, stop $B$ will break if the force on it reaches 9000 lb. Find the critical water depth.

$$F = \gamma h_{cg} A = (62.4)(h_{cg})[(3)(5)] = 936h_{cg} \qquad y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(5)(3)^3/12](\sin 90°)}{(h_{cg})[(3)(5)]} = -\frac{0.750}{h_{cg}}$$

$$\sum M_A = 0 \qquad (9000)(3) - (936h_{cg})(1.5 + 0.750/h_{cg}) = 0$$

$$h_{cg} = 18.73\text{ ft} \qquad h_{crit} = 18.73 + 1.5 = 20.23\text{ ft}$$

**3.25** In Fig. 3-18, hinge $A$ will break if its horizontal reaction becomes equal to 8000 lb. Find the critical water depth.

From Prob. 3.24, $F = 936h_{cg}$ and $y_{cp} = -0.750/h_{cg}$; $\sum M_B = 0$; $(936h_{cg})(1.5 - 0.750/h_{cg}) - (8000)(3) = 0$, $h_{cg} = 17.59$ ft; $h_{crit} = 17.59 + 1.5 = 19.09$ ft.

**3.26** Calculate the resultant force on triangular window $ABC$ in Fig. 3-19 and locate its center of pressure.

$$F = \gamma h_{cg} A = (10.08)[0.25 + (\tfrac{2}{3})(0.60)][(0.40)(0.60)/2] = 0.786\text{ kN}$$

$$I_{xx} = bh^3/36 = (0.40)(0.60)^3/36 = 0.00240\text{ m}^4$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-(0.00240)(\sin 90°)}{[0.25 + (\frac{2}{3})(0.60)][(0.40)(0.60)/2]} = -31\text{ mm} \qquad \text{(i.e., below the centroid)}$$

$$I_{xy} = b(b - 2s)(h)^2/72 = 0.40[0.40 - (2)(0.40)](0.60)^2/72 = -0.000800\text{ m}^4$$

$$x_{cp} = \frac{-I_{xy}\sin\theta}{h_{cg}A} = \frac{-(-0.000800)(\sin 90°)}{[0.25 + (\frac{2}{3})(0.60)][(0.40)(0.60)/2]} = +10\text{ mm} \qquad \text{(i.e., right of the centroid)}$$

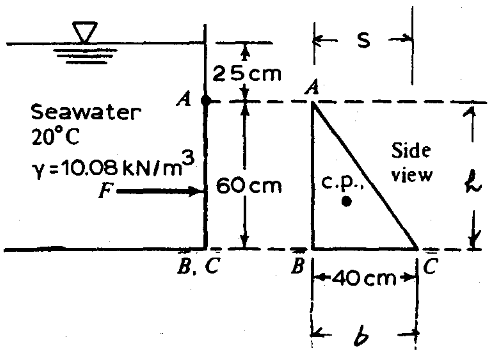



Fig. 3-19

**3.27** Freshly poured concrete approximates a fluid with s.g. = 2.40. Figure 3-20 shows a slab poured between wooden forms which are connected by four corner bolts $A$, $B$, $C$, and $D$. Neglecting end effects, compute the forces in the four bolts.

$$F = \gamma h_{cg} A = [(2.40)(62.4)](\tfrac{12}{2})[(9)(12)] = 97\,044 \text{ lb}$$

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg} A} = \frac{-[(9)(12)^3/12](\sin 90°)}{[(\tfrac{12}{2})][(9)(12)]} = -2.00 \text{ ft}$$

$$\sum M_A = 0 \qquad (2)(F_C)(12) - (97\,044)(6 + 2.00) = 0 \qquad F_C = F_D = 32\,348 \text{ lb}$$

$$\sum M_C = 0 \qquad (97\,044)(6 - 2.00) - (2)(F_A)(12) = 0 \qquad F_A = F_B = 16\,174 \text{ lb}$$

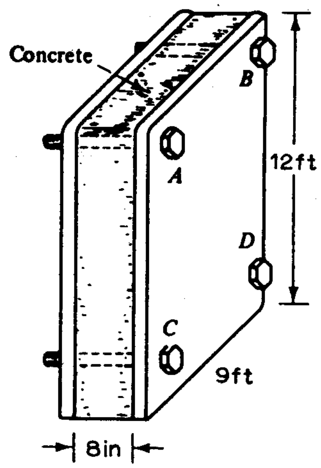



**Fig. 3-20**

**3.28** Find the net hydrostatic force per unit width on rectangular panel $AB$ in Fig. 3-21 and determine its line of action.

$$F_{H_2O} = (9.79)(2 + 1 + \tfrac{2}{2})[(2)(1)] = 78.32 \text{ kN} \qquad F_{glyc} = (12.36)(1 + \tfrac{2}{2})[(2)(1)] = 49.44 \text{ kN}$$

$$F_{net} = F_{H_2O} - F_{glyc} = 78.32 - 49.44 = 28.88 \text{ kN}$$

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg} A}$$

$$(y_{cp})_{H_2O} = \frac{-[(1)(2)^3/12](\sin 90°)}{(2 + 1 + \tfrac{2}{2})[(2)(1)]} = -0.0833 \text{ m}$$

$$(y_{cp})_{glyc} = \frac{-[(1)(2)^3/12](\sin 90°)}{[(1 + \tfrac{2}{2})][(2)(1)]} = -0.1667 \text{ m}$$

$$\sum M_B = 0 \qquad (78.32)(1 - 0.0833) - (49.44)(1 - 0.1667) = 28.88D$$

$D = 1.059$ m (above point $B$, as shown in Fig. 3-21$c$)

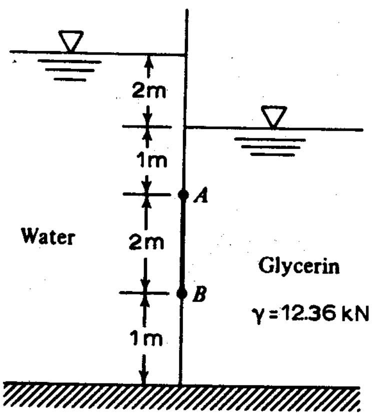



**Fig. 3-21(a)**

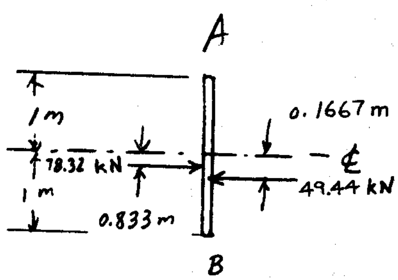



**Fig. 3-21(b)**

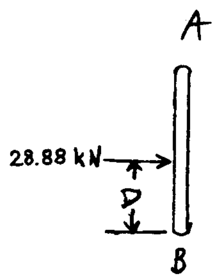



**Fig. 3-21(c)**

**3.29** A cylindrical, wooden-stave barrel is 3.5 ft in diameter and 5 ft high, as shown in Fig. 3-22. It is held together by steel hoops at the top and bottom, each with a cross section of 0.40 in². If the barrel is filled with orange juice (s.g. = 1.04), compute the tension stress in each hoop.

▌

$$F = \gamma h_{cg}A = [(1.04)(62.4)](\tfrac{5}{2})[(3.5)(5)] = 2839 \text{ lb}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(3.5)(5)^3/12](\sin 90^\circ)}{\frac{5}{2}[(3.5)(5)]} = -0.833 \text{ ft}$$

$$\sum M_B = 0 \qquad 2839(\tfrac{5}{2} - 0.833) - 2(F_{\text{upper}})(5) = 0 \qquad F_{\text{upper}} = 473 \text{ lb}$$

$$\sum M_A = 0 \qquad 2(F_{\text{lower}})(5) - 2839(\tfrac{5}{2} + 0.833) = 0 \qquad F_{\text{lower}} = 946 \text{ lb}$$

$$\sigma_{\text{upper}} = 473/0.40 = 1182 \text{ psi} \qquad \sigma_{\text{lower}} = 946/0.40 = 2365 \text{ psi}$$

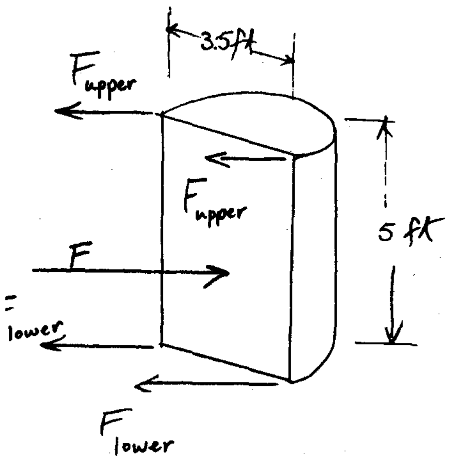


Fig. 3-22(a)

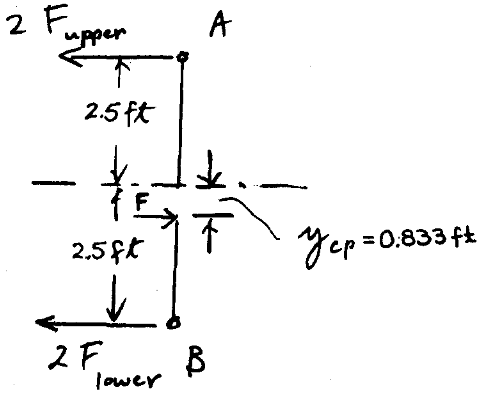


Fig. 3-22(b)

**3.30** Gate $AB$ in Fig. 3-23$a$ is 16 ft long and 8 ft wide. Neglecting the weight of the gate, compute the water level $h$ for which the gate will start to fall.

▌

$$F = \gamma h_{cg}A = (62.4)(h/2)[(8)(h/\sin 60^\circ)] = 288.2h^2$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[8(h/\sin 60^\circ)^3/12](\sin 60^\circ)}{(h/2)[8(h/\sin 60^\circ)]} = -0.1925h$$

$$\sum M_B = 0 \qquad (11\,000)(16) - (288.2h^2)[(h/\sin 60^\circ)/2 - 0.1925h] = 0 \qquad h = 11.7 \text{ ft}$$

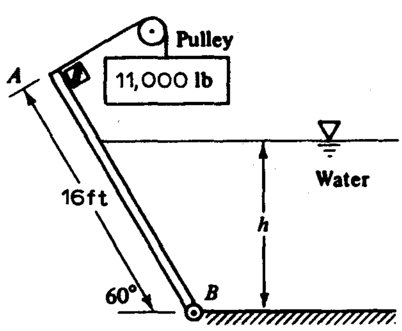


Fig. 3-23(a)

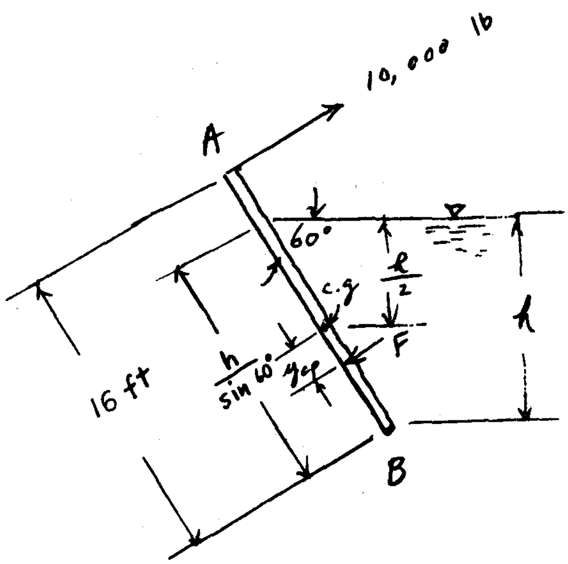


Fig. 3-23(b)

**3.31** Repeat Prob. 3.30, including the weight of the 2-in-thick steel (s.g. = 7.85) gate. (See Fig. 3-24.)

▌ $W_{\text{gate}} = [(7.85)(62.4)][(16)(8)(\frac{2}{12})] = 10\,450$ lb. From Prob. 3.30, $F = 288.2h^2$; $\sum M_B = 0$, $(11\,000)(16) - (288.2h^2)[(h/\sin 60°)/2 - 0.1925h] - 10\,450(\frac{16}{2}\cos 60°) = 0$, $h = 10.7$ ft.

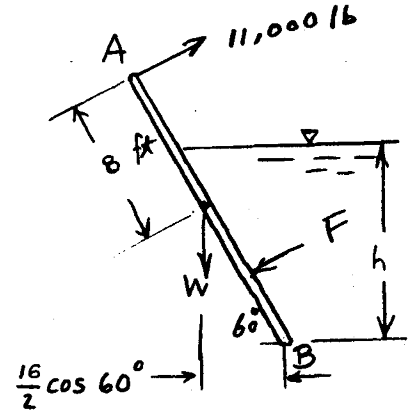



Fig. 3-24

**3.32** A horizontal duct coming from a large dam is 2.5 m in diameter; it is closed by a circular door whose center or centroid is 45 m below the dam's water level. Compute the force on the door and locate its center of pressure.

▌ $F = \gamma h_{cg}A = (9.79)(45)[\pi(2.5)^2/4)] = 2163$ kN $\qquad I_{xx} = \pi r^4/4 = \pi(\frac{2.5}{2})^4/4 = 1.917$ m$^4$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-(1.917)(\sin 90°)}{(45)[\pi(2.5)^2/4)]} = -0.0087 \text{ m}$$

Line of action of $F$ is 8.7 mm below the centroid of the door.

**3.33** Gate $AB$ in Fig. 3-25 is semicircular, hinged at $B$. What horizontal force $P$ is required at $A$ for equilibrium?

▌ $4r/(3\pi) = (4)(4)/(3\pi) = 1.698$ m $\qquad F = \gamma h_{cg}A = (9.79)(6 + 4 - 1.698)[\pi(4)^2/2)] = 2043$ kN

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(0.10976)(4)^4](\sin 90°)}{(6+4-1.698)[\pi(4)^2/2)]} = -0.1347 \text{ m}$$

$$\sum M_B = 0 \qquad (2043)(1.698 - 0.1347) - 4P = 0 \qquad P = 798 \text{ kN}$$

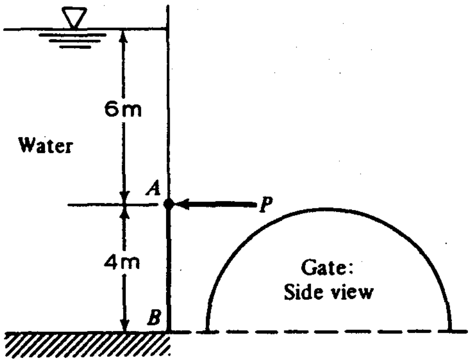



Fig. 3-25(a)

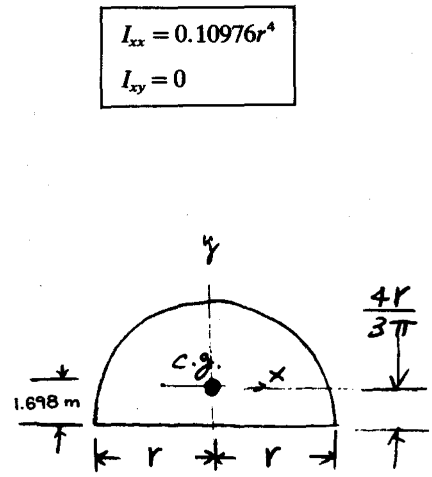



Fig. 3-25(b)

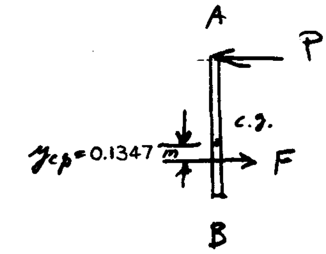



Fig. 3-25(c)

**3.34** Dam $ABC$ in Fig. 3-26 is 38 m wide and made of concrete weighing 22 kN/m$^3$. Find the hydrostatic force on surface $AB$ and its moment about $C$. Could this force tip the dam over?

▌ $F = \gamma h_{cg}A = (9.79)(\frac{64}{2})[(38)(80)] = 952\,371$ kN. $F$ acts at $(\frac{2}{3})(80)$, or 53.33 m from $A$ along surface $AB$ (see Fig. 3-26$b$). For the given triangular shape, the altitude from $C$ to $AB$ intersects $AB$ 51.2 m from $A$ (see Fig. 3-26$b$). Hence, $M_C = (952\,371)(53.33 - 51.2) = 2\,028\,550$ kN. Since the moment of $F$ about point $C$ is counterclockwise, there is no danger of tipping.

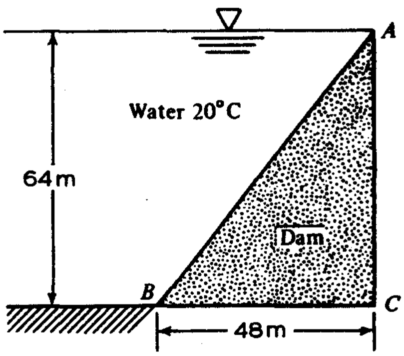



**Fig. 3-26(a)**

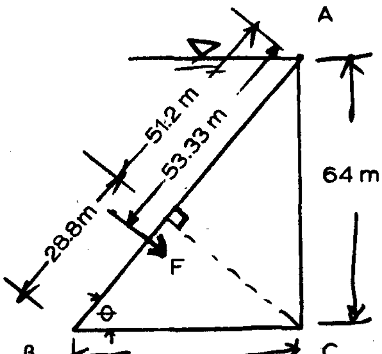



**Fig. 3-26(b)**

**3.35** Isosceles triangular gate $AB$ in Fig. 3-27 is hinged at $A$. Compute the horizontal force $P$ required at point $B$ for equilibrium, neglecting the weight of the gate.

▌ $AB = 3/\sin 60° = 3.464$ m $\qquad F = \gamma h_{cg}A = [(0.82)(9.79)](2 + 1.00)[(1.2)(3.464)/2] = 50.05$ kN

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(1.2)(3.464)^3/36](\sin 60°)}{(2+1.00)[(1.2)(3.464)/2]} = -0.1924 \text{ m}$$

$$\sum M_A = 0 \qquad 3P - (50.05)(3.464/3 + 0.1924) = 0 \qquad P = 22.47 \text{ kN}$$

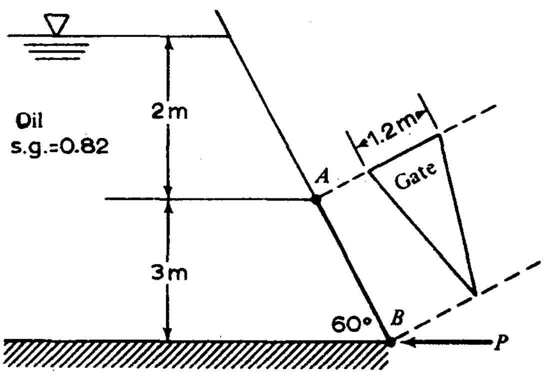



**Fig. 3-27(a)**

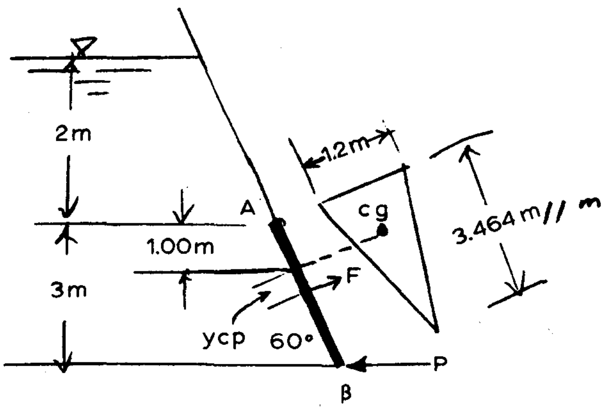



**Fig. 3-27(b)**

**3.36** The tank in Fig. 3-28 is 40 cm wide. Compute the hydrostatic forces on horizontal panels $BC$ and $AD$. Neglect atmospheric pressure.

▌ $p = \gamma h \qquad p_{BC} = [(0.84)(9.79)](0.35 + 0.40) + (9.79)(0.25) = 8.615$ kPa

$$F = pA \qquad F_{BC} = (8.615)[(1.20)(0.40)] = 4.135 \text{ kN}$$

$$p_{AD} = [(0.84)(9.79)](0.40) = 3.289 \text{ kPa} \qquad F_{AD} = (3.289)[(0.55)(0.40)] = 0.724 \text{ kN}$$

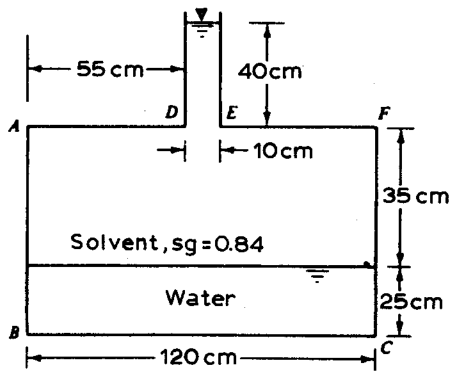



**Fig. 3-28**

**3.37** **Water in a tank is pressurized to 85 cmHg (Fig. 3-29). Determine the hydrostatic force per meter width on panel *AB*.**

▌ On panel $AB$, $p_{cg} = [(13.6)(9.79)](0.85) + (9.79)(4 + \frac{3}{2}) = 167.0$ kPa, $F_{AB} = (167.0)[(3)(1)] = 501$ kN.

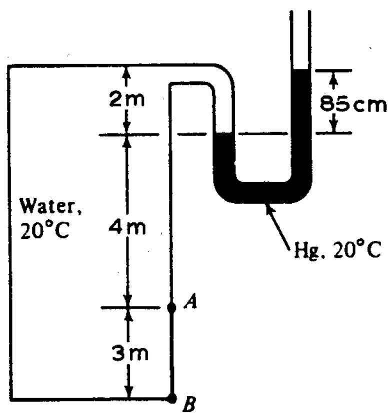



**Fig. 3-29**

**3.38** **Calculate the force and center of pressure on one side of the vertical triangular panel *ABC* in Fig. 3-30.**

▌ $$F = \gamma h_{cg} A = (62.4)(1+6)[(9)(6)/2] = 11\,794 \text{ lb} \qquad I_{xx} = (6)(9)^3/36 = 121.5 \text{ ft}^4$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-(121.5)(\sin 90°)}{(1+6)[(9)(6)/2]} = -0.64 \text{ ft}$$

$$I_{xy} = \frac{6[6-(2)(6)](9)^2}{72} = -40.5 \text{ ft}^4 \qquad x_{cp} = \frac{-I_{xy}\sin\theta}{h_{cg}A} = \frac{-(-40.5)(\sin 90°)}{(1+6)[(9)(6)/2]} = 0.21 \text{ ft}$$

Thus, the center of pressure is 6 + 0.64, or 6.64 ft below point $A$ and 2 + 0.21, or 2.21 ft to the right of point $B$.

**3.39** **In Fig. 3-31, gate *AB* is 4 m wide and is connected by a rod and pulley to a massive sphere (s.g. = 2.40). What is the smallest radius that will keep the gate closed?**

▌ $$F = \gamma h_{cg} A = (9.79)(9 + \tfrac{3}{2})[(4)(3)] = 1234 \text{ kN}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(4)(3)^3/12](\sin 90°)}{(9+\frac{3}{2})[(4)(3)]} = -0.071 \text{ m}$$

$$\sum M_B = 0 \qquad (W_{sphere})(7+9+3) - (1234)(3 - 1.5 - 0.071) = 0 \qquad W_{sphere} = 92.8 \text{ kN}$$

$$W_{sphere} = \gamma(4\pi r^3/3) \qquad 92.8 = [(2.40)(9.79)](4\pi r^3/3) \qquad r = 0.98 \text{ m}$$

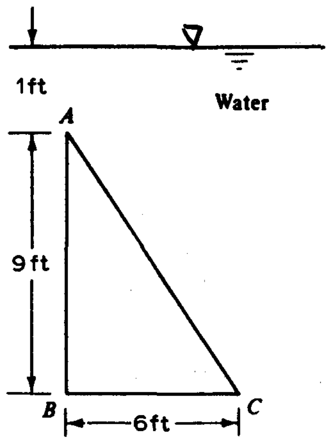



Fig. 3-30(a)

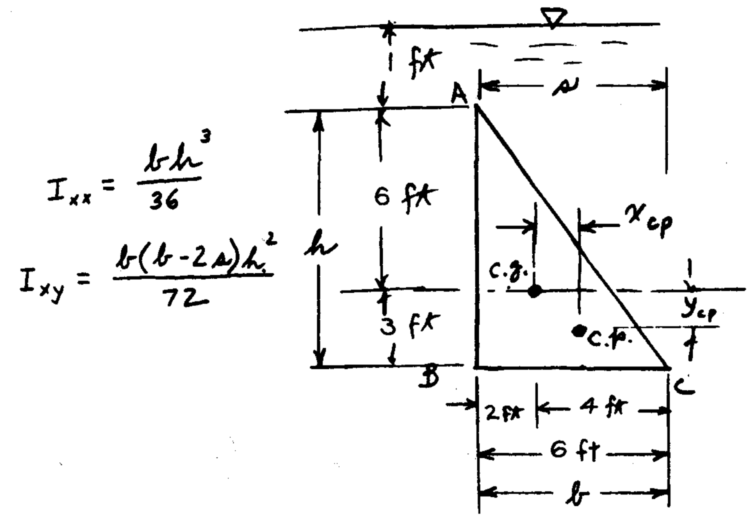



Fig. 3-30(b)

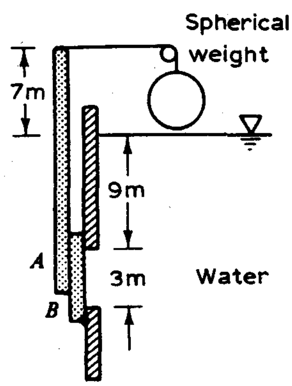



Fig. 3-31

**3.40** The triangular trough in Fig. 3-32 is hinged at $A$ and held together by cable $BC$ at the top. If cable spacing is 1 m into the paper, what is the cable tension?

■

$$F = \gamma h_{cg} A = (9.79)(\tfrac{5}{2})[(8.717)(1)] = 213.3 \text{ kN}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(1)(8.717)^3/12](\sin 35°)}{\frac{5}{2}[(8.717)(1)]} = -1.453 \text{ m}$$

$$\sum M_A = 0 \qquad (T)(2+5) - (213.3)(4.359 - 1.453) = 0 \qquad T = 88.5 \text{ kN}$$

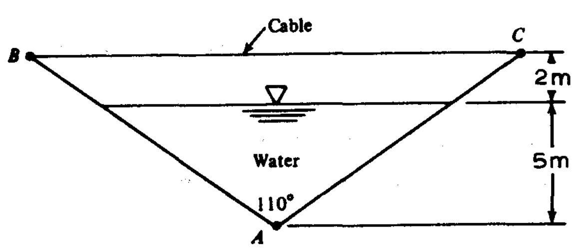



Fig. 3-32(a)

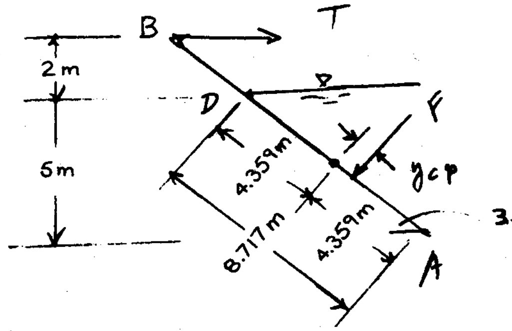



Fig. 3-32(b)

**3.41** In Fig. 3-33, gate $AB$ is 4 ft wide and opens to let fresh water out when the ocean tide is falling. The hinge at $A$ is 3 ft above the fresh-water surface. At what ocean depth $h$ will the gate open? Neglect the gate's weight.

▌ $F = \gamma h_{cg} A \qquad F_1 = (62.4)(\tfrac{12}{2})[(12)(4)] = 17\,971 \text{ lb} \qquad F_2 = [(1.025)(62.4)](h/2)[(4)(h)] = 127.9h^2$

$$\sum M_A = 0 \qquad (127.9h^2)(12 + 3 - h/3) - (17\,971)(3 + 8) = 0 \qquad h = 11.8 \text{ ft}$$

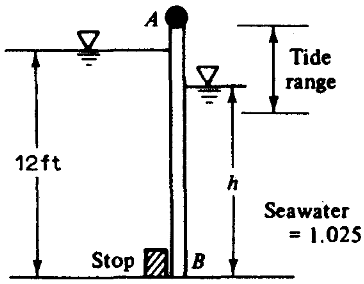



Fig. 3-33(*a*)

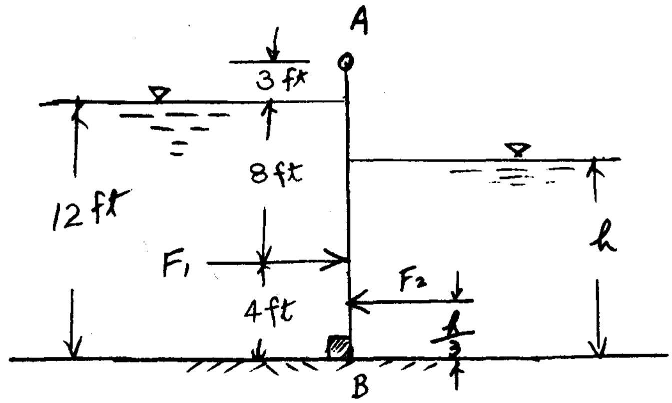



Fig. 3-33(*b*)

**3.42** Show that depth $h$ in Prob. 3.41 is independent of gate width $b$ (perpendicular to the paper).

▌ Areas, and hence pressure forces, are directly proportional to $b$. Thus $b$ will cancel out of the equation $\sum M_A = 0$ that determines $h$.

**3.43** Compute the force on one side of parabolic panel $ABC$ in Fig. 3-34 and the vertical distance down to the center of pressure.

▌
$$F = \gamma h_{cg} A = (9.79)(1 + 6)[(\tfrac{2}{3})(10)(6)] = 2741 \text{ kN}$$

$$I_{xx} = I_{x'} - A(\Delta h)^2 = \tfrac{2}{7}(bh^3) - [\tfrac{2}{3}(bh)][\tfrac{3}{5}(h)]^2 = (\tfrac{2}{7})(6)(10)^3 - [(\tfrac{2}{3})(6)(10)][(\tfrac{3}{5})(10)]^2 = 274.3 \text{ m}^4$$

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg} A} = \frac{-(274.3)(\sin 90°)}{(1 + 6)[(\tfrac{2}{3})(10)(6)]} = -0.980 \text{ m}$$

Hence, the center of pressure is 6 + 0.980, or 6.980 m below point $A$.

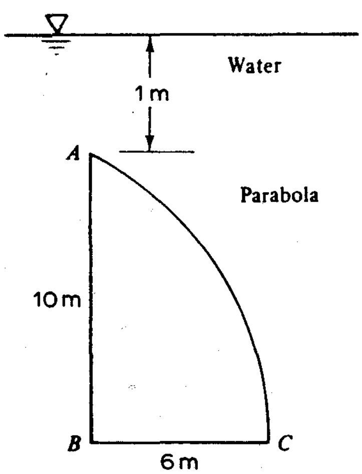



Fig. 3-34(*a*)

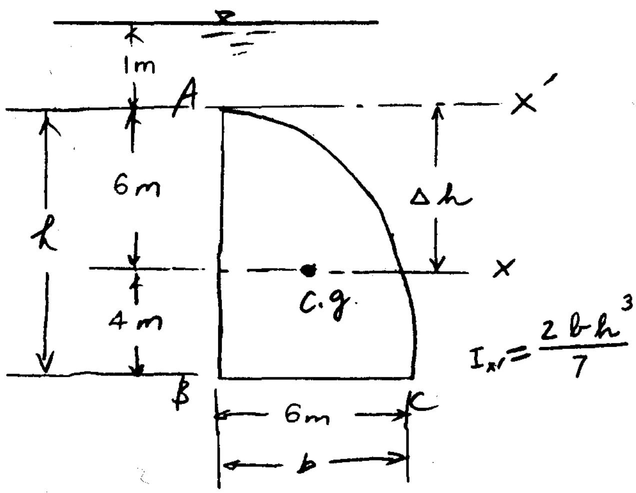



Fig. 3-34(*b*)

**3.44** Circular gate $ABC$ in Fig. 3-35 is 4 m in diameter and is hinged at $B$. Compute the force $P$ just sufficient to keep the gate from opening when $h$ is 8 m.

▌ $$F = \gamma h_{cg}A = (9.79)(8)[\pi(4)^2/4] = 984.2\ \text{kN} \qquad I_{xx} = \pi d^4/64 = \pi(4)^4/64 = 12.57\ \text{m}^4$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-(12.57)(\sin 90°)}{(8)[(\pi)(2)^2]} = -0.125\ \text{m}$$

$$\sum M_B = 0 \qquad (P)(2) - (984.2)(0.125) = 0 \qquad P = 61.5\ \text{kN}$$

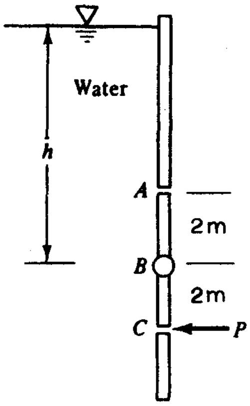



**Fig. 3-35(a)**

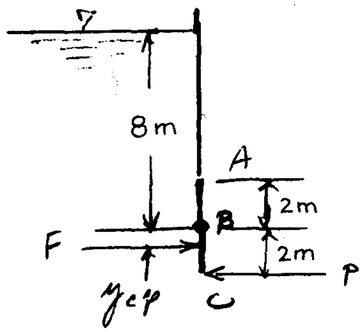



**Fig. 3-35(b)**

**3.45** For the conditions given in Prob. 3.44, derive an analytical expression for $P$ as a function of $h$.

▌ $$F = \gamma h_{cg}A = \gamma h_{cg}[\pi(r)^2] \qquad I_{xx} = \pi(r)^4/4$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(\pi)(r)^4/4](\sin 90°)}{h[(\pi)(r)^2]} = \frac{-r^2}{4h}$$

$$\sum M_B = 0 \qquad Pr - [\gamma h_{cg}(\pi)(r)^2][(r)^2/(4r)] = 0 \qquad P = \gamma\pi r^3/4$$

(Note that force $P$ is independent of depth $h$.)

**3.46** Gate $ABC$ in Fig. 3-36 is 2 m square and hinged at $B$. How large must $h$ be for the gate to open?

▌ The gate will open when resultant force $F$ acts above point $B$—i.e., when $|y_{cp}| < 0.2$ m. (Note in Fig. 3-36$b$ that $y_{cp}$ is the distance between $F$ and the centroid of gate $ABC$.)

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(2)(2)^3/12](\sin 90°)}{(h+1.0)[(2)(2)]} = \frac{-1.333}{4h+4}$$

For $|y_{cp}| < 0.2$, $1.333/(4h+4) < 0.2$, $h > 0.666$ m. (Note that this result is independent of fluid weight.)

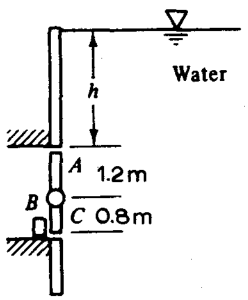



**Fig. 3-36(a)**

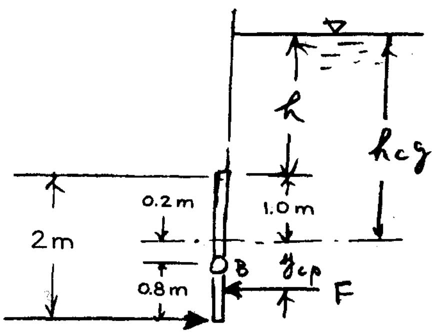



**Fig. 3-36(b)**

**3.47** Gate $AB$ in Fig. 3-37 is 6 ft wide and weighs 2000 lb when submerged. It is hinged at $B$ and rests against a smooth wall at $A$. Determine the water level $h$ which will just cause the gate to open.

▌ $F = \gamma h_{cg} A \qquad F_1 = 62.4(h + \frac{8}{2})[(10)(6)] = 3744h + 14\,976 \qquad F_2 = 62.4(5 + \frac{8}{2})[(10)(6)] = 33\,696 \text{ lb}$

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg} A} \qquad (y_{cp})_1 = \frac{-[(6)(10)^3/12](\frac{8}{10})}{(h + \frac{8}{2})[(10)(6)]} = \frac{-6.67}{h+4}$$

$$(y_{cp})_2 = \frac{-[(6)(10)^3/12](\frac{8}{10})}{(5 + \frac{8}{2})[(10)(6)]} = -0.741 \text{ ft}$$

$\sum M_B = 0 \qquad (3744h + 14\,976)[5 - 6.67/(h+4)] - (33\,696)(5 - 0.741) - (2000)(\frac{6}{2}) = 0 \qquad h = 5.32 \text{ ft}$

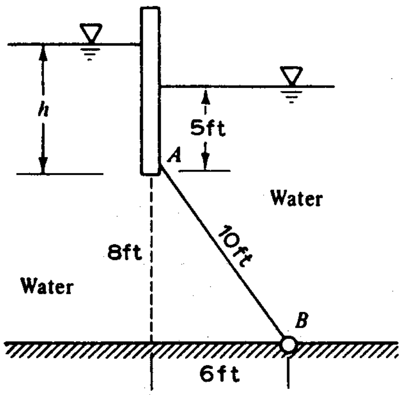



**Fig. 3-37(a)**

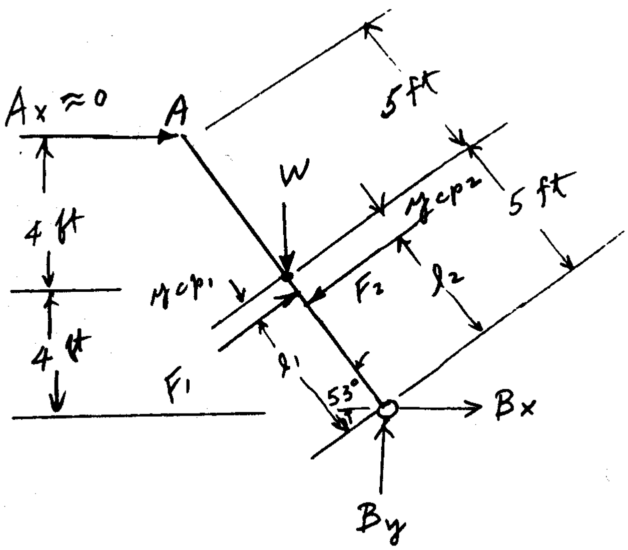



**Fig. 3-37(b)**

**3.48** The tank in Fig. 3-38 contains oil and water as shown. Find the resultant force on side $ABC$, which is 4 ft wide.

▌ $F = \gamma h_{cg} A \qquad F_{AB} = [(0.80)(62.4)](\frac{10}{2})[(10)(4)] = 9980 \text{ lb}$

$F_{AB}$ acts at a point $(\frac{2}{3})(10)$, or 6.67 ft below point $A$. Water is acting on area $BC$, and any superimposed liquid can be converted to an equivalent depth of water. Employ an imaginary water surface (IWS) for this calculation, locating IWS by changing 10 ft of oil to (0.80)(10), or 8 ft of water. Thus, $F_{BC} = (62.4)(8 + \frac{6}{2})[(6)(4)] = 16\,470$ lb.

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg} A} = \frac{-[(4)(6)^3/12](\sin 90^\circ)}{(8 + \frac{6}{2})[(6)(4)]} = -0.27 \text{ ft} \qquad \text{(i.e., below the centroid of } BC\text{)}$$

$F_{BC}$ acts at a point $(2 + 8 + \frac{6}{2} + 0.27)$, or 13.27 ft below $A$. $\sum M_A = 0$; $(9980 + 16\,470)(h_{cp}) - (9980)(6.67) - (16\,470)(13.27) = 0$, $h_{cp} = 10.78$ ft from $A$. Thus, the total resultant force on side $ABC$ is 9980 + 16 470, or 26 450 lb acting 10.78 ft below $A$.

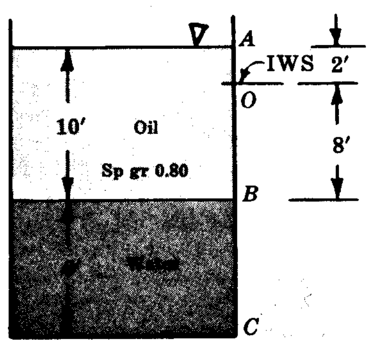



**Fig. 3-38**

**3.49** Gate $AB$ in Fig. 3-39 is 4 ft wide and hinged at $A$. Gage $G$ reads $-2.17$ psi, while oil (s.g. = 0.75) is in the right tank. What horizontal force must be applied at $B$ for equilibrium of gate $AB$?

▌ $$F = \gamma h_{cg} A \qquad F_{oil} = [(0.75)(62.4)](\tfrac{6}{2})[(6)(4)] = 3370 \text{ lb}$$

$F_{oil}$ acts $(\frac{2}{3})(6)$, or 4.0 ft from $A$. For the left side, the negative pressure due to the air can be converted to its equivalent head in feet of water. $h = p/\gamma = (-2.17)(144)/62.4 = -5.01$ ft. This negative pressure head is equivalent to having 5.01 ft less water above $A$. Hence, $F_{H_2O} = (62.4)(6.99 + \frac{6}{2})[(6)(4)] = 14\,960$ lb.

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg}A} = \frac{-[(4)(6)^3/12](\sin 90°)}{(6.99 + \frac{6}{2})[(6)(4)]} = -0.30 \text{ ft}$$

$F_{H_2O}$ acts at $(0.30 + \frac{6}{2})$, or 3.30 ft below $A$. $\sum M_A = 0$; $(3370)(4.0) + 6F - (14\,960)(3.30) = 0$, $F = 5980$ lb (acting leftward).

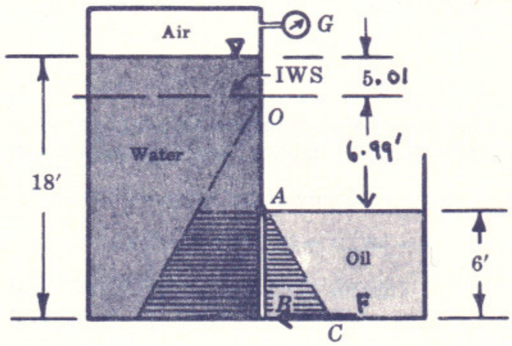



**Fig. 3-39**

**3.50** A vertical circular disk 1.1 m in diameter has its highest point 0.4 m below the surface of a pond. Find the magnitude of the hydrostatic force on one side and the depth to the center of pressure.

▌ $$F = \gamma h_{cg} A = (9.79)(0.4 + 1.1/2)[(\pi)(1.1)^2/4] = 8.84 \text{ kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \left(0.4 + \frac{1.1}{2}\right) + \frac{(\pi)(1.1)^4/64}{(0.4 + 1.1/2)[(\pi)(1.1)^2/4]} = 1.03 \text{ m}$$

**3.51** The vertical plate shown in Fig. 3-40 is submerged in vinegar (s.g. = 0.80). Find the magnitude of the hydrostatic force on one side and the depth to the center of pressure.

▌ $$F = \gamma h_{cg} A \qquad F_1 = [(0.80)(9.79)](2 + \tfrac{7}{2})[(3)(7)] = 905 \text{ kN} \qquad h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A}$$

$$(h_{cp})_1 = 2 + \tfrac{7}{2} + \frac{(3)(7)^3/12}{(2 + \frac{7}{2})[(3)(7)]} = 6.24 \text{ m} \qquad F_2 = [(0.80)(9.79)][2 + 3 + 4/2][(2)(4)] = 439 \text{ kN}$$

$$(h_{cp})_2 = [2 + 3 + 4/2] + \frac{(2)(4)^3/12}{(2 + 3 + 4/2)[(2)(4)]} = 7.19 \text{ m}$$

$$F = 905 + 439 = 1344 \text{ kN} \qquad 1344 h_{cp} = (905)(6.24) + (439)(7.19) \qquad h_{cp} = 6.55 \text{ m}$$

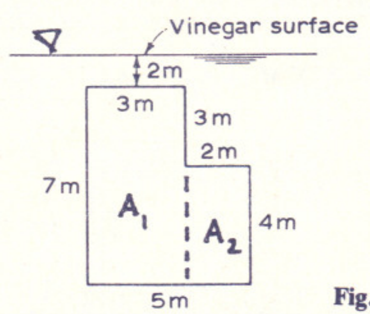



**Fig. 3-40**

**3.52** The irrigation head gate shown in Fig. 3-41*a* is a plate which slides over the opening to a culvert. The coefficient of friction between the gate and its sliding ways is 0.5. Find the force required to slide open this 1000-lb gate if it is set **(*a*)** vertically and **(*b*)** on a 2:1 slope ($n = 2$ in Fig. 3-41*a*), as is common.

**I** **(a)** $F = \gamma h_{cg} A = (62.4)[14 + (\frac{60}{12})/2][(\frac{60}{12})(\frac{60}{12})] = 25\,740$ lb. Let $T$ = force parallel to gate required to open it. $\sum F_y = 0$; $T - 1000 - (0.5)(25\,740) = 0$, $T = 13\,870$ lb. **(b)** See Fig. 3-41*b*. $F = (62.4)[14 + \frac{60}{12}(1/\sqrt{5})/2][(\frac{60}{12})(\frac{60}{12})] = 23\,584$ lb. Let $N$ = total force normal to gate; $N = 23\,584 + (1000)(2/\sqrt{5}) = 24\,478$ lb. $\sum F_y = 0$; $T - (1000)(1/\sqrt{5}) - (0.5)(24\,478) = 0$, $T = 12\,686$ lb.

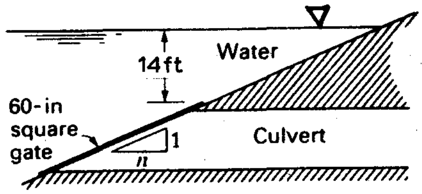
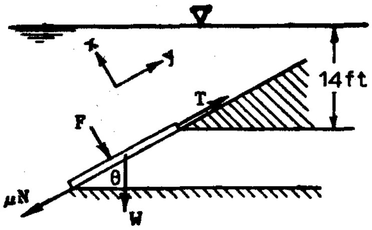



Fig. 3-41(a)



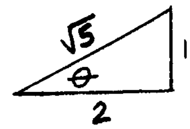




Fig. 3-41(b)

**3.53** A 65-in-square floodgate, weighing 2200 lb, is hinged 44.5 in above the center, as shown in Fig. 3-42, and the face is inclined 5° to the vertical. Find the depth to which water will rise behind the gate before it will open.

**I**

$$\text{Closing moment of gate about hinge} = (2200)[(\tfrac{44.5}{12})(\sin 5°)] = 711 \text{ lb}\cdot\text{ft}$$

$$F = \gamma h_{cg} A = (62.4)(h/2)[(\tfrac{65}{12})(h)/\cos 5°] = 169.6h^2$$

$$\sum M_{\text{hinge}} = 0 \qquad (169.6h^2)[(65 + 12)/12 - (h/\cos 5°)/3] - 711 = 0 \qquad h = 0.826 \text{ ft}$$

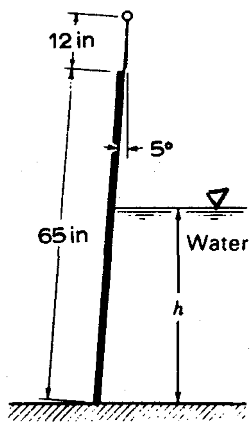



Fig. 3-42

**3.54** Gate $MN$ in Fig. 3-43 rotates about an axis through $N$. If the width of the gate is 5 ft, what torque applied to the shaft through $N$ is required to hold the gate closed?

**I** $F = \gamma h_{cg} A \qquad F_1 = 62.4[6 + (3+4)/2][(3+4)(5)] = 20\,748$ lb $\qquad F_2 = (62.4)(\frac{4}{2})[(5)(4)] = 2496$ lb

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} \qquad (y_{cp})_1 = \frac{-[(5)(3+4)^3/12](\sin 90°)}{[6 + (3+4)/2][(3+4)(5)]} = 0.430 \text{ ft}$$

$F_2$ acts at $(\frac{1}{3})(4)$, or 1.333 ft from $N$. $\sum M_N = 0$; $(20\,748)[(3+4)/2 - 0.430] - (2496)(1.333) - \text{torque}_N = 0$, $\text{torque}_N = 60\,369$ lb · ft.

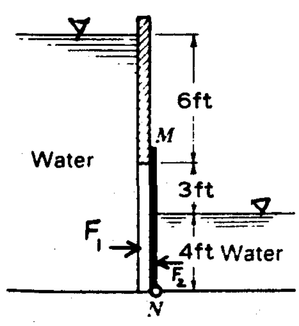



Fig. 3-43

**3.55** Find the minimum depth of $z$ for which the gate in Fig. 3-44 will open, if the gate is **(a)** square and **(b)** isosceles triangular, with base = height.

**(a)**

$$F = \gamma h_{cg}A \qquad F_{H_2O} = (62.4)(z - \tfrac{3}{2})[(3)(3)] = (561.6)(z - 1.5)$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} \qquad (y_{cp})_{H_2O} = \frac{-[(3)(3)^3/12](\sin 90°)}{(z - \frac{3}{2})[(3)(3)]} = \frac{-0.750}{z - 1.5}$$

$$\text{Moment due to water} = [(561.6)(z - 1.5)][\tfrac{3}{2} + 0.750/(z - 1.5)] = (561.6)(1.5z - 1.500)$$

$F_{gas} = pA = [(5)(144)][(3)(3)] = 6480$ lb. $F_{gas}$ acts at $\frac{3}{2}$, or 1.5 ft below hinge. Moment due to gas = (6480)(1.5) = 9720 lb · ft. Equating moments gives (561.6)(1.5$z$ − 1.500) = 9720, $z$ = 12.54 ft.

**(b)**

$$F_{H_2O} = (62.4)[z - (\tfrac{2}{3})(3)][(3)(3)/2] = (280.8)(z - 2.000)$$

$$(y_{cp})_{H_2O} = \frac{-[(3)(3)^3/36](\sin 90°)}{[z - (\frac{2}{3})(3)][(3)(3)/2]} = \frac{0.500}{z - 2.000}$$

$$\text{Moment due to water} = [(280.8)(z - 2.000)][\tfrac{3}{3} + 0.500/(z - 2.000)] = 280.8z - 421.2$$

$F_{gas} = [(5)(144)][(3)(3)/2] = 3240$ lb. $F_{gas}$ acts at $\frac{3}{3}$, or 1.000 ft below hinge. Moment due to gas = (3240)(1.000) = 3240 lb · ft. Equating moments gives (280.8$z$ − 421.2) = 3240, $z$ = 13.04 ft.

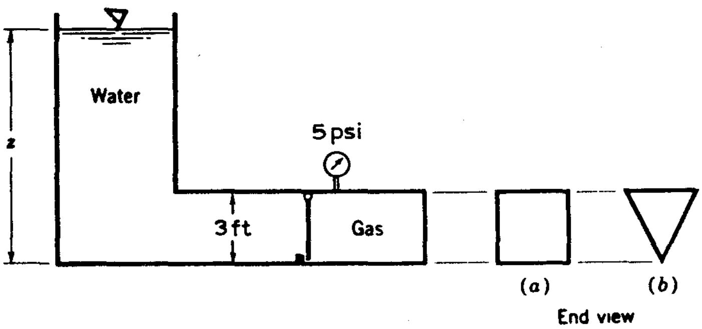



Fig. 3-44

**3.56** The triangular gate $CDE$ in Fig. 3-45 is hinged along $CD$ and is opened by a normal force $P$ applied at $E$. It holds a liquid of specific gravity 0.82 above it and is open to the atmosphere on its lower side. Neglecting the weight of the gate, find **(a)** the magnitude of force exerted on the gate, by direct integration; **(b)** the location of the center of pressure; and **(c)** the force $P$ needed to open the gate.

**(a)** $F = \int \gamma h\, dA = \int \gamma(y\sin\theta)(x\,dy)$. When $y = 8$, $x = 0$, and when $y = 8 + \frac{12}{2}$, or 14, $x = 6$, with $x$ varying linearly with $y$. Hence, $x = y - 8$. When $y = 14$, $x = 6$, and when $y = 8 + 12$, or 20, $x = 0$, with $x$ varying linearly with $y$. Hence, $x = 20 - y$.

$$F = \int_8^{14} [(0.82)(62.4)](y\sin 30°)[(y - 8)\,dy] + \int_{14}^{20} [(0.82)(62.4)](y\sin 30°)[(20 - y)\,dy]$$

$$= [(0.82)(62.4)](\sin 30°)\left\{\left[\frac{y^3}{3} - 4y^2\right]_8^{14} + \left[10y^2 - \frac{y^3}{3}\right]_{14}^{20}\right\} = 12\,894 \text{ lb}$$

**(b)**

$$x_{cp} = \frac{-I_{xy}\sin\theta}{h_{cg}A}$$

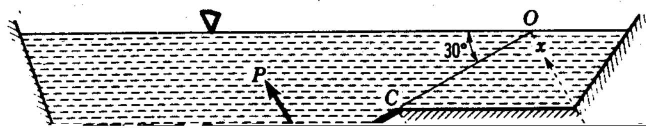
Since $I_{xy} = 0$, $x_{cp} = 0$,

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(2)(6)(\frac{12}{2})^3/12](\sin 30°)}{[(8 + \frac{12}{2})(\sin 30°)][(12)(6)/2]} = -0.43 \text{ ft}$$

(i.e., the pressure center is 0.43 ft below the centroid, measured in the plane of the area).

**(c)**

$$\sum M_{CD} = 0 \qquad 6P = (12\,894)(\tfrac{6}{3}) \qquad P = 4298 \text{ lb}$$


O
30°
x
P
C

**3.59** Determine $y$ in Fig. 3-48 so that the flashboards will tumble only when the water reaches their top.

▌ The flashboards will tumble when $y$ is at the center of pressure. Hence, $y = \frac{4}{3}$, or 1.333 m.

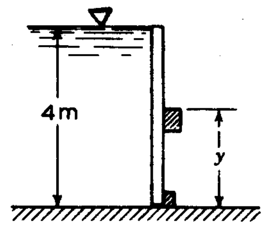



**Fig. 3-48**

**3.60** Determine the pivot location $y$ of the square gate in Fig. 3-49 so that it will rotate open when the liquid surface is as shown.

▌ The gate will open when the pivot location is at the center of pressure.

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = (3 - \tfrac{2}{2}) + \frac{(1)(2)^3/12}{(3 - \frac{2}{2})[(2)(1)]} = 2.167 \text{ m} \qquad y = 3 - 2.167 = 0.833 \text{ m}$$

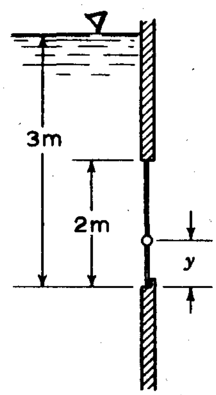



**Fig. 3-49**

**3.61** The gate in Fig. 3-50$a$ (shown in raised position) weighs 350 lb for each foot normal to the paper. Its center of gravity is 1.5 ft from the left face and 2.0 ft above the lower face. For what water level below the hinge at $O$ does the gate just begin to swing up (rotate counterclockwise)?

▌ Refer to Fig. 3-50$b$ and consider 1 ft of length. $F = \gamma hA = (62.4)[(h_O/2)][(h_O)(1)] = 31.2h_O^2$; $\Sigma M_O = 0$; $(2)(350) - (5 - h_O/3)(31.2h_O^2) = 0$, $h_O = 2.30$ ft.

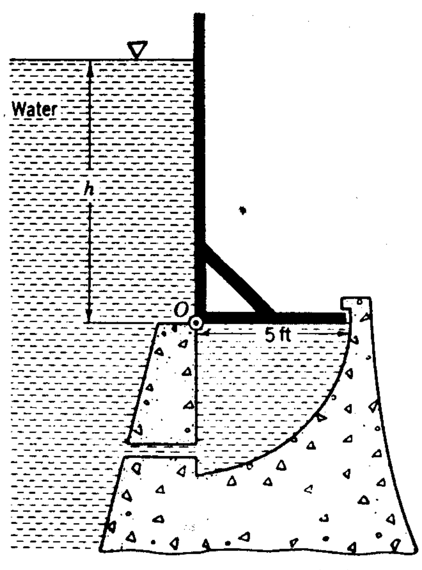



**Fig. 3-50(a)**

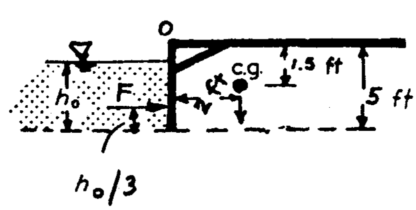



**Fig. 3-50(b)**

**3.62** For the gate described in Prob. 3.61 and Fig. 3-50*a*, find $h$ for the gate just to come up to the vertical position shown in Fig. 3-50*a*.

▌ See Fig. 3-51. $F_1 = \gamma h A = (62.4)(h)[(5)(1)] = 312h$, $F_2 = (62.4)(h/2)[(h)(1)] = 31.2h^2$; $\sum M_O = 0$; $(1.5)(350) + (h/3)(31.2h^2) - (2.5)(312h) = 0$, $h = 0.68$ ft.

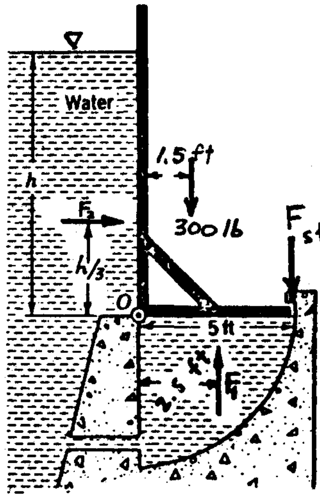



Fig. 3-51

**3.63** For the gate described in Prob. 3.61 and Fig. 3-50*a*, find $h$ and the force against the stop when this force is a maximum for the gate.

▌ See Fig. 3-51. $F_1 = \gamma h A = (62.4)(h)[(5)(1)] = 312h$, $F_2 = (62.4)(h/2)[(h)(1)] = 31.2h^2$; $\sum M_O = 0$; $(1.5)(350) + (h/3)(31.2h^2) - (2.5)(312h) + (5)(F_{stop}) = 0$, $F_{stop} = 156h - 2.08h^3 - 105$.

$$\frac{dF_{stop}}{dh} = 156 - 6.24h^2 = 0 \qquad h = 5.00\text{ ft}$$

$$F_{stop} = (156)(5.00) - (2.08)(5.00)^3 - 105 = 415\text{ lb}$$

**3.64** Compute the air pressure required to keep the gate of Fig. 3-52 closed. The gate is a circular plate of diameter 0.8 m and weight 2.0 kN.

▌ $$F = \gamma h A \qquad F_{liq} = [(2)(9.79)][1.7 + (\tfrac{1}{2})(0.8)(\sin 45°)][\pi(0.8)^2/4] = 19.52\text{ kN}$$

$$z_{cp} = z_{cg} + \frac{I_{cg}}{z_{cg}A} = \left[\frac{1.7}{\cos 45°} + \left(\frac{1}{2}\right)(0.8)\right] + \frac{\pi[(\frac{1}{2})(0.8)]^4/4}{[1.7/\cos 45° + (\frac{1}{2})(0.8)][\pi(0.8)^2/4]} = 2.818\text{ m}$$

$$\sum M_{hinge} = 0 \qquad (19.52)(2.818 - 1.7/\cos 45°) + 2.0[(\tfrac{1}{2})(0.8)(\cos 45°)] - [\pi(0.8)^2/4](p_{air})[(\tfrac{1}{2})(0.8)] = 0$$

$$p_{air} = 42.99\text{ kPa}$$

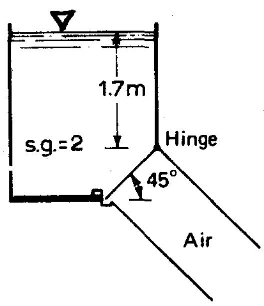



Fig. 3-52

# CHAPTER 4
# Dams

**4.1** In Fig. 4-1, calculate the width of concrete dam that is necessary to prevent the dam from sliding. The specific weight of the concrete is 150 lb/ft$^3$, and the coefficient of friction between the base of the dam and the foundation is 0.42. Use 1.5 as the factor of safety (F.S.) against sliding. Will it also be safe against overturning?

Working with a 1-ft "slice" (i.e., dimension perpendicular to the paper) of the dam, $W_{\text{dam}} = (20)(w)(1)(150) = 3000w$, $F = \gamma hA$, $F_H = (62.4)[(0+15)/2][(15)(1)] = 7020$ lb.

$$\text{F.S.}_{\text{sliding}} = \frac{\text{sliding resistance}}{\text{sliding force}} \qquad 1.5 = \frac{(0.42)(3000w)}{7020} \qquad w = 8.36 \text{ ft}$$

$$\text{F.S.}_{\text{overturning}} = \frac{\text{total righting moment}}{\text{overturning moment}} = \frac{[(3000)(8.36)](8.36/2)}{(7020)(\frac{15}{3})} = 2.99$$

Therefore, it should be safe against overturning.

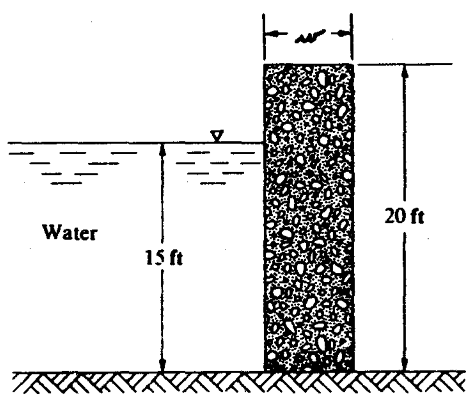


Fig. 4-1

**4.2** Figure 4-2 is the cross section of an earthwork (s.g. = 2.5) dam. Assuming that hydrostatic uplift varies linearly from one-half the hydrostatic head at the upstream edge of the dam to zero at the downstream edge, find the maximum and minimum pressure intensity in the base of the dam.

$$F = \gamma hA \qquad F_H = (62.4)[(0+97)/2][(97)(1)] = 293\,561 \text{ lb}$$

For equilibrium, $R_x = 293\,561$ lb.

$$W_1 = [(2.5)(62.4)][(1)(10)(90+30)] = 187\,200 \text{ lb} \qquad W_2 = [(2.5)(62.4)][(1)(60)(90)/2] = 421\,200 \text{ lb}$$

$$F_U = [(62.4)(48.5+0)/2][(60+10)(1)] = 105\,924 \text{ lb} \qquad R_y = 187\,200 + 421\,200 - 105\,924 = 502\,476 \text{ lb}$$

$$\sum M_0 = 0 \qquad (293\,561)(32.33) + (187\,200)(5) + (421\,200)(30) - (105\,924)[(60+10)/3] - 502\,476x = 0$$

$$x = 40.98 \text{ ft} \qquad \text{Eccentricity} = 40.98 - (60+10)/2 = 5.98 \text{ ft}$$

Since the eccentricity is less than one-sixth the base of the dam, the resultant acts within the middle third of the base.

$$p = \frac{F}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} = \frac{502\,476}{(60+10)(1)} \pm \frac{[(502\,476)(5.98)](60+10)/2}{(1)(60+10)^3/12} \pm 0 = 7178 \pm 3679$$

$$p_{\max} = 7178 + 3679 = 10\,857 \text{ lb/ft}^2 \qquad p_{\min} = 7178 - 3679 = 3499 \text{ lb/ft}^2$$

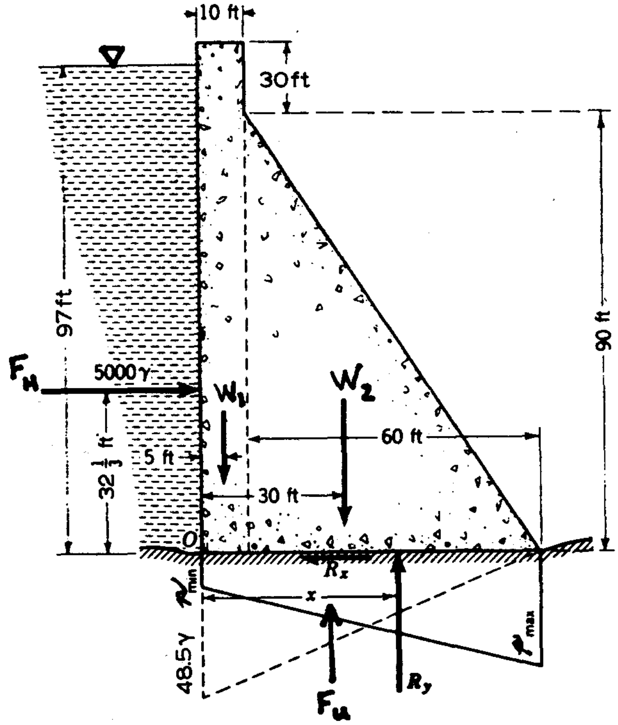



Fig. 4-2

**4.3** For linear stress variation over the base of the dam of Fig. 4-3*a*, find where the resultant crosses the base and compute the maximum and minimum pressure intensity at the base. Neglect hydrostatic uplift.

▌ Figure 4-3*b* shows the forces acting on the dam. $F_1 = \gamma[(19+6)/2][(19+6)(1)] = 312\gamma$, $F_2 = \gamma[(6)(3)(1)] = 18\gamma$, $F_3 = \gamma[(1)(19)(3)/2] = 28.5\gamma$, $F_4 = [(2.5)(\gamma)][(4)(19+6)(1)] = 250\gamma$, $F_5 = [(2.5)(\gamma)][(1)(19)(3)/2] = 71.25\gamma$, $F_6 = [(2.5)(\gamma)][(1)(19)(11)/2] = 261\gamma$; $R_y = 18\gamma + 28.5\gamma + 250\gamma + 71.25\gamma + 261\gamma = 628.75\gamma$. $\sum M_A = 0$; $(628.75\gamma)(x) - (312\gamma)[(19+6)/3] - (18\gamma)(1.5) - (28.5\gamma)(1) - (250\gamma)(3+2) - (71.25\gamma)(3-1) - (261\gamma)(4+3+\frac{11}{3})$; $x = 10.87$ m. Eccentricity $= 10.87 - (11+4+3)/2 = 1.87$ ft. Since the eccentricity is less than one-sixth the base of the dam, the resultant acts within the middle third of the base.

$$p = \frac{F}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} = \frac{(628.75)(9.79)}{(11+4+3)(1)} \pm \frac{[(628.75)(9.79)(1.87)](11+4+3)/2}{(1)(11+4+3)^3/12} \pm 0 = 342 \pm 213 \text{ kPa}$$

$$p_{max} = 342 + 213 = 555 \text{ kPa} \qquad p_{min} = 342 - 213 = 129 \text{ kPa}$$

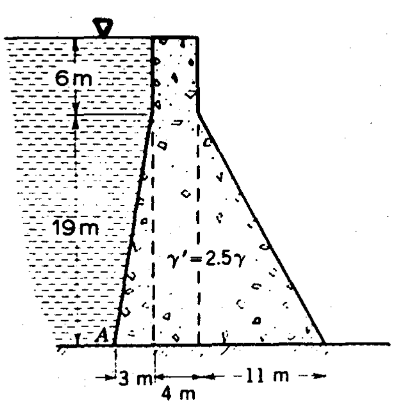



Fig. 4-3(*a*)

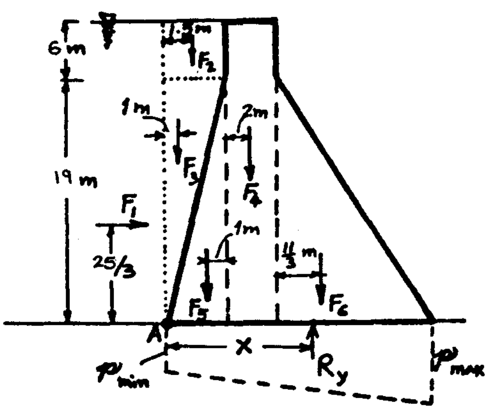



Fig. 4-3(*b*)

**4.4** For the conditions given in Prob. 4.3 with the addition that hydrostatic uplift varies linearly from 19 m at $A$ to zero at the toe of the dam, would the resultant still act within the middle third of the base?

▌ $F_U = \gamma[(19+0)/2][(4+3+11)(1)] = 171\gamma \qquad R_y = 18\gamma + 28.5\gamma + 250\gamma + 71.25\gamma + 261\gamma - 171\gamma = 457.75\gamma$

$$\sum M_A = 0 \qquad (457.75\gamma)(x) - (312\gamma)[(19+6)/3] - (18\gamma)(1.5) - (28.5\gamma)(1) - (250\gamma)(3+2)$$
$$- (71.25\gamma)(3-1) - (261\gamma)(4+3+\tfrac{11}{3}) + (171\gamma)[(4+3+11)/3] = 0$$
$$x = 12.68\text{ m} \qquad \text{Eccentricity} = 12.68 - (11+4+3)/2 = 3.68\text{ ft}$$

Since the eccentricity is greater than one-sixth the base of the dam, the resultant acts outside the middle third of the base.

**4.5** A concrete dam retaining water is shown in Fig. 4-4$a$. If the specific weight of the concrete is 150 lb/ft$^3$, find the factor of safety against sliding, the factor of safety against overturning, and the pressure intensity on the base. Assume the foundation soil is impermeable and that the coefficient of friction between dam and foundation soil is 0.45.

▌ The forces acting on the dam are shown in Fig. 4.4$b$. $F = \gamma h A$, $F_x = (62.4)[(0+42)/2][(42)(1)] = 55\,040$ lb. From Fig. 4-4$b$, $CD/42 = \frac{10}{50}$, $CD = 8.40$ ft; $F_y = (62.4)[(8.40)(42)/2](1) = 11\,010$ lb.

| component | weight of component (kips) | moment arm from toe, $B$ (ft) | righting moment about toe, $B$ (kip · ft) |
|---|---|---|---|
| 1 | $(\frac{1}{2})(10\times 50)(0.15)(1) = 37.50$ | $20 + \frac{10}{3} = 23.33$ | 875 |
| 2 | $(10\times 50)(0.15)(1) = 75.00$ | $10 + \frac{10}{2} = 15.00$ | 1125 |
| 3 | $(\frac{1}{2})(10\times 50)(0.15)(1) = 37.50$ | $(\frac{2}{3})(10) = 6.67$ | 250 |
| $F_y$ | 11.01 | $30 - (\frac{1}{3})(8.40) = 27.20$ | 299 |
| | $\sum V = 161.01$ **kips** | | $\sum M_r = 2549$ **kip · ft** |

$$M_{\text{overturning}} = (55.04)(\tfrac{42}{3}) = 771\text{ kip-ft} \qquad \text{F.S.}_{\text{sliding}} = \frac{\text{sliding resistance}}{\text{sliding force}} = \frac{(0.45)(161.01)}{55.04} = 1.32$$

$$\text{F.S.}_{\text{overturning}} = \frac{\text{total righting moment}}{\text{overturning moment}} = \frac{2549}{771} = 3.31$$

$R_x = F_x = 55.04$ kips and $R_y = \sum V = 161.01$ kips; hence, $R = \sqrt{55.04^2 + 161.01^2} = 170.16$ kips.

$$x = \frac{\sum M_B}{R_y} = \frac{\sum M_r - M_0}{\sum V} = \frac{2549 - 771}{161.01} = 11.04\text{ ft} \qquad \text{Eccentricity} = \tfrac{30}{2} - 11.04 = 3.96\text{ ft}$$

Since the eccentricity is less than one-sixth the base of the dam, the resultant acts within the middle third of the base.

$$p = \frac{F}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} = \frac{161.01}{(30)(1)} \pm \frac{[(161.01)(3.96)](15)}{(1)(30)^3/12} \pm 0 = 5.37 \pm 4.25$$

$$p_B = 5.37 + 4.25 = 9.62\text{ kips/ft}^2 \qquad p_A = 5.37 - 4.25 = 1.12\text{ kips/ft}^2$$

The complete pressure distribution on the base of the dam is given in Fig. 4-4$c$.

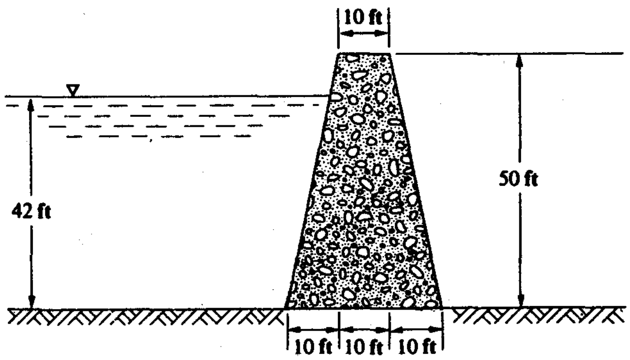



Fig. 4-4(a)

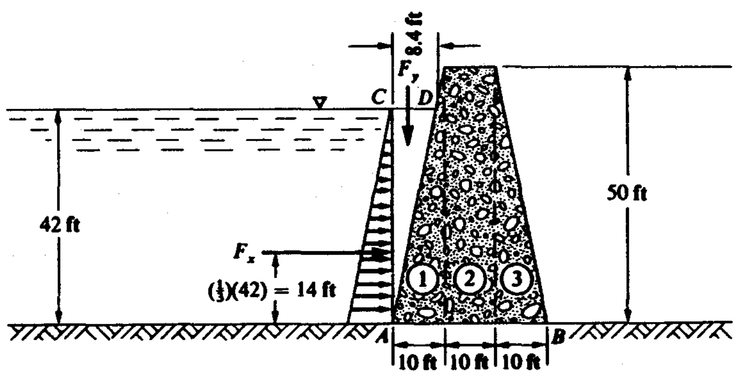



Fig. 4-4(b)

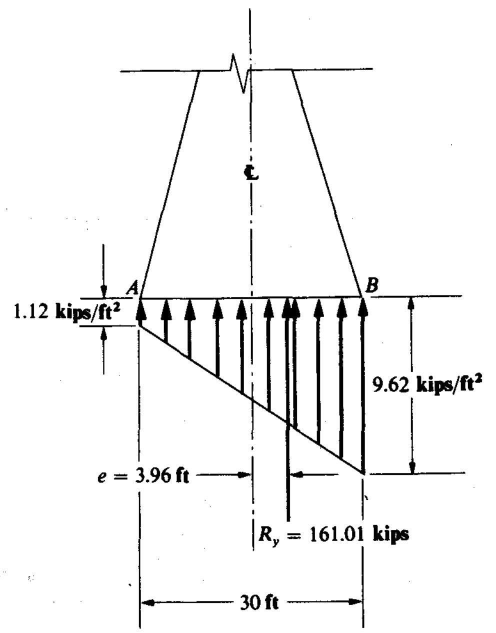



Fig. 4-4(c)

**4.6** A concrete dam retaining water is shown in Fig. 4-5*a*. If the specific weight of the concrete is 23.5 kN/m$^3$, find the factor of safety against sliding, the factor of safety against overturning, and the pressure intensity on the base. Assume there is a hydrostatic uplift that varies uniformly from full hydrostatic head at the heel of the dam to zero at the toe and that the coefficient of friction between dam and foundation soil is 0.45.

▮ The forces acting on the dam are shown in Fig. 4-5*b*. $F = \gamma hA$, $F_x = (9.79)[(0+14)/2][(14)(1)] = 959.4$ kN, $F_y = (9.79)[(3)(14-3)(1)] = 323.1$ kN. Hydrostatic uplift varies from (14)(9.79), or 137.1 kN/m$^2$ at the heel to zero at the toe, as shown in Fig. 4-5*b*. $F_U = (137.1/2)(15)(1) = 1028$ kN. It acts at $(\frac{1}{3})(15)$, or 5.0 m from point *A*, as shown in Fig. 4-5*b*.

| component | weight of component (kN) | moment arm from toe, $B$ (m) | righting moment about toe, $B$ (kN · m) |
|---|---|---|---|
| 1 | $(\frac{1}{2})(15-3-4)(12)(23.5)(1)=1128$ | $(\frac{2}{3})(15-3-4)=5.333$ | 6 016 |
| 2 | $(4)(12+3)(23.5)(1)=1410$ | $(15-3-\frac{4}{2})=10.000$ | 14 100 |
| 3 | $(15)(3)(23.5)(1)=1058$ | $\frac{15}{2}=7.500$ | 7 935 |
| $F_y$ | $=323$ | $(15-\frac{3}{2})=13.500$ | 4 360 |
| | $\sum V=3919$ **kN** | | $\sum M_r=32\,411$ **kN · m** |

$$M_{\text{overturning}}=(959.4)(\tfrac{14}{3})+(1028)(10)=14\,760\text{ kN}$$

$$\text{F.S.}_{\text{sliding}}=\frac{\text{sliding resistance}}{\text{sliding force}}=\frac{(0.45)(3919-1028)}{959.4}=1.36$$

$$\text{F.S.}_{\text{overturning}}=\frac{\text{total righting moment}}{\text{overturning moment}}=\frac{32\,411}{14\,760}=2.20$$

$R_x=F_x=959.4$ kN and $R_y=\sum V-F_U=3919-1028=2891$ kN; hence, $R=\sqrt{959.4^2+2891^2}=3046$ kN.

$$x=\frac{\sum M_B}{R_y}=\frac{\sum M_r-M_0}{\sum V}=\frac{32\,411-14\,760}{2891}=6.105\text{ m}\qquad \text{Eccentricity}=\tfrac{15}{2}-6.105=1.395\text{ m}$$

Since the eccentricity is less than one-sixth the base of the dam, the resultant acts within the middle third of the base.

$$p=\frac{F}{A}\pm\frac{M_y x}{I_y}\pm\frac{M_x y}{I_x}=\frac{2891}{(15)(1)}\pm\frac{[(2891)(1.395)](\frac{15}{2})}{(1)(15)^3/12}\pm 0=192.7\pm 107.5$$

$$p_B=192.7+107.5=300.2\text{ kN/m}^2\qquad p_A=192.7-107.5=85.2\text{ kN/m}^2$$

The complete pressure distribution on the base of the dam is given in Fig. 4-5*c*.

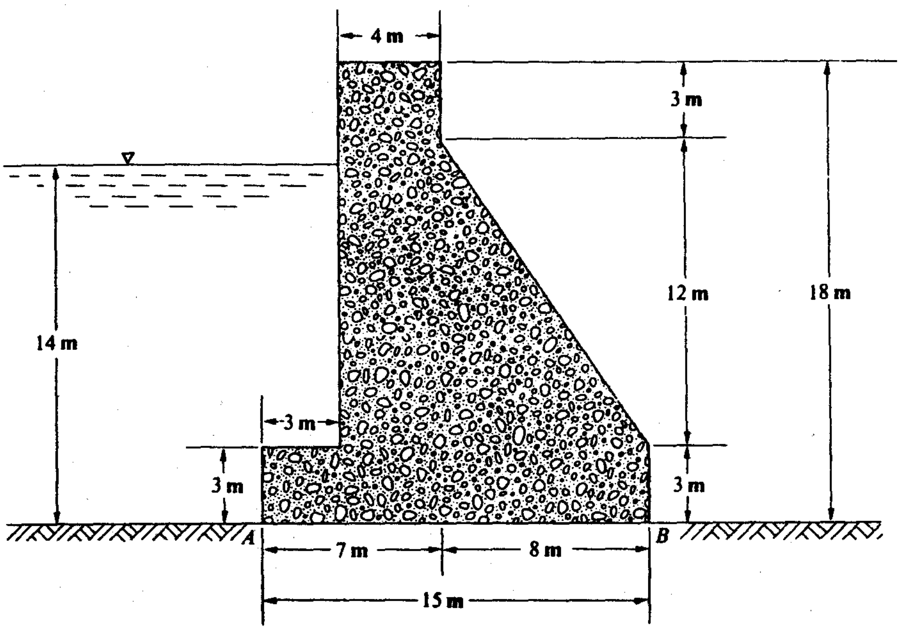



**Fig. 4-5(*a*)**

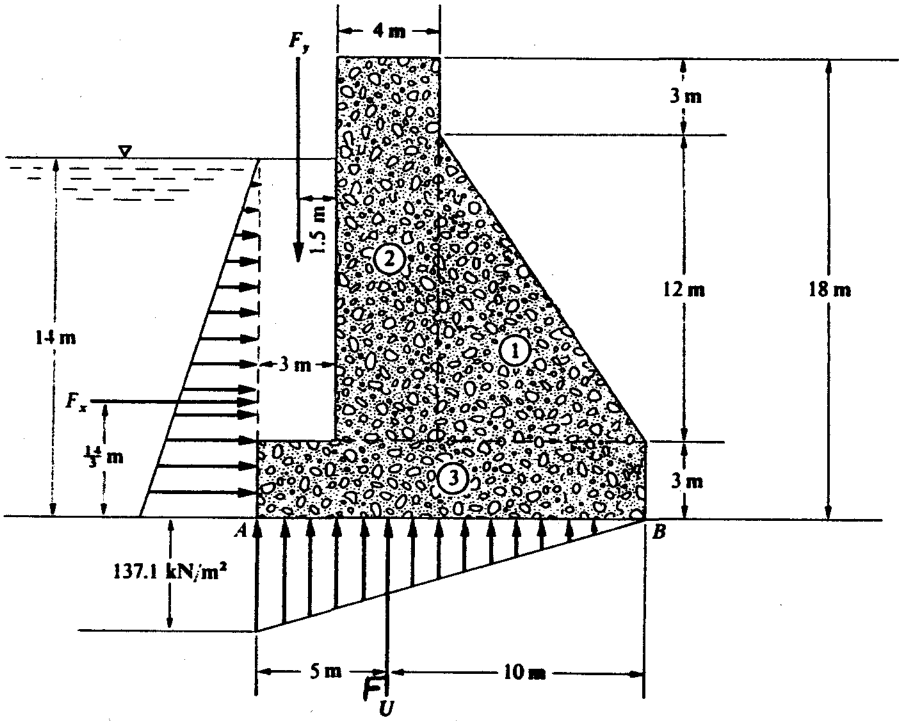



Fig. 4-5(*b*)

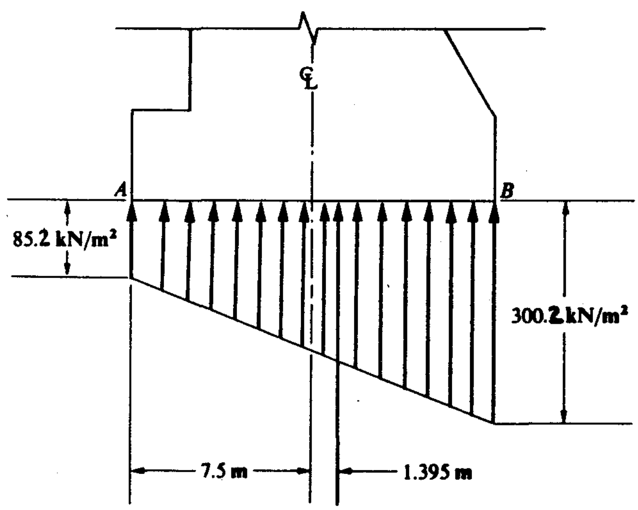



Fig. 4-5(*c*)

**4.7** A concrete dam retaining water is shown in Fig. 4-6*a*. If the specific weight of the concrete is 23.5 kN/m³, find the factor of safety against sliding, the factor of safety against overturning, and the maximum and minimum pressure intensity on the base. Assume there is no hydrostatic uplift and that the coefficient of friction between dam and foundation soil is 0.48.

▮ The forces acting on the dam are shown in Fig. 4-6*b*. $F = \gamma h A$, $F_H = (9.79)[(0+6)/2][(6)(1)] = 176.2$ kN.

| component | weight of component (kN) | moment arm from toe, $A$ (m) | righting moment about toe, $A$ (kN·m) |
|---|---|---|---|
| 1 | $(\frac{1}{2})(2)(7)(23.5) = 164.5$ | $(\frac{2}{3})(2) = 1.333$ | 219 |
| 2 | $(2)(7)(23.5) = 329.0$ | $2 + \frac{2}{2} = 3.000$ | 987 |
| | $\sum V = 493.5$ kN | | $\sum M_r = 1206$ kN·m |

$$M_{\text{overturning}} = (176.2)(\tfrac{6}{3}) = 352.4 \text{ kN}$$

$$\text{F.S.}_{\text{sliding}} = \frac{\text{slinding resistance}}{\text{sliding force}} = \frac{(0.48)(493.5)}{176.2} = 1.34$$

$$\text{F.S.}_{\text{overturning}} = \frac{\text{total righting moment}}{\text{overturning moment}} = \frac{1206}{352.4} = 3.42$$

$R_x = F_H = 176.2$ kN and $R_y = \sum V = 493.5$ kN; hence, $R = \sqrt{176.2^2 + 493.5^2} = 524$ kN.

$$x = \frac{\sum M_A}{R_y} = \frac{\sum M_r - M_0}{\sum V} = \frac{1206 - 352.4}{493.5} = 1.730 \text{ m} \qquad \text{Eccentricity} = \tfrac{4}{2} - 1.730 = 0.270 \text{ m}$$

Since the eccentricity is less than one-sixth the base of the dam, the resultant acts within the middle third of the base.

$$p = \frac{F}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} = \frac{493.5}{(4)(1)} \pm \frac{[(493.5)(0.270)](\tfrac{4}{2})}{(1)(4)^3/12} \pm 0 = 123.4 \pm 50.0$$

$$p_{\max} = 123.4 + 50.0 = 173.4 \text{ kN/m}^2 \qquad p_{\min} = 123.4 - 50.0 = 73.4 \text{ kN/m}^2$$

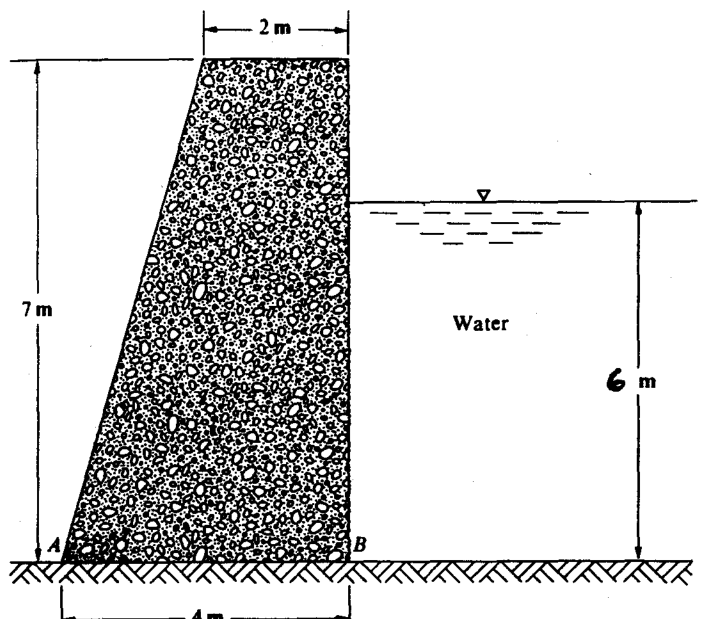



Fig. 4-6(*a*)

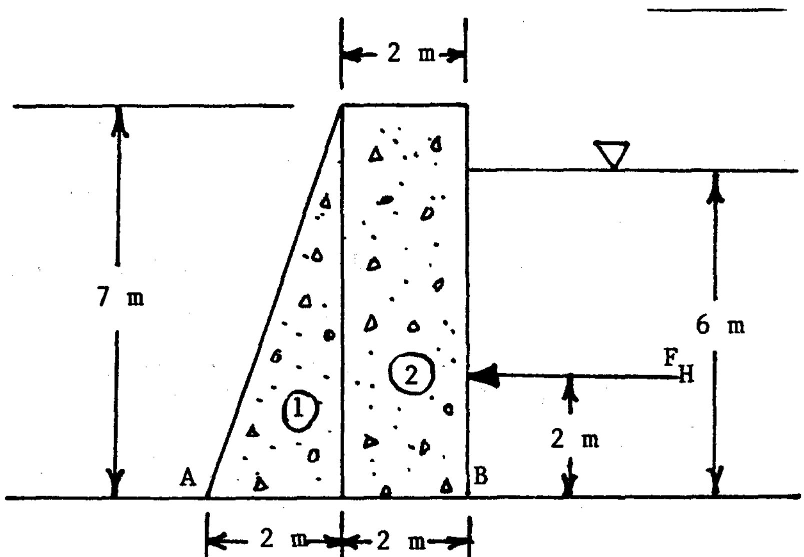



Fig. 4-6(*b*)

**4.8** For the dam shown in Fig. 4-7, what is the minimum width $b$ for the base of a dam 100 ft high if hydrostatic uplift is assumed to vary uniformly from full hydrostatic head at the heel to zero at the toe, and also assuming an ice thrust $P_1$ of 12 480 lb per linear foot of dam at the top? For this study, make the resultant of the reacting forces cut the base at the downstream edge of the middle third of the base (i.e., at $O$ in Fig. 4-7) and take the weight of the masonry as $2.50\gamma$.

▌ $F = \gamma h A \qquad F_H = (62.4)[(100+0)/2][(100)(1)] = 312\,000\text{ lb} \qquad F_U = [(100)(62.4)/2][(1)(b)] = 3120b$

$W_1 = [(2.50)(62.4)][(20)(100)(1)] = 312\,000\text{ lb} \qquad W_2 = [(2.50)(62.4)][(b-20)(100)(1)/2] = 7800b - 156\,000$

$$\sum M_O = 0$$

$$(312\,000)(\tfrac{100}{3}) + (3120b)(b/3) - (312\,000)[(\tfrac{2}{3})(b) - \tfrac{20}{2}] - (7800b - 156\,000)[(\tfrac{2}{3})(b-20) - b/3] + (12\,480)(100) = 0$$

$$3b^2 + 100b - 24\,400 = 0 \qquad b = 75.0\text{ ft}$$

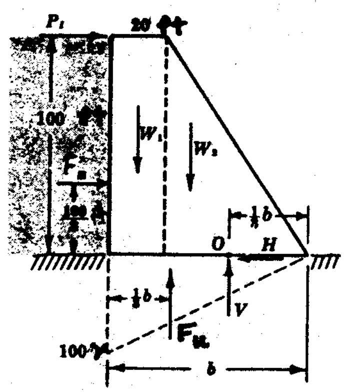



Fig. 4-7

# CHAPTER 5
# Forces on Submerged Curved Areas

**5.1** The submerged, curved surface $AB$ in Fig. 5-1$a$ is one-quarter of a circle of radius 4 ft. The tank's length (distance perpendicular to the plane of the figure) is 6 ft. Find the horizontal and vertical components of the total resultant force acting on the curved surface and their locations.

▌ The horizontal component of the total resultant force acting on the curved surface is equal to the total resultant force, $F_H$, acting on the vertical projection of curved surface $AB$ (i.e., $BF$ in Fig. 5-1$b$). This projection is a rectangle 6 ft long and 4 ft high. For the portion of $F_H$ resulting from horizontal pressure of $BHEF$ in Fig. 5-1$b$, $p_1 = (8)(62.4) = 499\ \text{lb/ft}^2$, $A = (6)(4) = 24\ \text{ft}^2$, $F_1 = (499)(24) = 11\,980\ \text{lb}$. For the portion of $F_H$ resulting from horizontal pressure of $HGE$ in Fig. 5-1$b$, $p_2 = (62.4)[(0+4)/2] = 125\ \text{lb/ft}^2$, $F_2 = (125)(24) = 3000\ \text{lb}$; $F_H = F_1 + F_2 = 11\,980 + 3000 = 14\,980\ \text{lb}$. The vertical component of the total resultant force acting on the curved surface is equal to the weight of the volume of water vertically above curved surface $AB$. This volume consists of a rectangular area ($AFCD$ in Fig. 5-1$c$) 4 ft by 8 ft and a quarter-circular area ($ABF$ in Fig. 5-1$c$) of radius 4 ft, both areas being 6 ft long. This volume ($V$) is $V = [(4)(8) + (\pi)(4)^2/4](6) = 267.4\ \text{ft}^3$, $F_V$ = weight of water in $V = (267.4)(62.4) = 16\,690\ \text{lb}$. The location of the horizontal component ($F_H$) is along a (horizontal) line through the center of pressure for the vertical projection (i.e., the center of gravity of $EFBG$ in Fig. 5-1$b$). This can be determined by equating the sum of the moments of $F_1$ and $F_2$ about point $C$ to the moment of $F_H$ about the same point. $(11\,980)(8 + \frac{4}{2}) + (3000)[8 + (\frac{2}{3})(4)] = 14\,980h_{cp}$, $h_{cp} = 10.13\ \text{ft}$. (This is the depth from the water surface to the location of the horizontal component. Stated another way, the horizontal component acts at a distance of $12 - 10.13$, or 1.87 ft above point $B$ in Fig. 5-1$b$.) The location of the vertical component ($F_V$) is

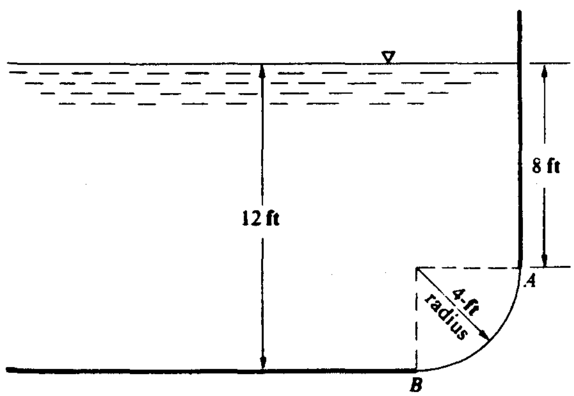



Fig. 5-1($a$)

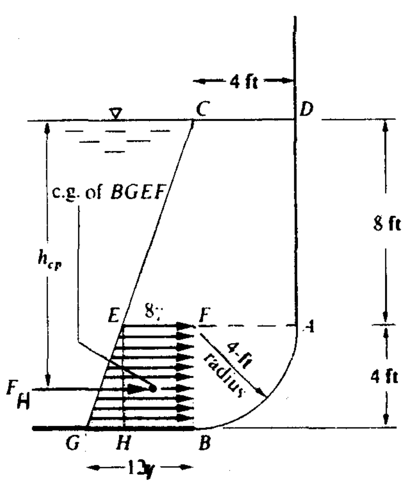



Fig. 5-1($b$)

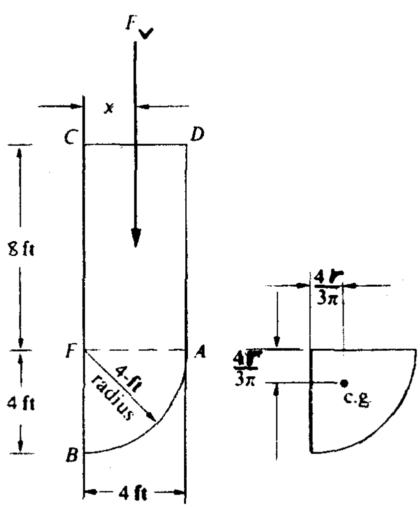



Fig. 5-1($c$)

along a (vertical) line through the center of gravity of the liquid volume vertically above surface $AB$ (i.e., the center of gravity of $ABCD$ in Fig. 5-1$c$). This can be determined by referring to Fig. 5-1$c$ and equating the sum of the moments of the rectangular area ($AFCD$ in Fig. 5-1$c$) and of the quarter-circular area ($ABF$ in Fig. 5-1$c$) about a vertical line through point $B$ to the moment of the total area about the same line. $(x)[(8)(4) + (\pi)(4)^2/4] = [(8)(4)](\frac{4}{2}) + [(\pi)(4)^2/4][(4)(4)/(3\pi)]$, $x = 1.91$ ft. (This is the distance from point $B$ to the line of action of the vertical component.)

**5.2** Solve Prob. 5.1 for the same given conditions except that water is on the other side of curved surface $AB$, as shown in Fig. 5-2.

▌ If necessary, refer to the solution of Prob. 5.1 for a more detailed explanation of the general procedure for solving this type of problem. $p = p_{avg} = (\gamma)[(h_1 + h_2)/2] = (62.4)[(8 + 12)/2] = 624\ \text{lb/ft}^2$, $A = (6)(4) = 24\ \text{ft}^2$, $F_H = pA = (624)(24) = 14\,980$ lb. The vertical component ($F_V$) is equal to the weight of the imaginary volume of water vertically above surface $AB$. Hence, $F_V = [(4)(8) + (\pi)(4)^2/4](6)(62.4) = 16\,690$ lb. The location of the horizontal component is 10.13 ft below the water surface (same as in Prob. 5.1 except that $F_H$ acts toward the left). The location of the vertical component is 1.91 ft from point $B$ (same as in Prob. 5.1 except that $F_V$ acts upward).

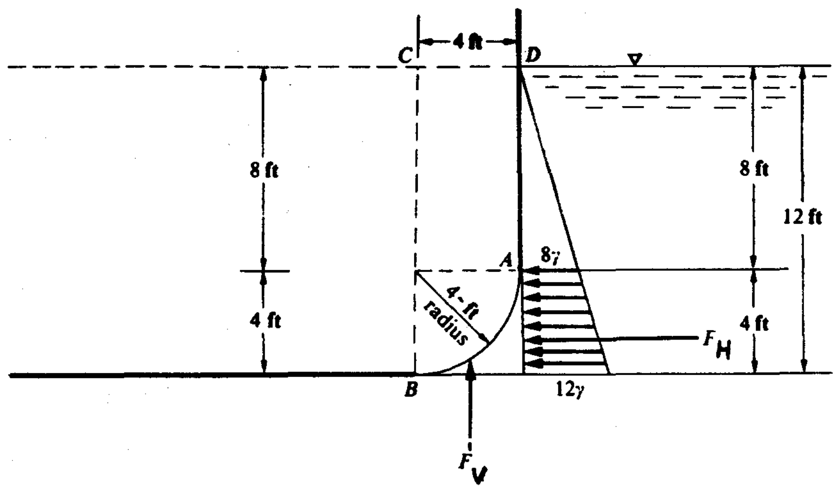
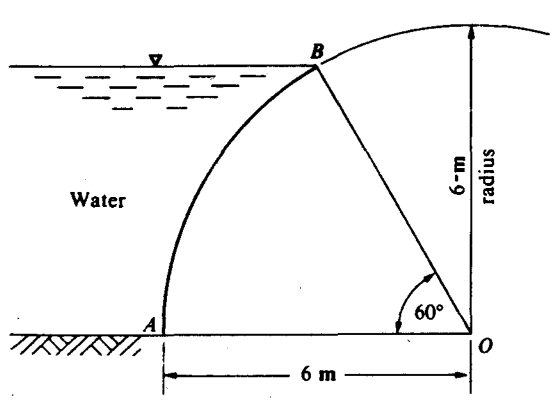



**Fig. 5-2**

**5.3** The submerged sector gate $AB$ shown in Fig. 5-3$a$ is one-sixth of a circle of radius 6 m. The length of the gate is 10 m. Determine the amount and location of the horizontal and vertical components of the total resultant force acting on the gate.

▌ If necessary, refer to the solution of Prob. 5.1 for a more detailed explanation of the general procedure for solving this type of problem. Refer to Fig. 5-3$b$. $F_H = \gamma \bar{h} A = (9.79)[(0 + 5.196)/2][(10)(5.196)] = 1322$ kN, $\text{Area}_{ABC} = \text{area}_{ACBD} + \text{area}_{BDO} - \text{area}_{ABO} = (5.196)(3) + (3.000)(5.196)/2 - (\pi)(6)^2/6 = 4.532\ \text{m}^2$, $F_V = (\text{area}_{ABC})(\text{length of gate})(\gamma) = (4.532)(10)(9.79) = 444$ kN. The location of the horizontal component ($F_H$) is along a (horizontal) line 5.196/3, or 1.732 m above the bottom of the gate ($A$). The location of the vertical component ($F_V$) is along a (vertical) line through the center of gravity of section $ABC$. Taking area moments about $AC$, $4.532x = [(5.196)(3)](\frac{3}{2}) + [(\frac{1}{2})(3.000)(5.196)](3 + 3.000/3) - [(\pi)(6)^2/6]\{6 - [\cos(60°/2)](2)(6)/\pi\}$, $x = 0.842$ m.




($a$)

**Fig. 5-3($a$)**

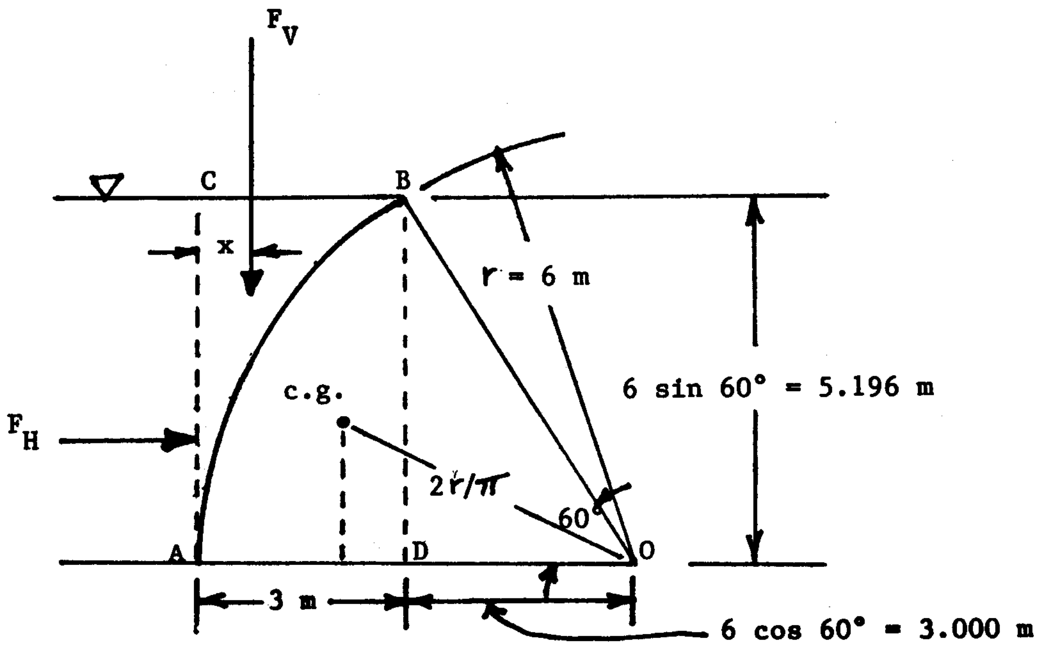



**Fig. 5-3(b)**

**5.4** The curved surface $AB$ shown in Fig. 5-4$a$ is a quarter of a circle of radius 5 ft. Determine, for an 8-ft length perpendicular to the paper, the amount and location of the horizontal and vertical components of the total resultant force acting on surface $AB$.

**I** If necessary, refer to the solution of Prob. 5.1 for a more detailed explanation of the general procedure for solving this type of problem. Refer to Fig. 5-4$b$. $F_H = \gamma\bar{h}A = (62.4)[(0+5)/2][(5)(8)] = 6240$ lb, $\text{area}_{ABD} = \text{area}_{ACBD} - \text{area}_{ABC} = (5)(5) - (\pi)(5)^2/4 = 5.365$ ft$^2$, $F_V = (\text{area}_{ABD})(\text{length})(\gamma) = (5.365)(8)(62.4) = 2678$ lb. $F_H$ is located at $\frac{5}{3}$, or 1.67 ft above $C$. $F_V$ is located at $x$ from line $AD$. $5.365x = [(5)(5)](\frac{5}{2}) - [(\pi)(5)^2/4][5 - (4)(5)/(3\pi)]$, $x = 1.12$ ft.

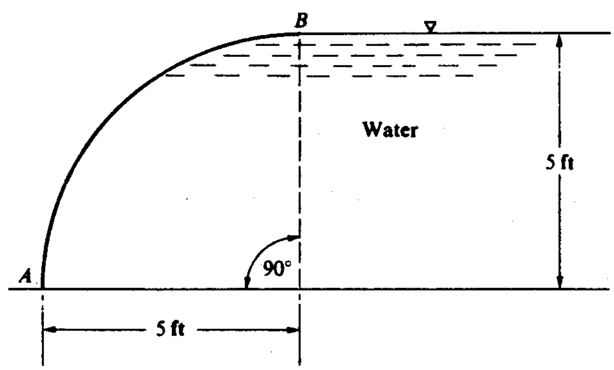



**Fig. 5-4(a)**

**5.5** Determine the value and location of the horizontal and vertical components of the force due to water acting on curved surface $AB$ in Fig. 5-5, per foot of its length.

**I** If necessary, refer to the solution of Prob. 5.1 for a more detailed explanation of the general procedure for solving this type of problem. $F_H = \gamma\bar{h}A = (62.4)[(0+6)/2][(6)(1)] = 1123$ lb, $F_V = (\text{area})(\text{length})(\gamma) = [(\pi)(6)^2/4](1)(62.4) = 1764$ lb. $F_H$ is located at $(\frac{2}{3})(6)$, or 4.00 ft below $C$. $F_V$ is located at the center of gravity of area $ABC$, or distance $x$ from line $CB$. $x = 4r/(3\pi) = (4)(6)/(3\pi) = 2.55$ ft.

**5.6** The 6-ft-diameter cylinder in Fig. 5-6 weighs 5000 lb and is 5 ft long. Determine the reactions at $A$ and $B$, neglecting friction.

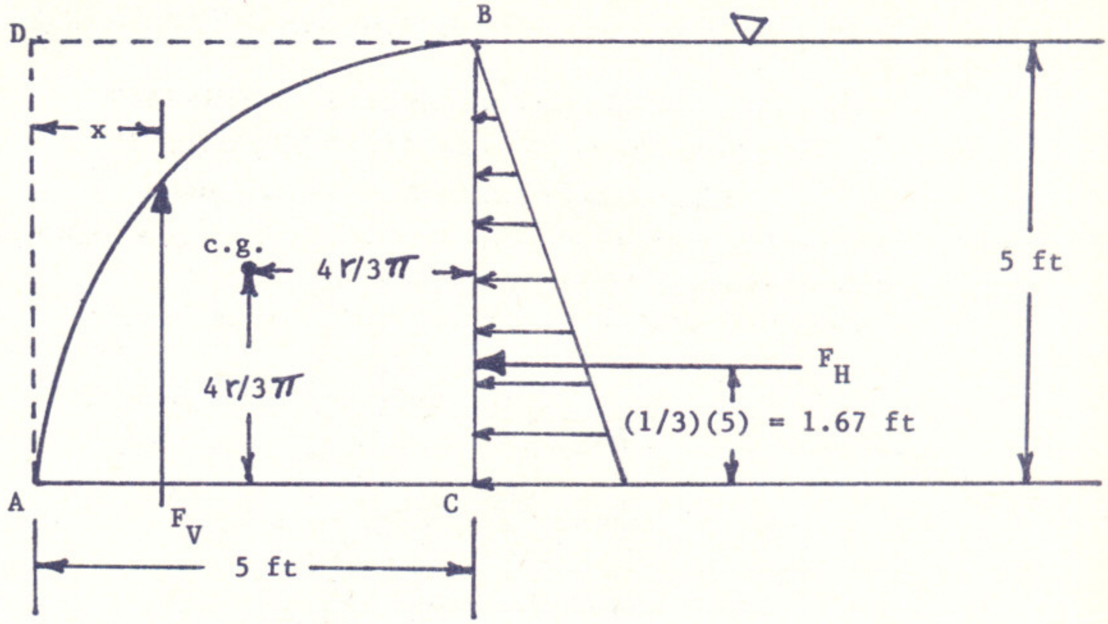



Fig. 5-4(b)

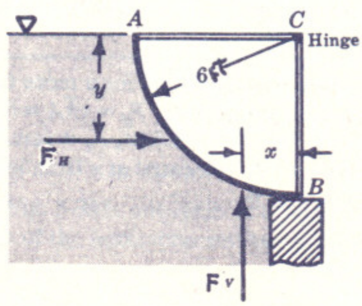



Fig. 5-5

▌ The reaction at $A$ is due to the horizontal component of the liquid force acting on the cylinder ($F_H$). $F_H = \gamma \bar{h} A = [(0.800)(62.4)][(0+6)/2][(3+3)(5)] = 4493$ lb. $F_H$ acts to the right; hence, the reaction at $A$ is 4493 lb to the left. The reaction at $B$ is the algebraic sum of the weight of the cylinder and the net vertical component of the force due to the liquid. $(F_V)_{up} = (\text{area}_{ECOBDE})(\text{length})(\gamma)$, $(F_V)_{down} = (\text{area}_{ECDE})(\text{length})(\gamma)$, $(F_V)_{net} = (F_V)_{up} - (F_V)_{down} = (\text{area}_{COBDC})(\text{length})(\gamma) = [(\pi)(3)^2/2](5)[(0.800)(62.4)] = 3529$ lb (upward). The reaction at $B$ is $5000 - 3529$, or 1471 lb upward.

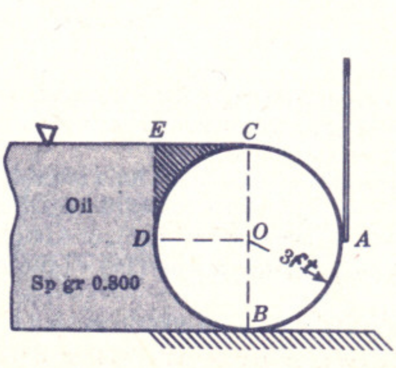



Fig. 5-6

**5.7** Referring to Fig. 5-7, determine the horizontal and vertical forces due to the water acting on the cylinder per foot of its length.

▌

$$(F_H)_{CDA} = 62.4\{[4 + (4 + 4.24 + 0.88)]/2\}[(2.12 + 3)(1)] = 2096 \text{ lb}$$

$$(F_H)_{AB} = (62.4)\{[(4 + 4.24) + (4 + 4.24 + 0.88)]/2\}[(0.88)(1)] = 477 \text{ lb}$$

$$(F_H)_{\text{net}} = (F_H)_{CDA} - (F_H)_{AB} = 2096 - 477 = 1619 \text{ lb (right)}$$

$$(F_V)_{\text{net}} = (F_V)_{DAB} - (F_V)_{DC} = \text{weight of volume}_{DABFED} - \text{weight of volume}_{DCGED} = \text{weight of volume}_{DABFGCD}$$

$$= \text{weight of (rectangle}_{GFJC} + \text{triangle}_{CJB} + \text{semicircle}_{CDAB})$$

$$= 62.4[(4)(4.24) + (4.24)(4.24)/2 + (\pi)(3)^2/2](1) = 2501 \text{ lb (upward)}$$

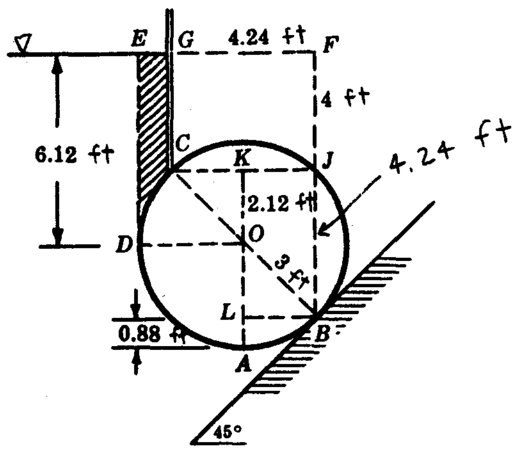



**Fig. 5-7**

**5.8** In Fig. 5-8, an 8-ft-diameter cylinder plugs a rectangular hole in a tank that is 3 ft long. With what force is the cylinder pressed against the bottom of the tank due to the 9-ft depth of water?

▌

$$(F_V)_{\text{net}} = (F_V)_{CDE} - (F_V)_{CA} - (F_V)_{BE} = 62.4[(4 + 4)(7) - (\pi)(4)^2/2](3)$$

$$- 62.4[(7)(0.54) + (\tfrac{30}{360})(\pi)(4)^2 - (2)(3.46)/2](3)$$

$$- 62.4[(7)(0.54) + (\tfrac{30}{360})(\pi)(4)^2 - (2)(3.46)/2](3) = 4090 \text{ lb (downward)}$$

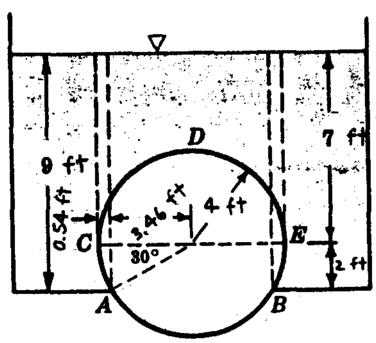



**Fig. 5-8**

**5.9** In Fig. 5-9, the 8-ft-diameter cylinder weighs 500 lb and rests on the bottom of a tank that is 3 ft long. Water and oil are poured into the left- and right-hand portions of the tank to depths of 2 ft and 4 ft, respectively. Find the magnitudes of the horizontal and vertical components of the force that will keep the cylinder touching the tank at $B$.

▌

$$(F_H)_{\text{net}} = (F_H)_{AB} - (F_H)_{CB} = [(0.750)(62.4)][(0 + 4)/2][(4)(3)] - (62.4)[(0 + 2)/2][(2)(3)] = 749 \text{ lb (left)}$$

$$(F_V)_{\text{net}} = (F_V)_{AB} + (F_V)_{CB} = [(0.750)(62.4)][(\pi)(4)^2/4](3) + (62.4)[(\tfrac{60}{360})(\pi)(4)^2 - (2)(\sqrt{12})/2](3)$$

$$= 2684 \text{ lb (upward)}$$

The components to hold the cylinder in place are 749 lb to the right and 2684 − 500, or 2184 lb down.

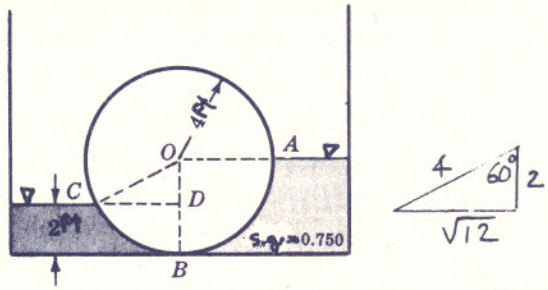



Fig. 5-9

**5.10** The half-conical buttress $ABE$ shown in Fig. 5-10 is used to support a half-cylindrical tower $ABCD$. Calculate the horizontal and vertical components of the force due to water acting on the buttress.

▮

$$F_H = \gamma h_{cg} A = (62.4)(3 + \tfrac{6}{3})[(6)(2+2)/2] = 3744 \text{ lb (right)}$$

$$F_V = \text{weight of (imaginary) volume of water above curved surface}$$
$$= (62.4)[(\tfrac{1}{2})(6)(\pi)(2)^2/3 + (\tfrac{1}{2})(\pi)(2)^2(3)] = 1960 \text{ lb (up)}$$

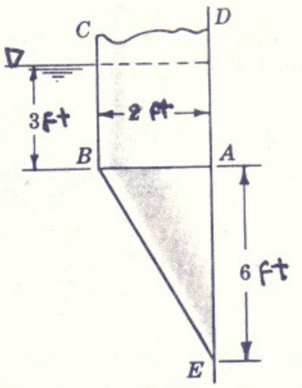



Fig. 5-10

**5.11** A dam has a parabolic shape $z = z_0(x/x_0)^2$, as shown in Fig. 5-11*a*. The fluid is water and atmospheric pressure may be neglected. If $x_0 = 10$ ft and $z_0 = 24$ ft, compute forces $F_H$ and $F_V$ on the dam and the position c.p. where they act. The width of the dam is 50 ft.

▮ $F_H = \gamma \bar{h} A = 62.4[(24+0)/2][(24)(50)] = 898\,600$ lb. The location of $F_H$ is along a (horizontal) line $\frac{24}{3}$, or 8.00 ft above the bottom of the dam. $F_V = (\text{area}_{0AB})(\text{width of dam})(\gamma)$. (See Fig. 5-11*b*.) $\text{Area}_{0AB} = 2x_0Z_0/3 = (2)(10)(24)/3 = 160 \text{ ft}^2$, $F_V = (160)(50)(62.4) = 499\,200$ lb. The location of $F_V$ is along a (vertical) line through the center of gravity of $\text{area}_{0AB}$. From Fig. 5-11*b*, $x = 3x_0/8 = (3)(10)/8 = 3.75$ ft, $z = 3z_0/5 = (3)(24)/5 = 14.4$ ft, $F_{\text{resultant}} = \sqrt{499\,200^2 + 898\,600^2} = 1\,028\,000$ lb. As seen in Fig. 5-11*c*, $F_{\text{resultant}}$ acts down and to the right at an angle of arctan (499 200/898 600), or 29.1°. $F_{\text{resultant}}$ passes through the point $(x, z) = (3.75 \text{ ft}, 8 \text{ ft})$. If we move down alone the 29.1° line until we strike the dam, we find an equivalent center of pressure on the dam at $x = 5.43$ ft and $z = 7.07$ ft. This definition of c.p. is rather artificial, but this is an unavoidable complication of dealing with a curved surface.

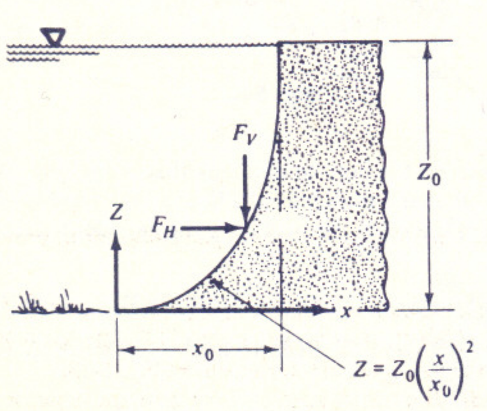



Fig. 5-11(*a*)

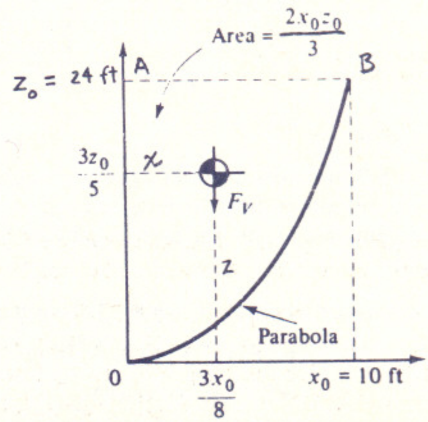



Fig. 5-11(*b*)

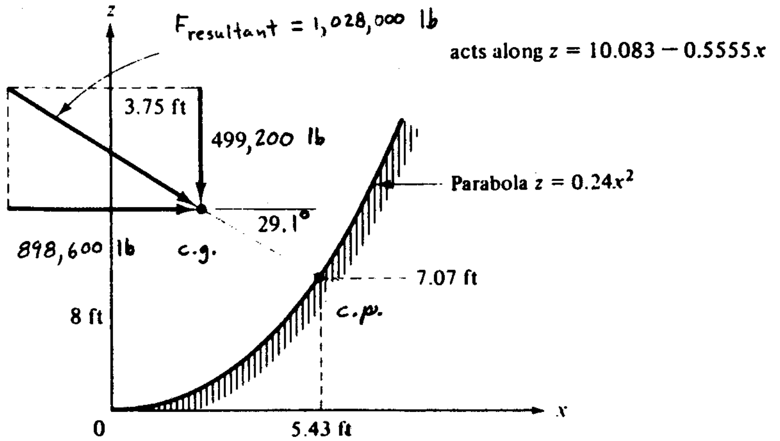



Fig. 5-11(c)

**5.12** The canal shown in cross section in Fig. 5-12*a* runs 40 m into the paper. Determine the horizontal and vertical components of the hydrostatic force against the quarter-circle wall and the point c.p. where the resultant strikes the wall.

▌ $F_H = \gamma \bar{h} A = 9.79[(18+0)/2][(18)(40)] = 63\,439$ kN. The location of $F_H$ is along a (horizontal) line $\frac{18}{3}$, or 6.00 m above the bottom of the wall. $F_V = 9.79[(40)(\pi)(18)^2/4] = 99\,650$ kN. The location of $F_V$ is along a (vertical) line through the center of gravity of area$_{OAB}$. $x = 4r/(3\pi) = (4)(18)/(3\pi) = 7.64$ m, $F_{resultant} = \sqrt{63\,439^2 + 99\,650^2} = 118\,130$ kN. As seen in Fig. 5-12*b*, $F_{resultant}$ acts down and to the right at an angle of arctan (99 650/63 439), or 57.5°. $F_{resultant}$ passes through the point $(x, z) = (7.64\text{ m}, 6.00\text{ m})$. If we move down along the 57.5° line until we strike the wall, we find an equivalent center of pressure at $x = 8.33$ m and $z = 2.82$ m.

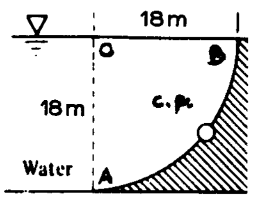



Fig. 5-12(*a*)

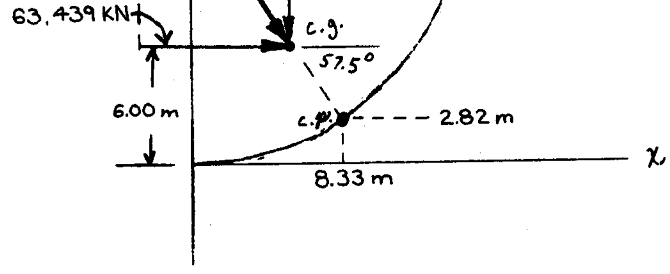



Fig. 5-12(*b*)

**5.13** Gate *AB* in Fig. 5-13*a* is a quarter circle 8 ft wide into the paper. Find the force *F* just sufficient to prevent rotation about hinge *B*. Neglect the weight of the gate.

▌ $F_H = \gamma \bar{h} A = 62.4[(7+0)/2][(7)(8)] = 12\,230$ lb (left). The location of $F_H$ is along a (horizontal) line $\frac{7}{3}$, or 2.333 ft above point *B*. (See Fig. 5-13*b*.) $F_V = F_1 - F_2 = 62.4[(8)(7)(7)] - 62.4[(8)(\pi)(7)^2/4] = 24\,461 - 19\,211 = 5250$ lb (up). The location of $F_V$ can be determined by taking moments about point *B* in Fig. 5-13*b*. $5250x = (24\,461)(\frac{7}{2}) - (19\,211)[7 - (4)(7)/(3\pi)]$, $x = 1.564$ ft. The forces acting on the gate are shown in Fig. 5-13*c*. $\sum M_B = 0$; $7F - (2.333)(12\,230) - (1.564)(5250) = 0$, $F = 5249$ lb (down).

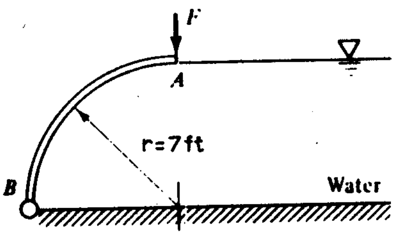



Fig. 5-13(a)

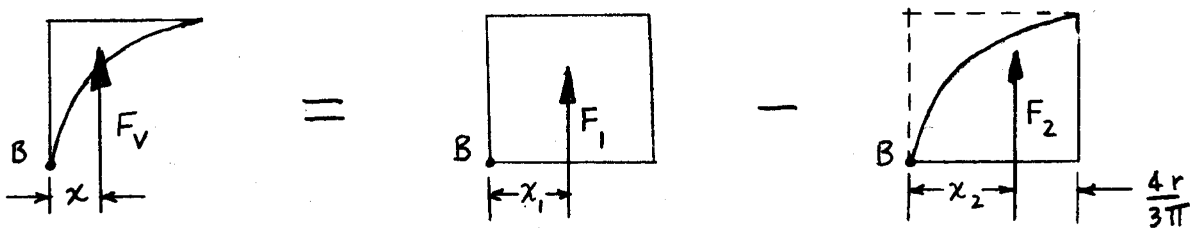



Fig. 5-13(b)

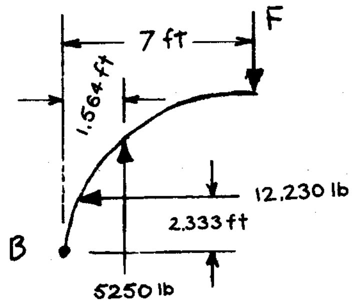



Fig. 5-13(c)

**5.14** Repeat Prob. 5.13 if the gate is steel weighing 3000 lb.

▌ The weight of the gate acts at the center of gravity of the gate shown in Fig. 5-14. $2r/\pi = (2)(7)/\pi = 4.456$ ft; $\sum M_B = 0$. From Prob. 5.14, $7F - (2.333)(12\,230) - (1.564)(5250) + (3000)(7 - 4.456) = 0$, $F = 4159$ lb.

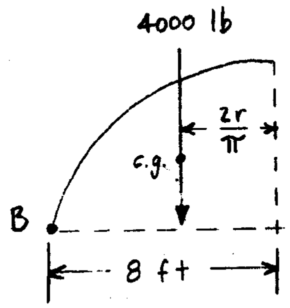



Fig. 5-14

**5.15** Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle face of the tank shown in Fig. 5-15*a*.

▌ $$F_H = \gamma h_{cg} A = 9.79[4 + \tfrac{1}{2}][(1)(7)] = 308 \text{ kN}$$

$$F_V = F_1 - F_2 = (9.79)[(7)(1)(5)] - (9.79)[(7)(\pi)(1)^2/4] = 289 \text{ kN} \qquad \text{(See Fig. 5-15}b.\text{)}$$

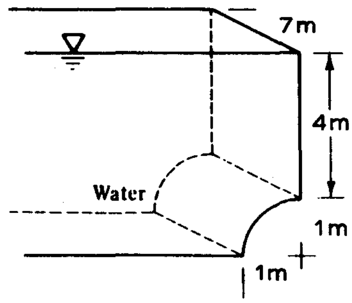



Fig. 5-15(a)

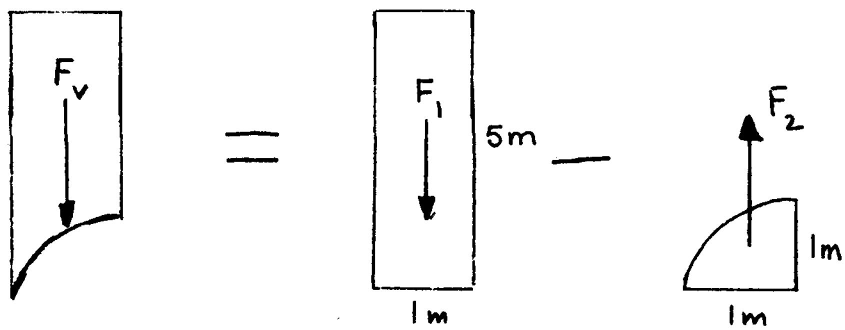



Fig. 5-15(b)

**5.16** Compute the horizontal and vertical components of the hydrostatic force on the hemispherical boulder shown in Fig. 5-16*a*.

▌ From symmetry, $F_H = 0$, $F_V = F_1 - F_2$ (see Fig. 5-16*b*). $F_V = 62.4[(\pi)(3)^2(12)] - (62.4)[(\frac{1}{2})(\frac{4}{3})(\pi)(3)^3] =$ 17 643 lb.

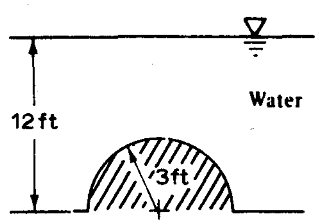



Fig. 5-16(a)

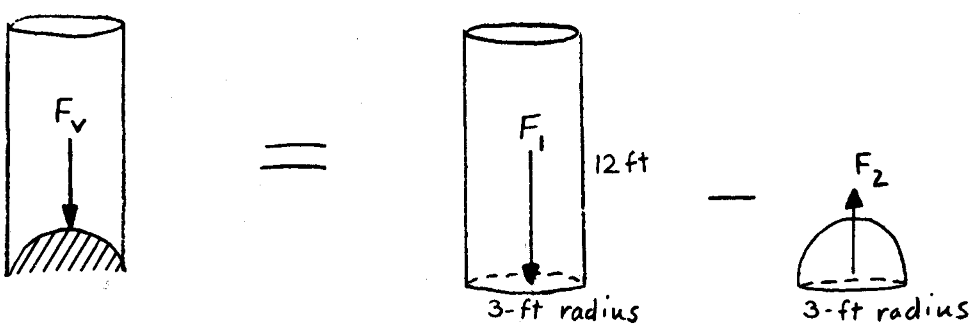



Fig. 5-16(b)

**5.17** The bottled cider (s.g. = 0.96) in Fig. 5-17 is under pressure, as shown by the manometer reading. Compute the net force on the 2-in-radius concavity in the bottom of the bottle.

▌ From symmetry, $F_H = 0$, $p_{AA} + [(0.96)(62.4)](\frac{3}{12}) - [(13.6)(62.4)](\frac{5}{12}) = p_{\text{atm}} = 0$, $p_{AA} = 339\ \text{lb/ft}^2$ (gage); $F_V = p_{AA}A_{\text{bottom}}$ + weight of liquid below $AA$ $= 339[(\pi)(\frac{4}{12})^2/4] + [(0.96)(62.4)][(\frac{7}{12})(\pi)(\frac{4}{12})^2/4] - [(0.96)(62.4)][(\frac{1}{2})(\frac{4}{3})(\pi)(\frac{2}{12})^3] = 32.1$ lb.

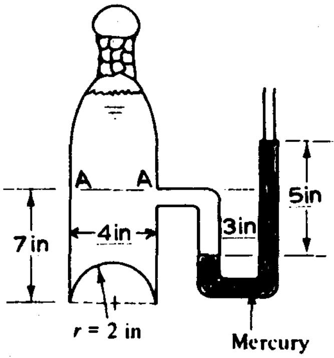


Fig. 5-17

**5.18** Half-cylinder $ABC$ in Fig. 5-18$a$ is 9 ft wide into the paper. Calculate the net moment of the pressure forces on the body about point $C$.

▮ From symmetry, the horizontal forces balance and produce no net moment about point $C$. (See Fig. 5-18$b$.) $F_V = F_1 - F_2 = F_{\text{buoyancy of body } ABC} = [(0.85)(62.4)][(9)(\pi)(\frac{9}{2})^2/2] = 15\,184$ lb, $x = 4r/(3\pi) = (4)(\frac{9}{2})/(3\pi) = 1.910$ ft, $M_C = (15\,184)(1.910) = 29\,001$ lb · ft (clockwise).

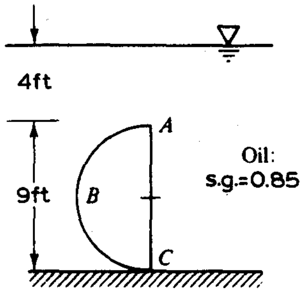


Fig. 5-18($a$)

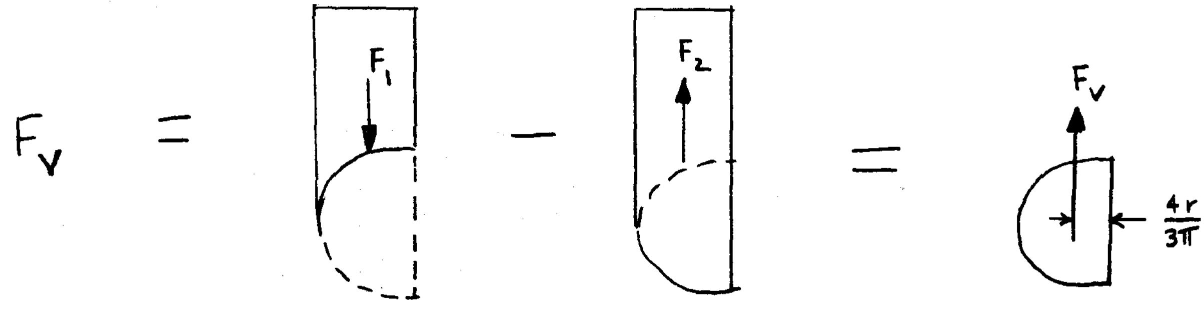


Fig. 5-18($b$)

**5.19** Compute the hydrostatic force and its line of action on semicylindrical indentation $ABC$ in Fig. 5-19$a$ per meter of width into the paper.

▮

$$F_H = \gamma h_{cg} A = [(0.88)(9.79)](2 + 2 + \tfrac{2.5}{2})[(2.5)(1)] = 113.1 \text{ kN}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(1)(2.5)^3/12](\sin 90°)}{(2 + 2 + \frac{2.5}{2})[(2.5)(1)]} = -0.099 \text{ m}$$

As demonstrated in Prob. 5.18, $F_V = F_{\text{buoyancy of body } ABC}$ and it acts at $4r/(3\pi)$ from point $C$. $F_V = [(0.88)(9.79)][(1)(\pi)(\frac{2.5}{2})^2/2] = 21.14$ kN, $x = 4r/(3\pi) = (4)(\frac{2.5}{2})/(3\pi) = 0.531$ m. The forces acting on the indentation are shown in Fig. 5-19$b$. $F_{\text{resultant}} = \sqrt{21.14^2 + 113.1^2} = 115.1$ kN. As shown in Fig. 5-19$b$, $F_{\text{resultant}}$ passes through point $O$ and acts up and to the right at an angle of arctan (21.14/113.1), or 10.59°.

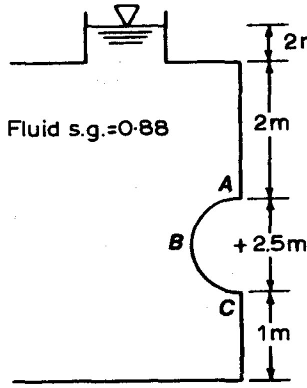



Fig. 5-19(a)

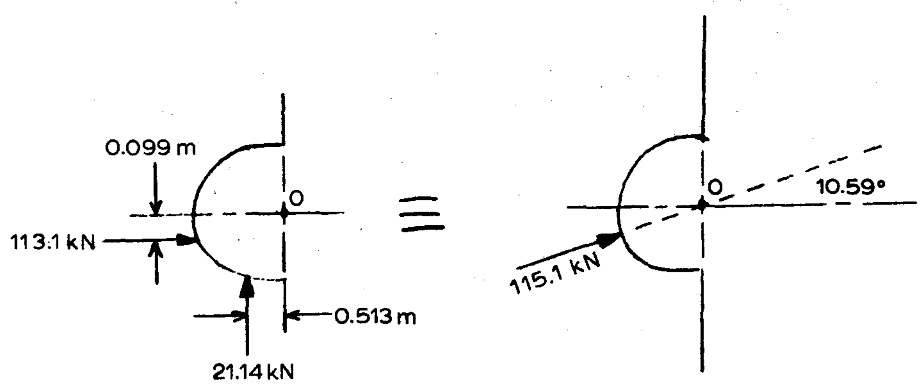



Fig. 5-19(b)

**5.20** Find the force on the conical plug in Fig. 5-20. Neglect the weight of the plug.

▮ $F_V = pA_{\text{hole}} + \text{weight of water above cone} = [(4.5)(144)][(\pi)(1)^2/4] + (62.4)[(4)(\pi)(1)^2/4]$
$- (62.4)[(\tfrac{1}{3})(1.207)(\pi)(1)^2/4] = 685\text{ lb}$

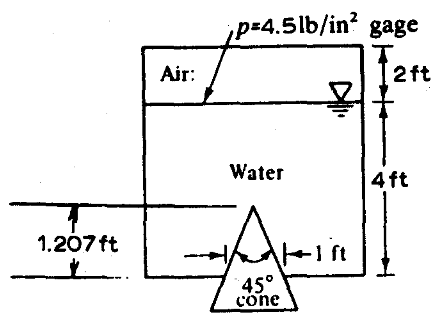



Fig. 5-20

**5.21** The hemispherical dome in Fig. 5-21 is filled with water and is attached to the floor by two diametrically opposed bolts. What force in either bolt is required to hold the dome down, if the dome weighs 25 kN?

▮ $F_V$ = weight of (imaginary) water above the container

$= 9.79[(5 + 1.5)(\pi)(1.5)^2] - 9.79[(5)(\pi)(0.04)^2/4] - 9.79[(\tfrac{1}{2})(\tfrac{4}{3})(\pi)(1.5)^3] = 380.5\text{ kN (up)}$

net upward force on dome $= 380.5 - 25 = 355.5$ kN

force per bolt $= 355.5/2 = 177.7$ kN

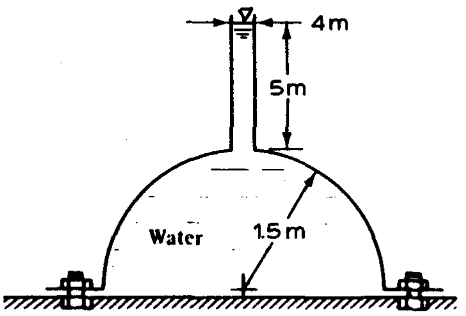



Fig. 5-21

**5.22** A 3-m-diameter water tank consists of two half-cylinders, each weighing 3.5 kN/m, bolted together as shown in Fig. 5-22*a*. If support of the end caps is neglected, determine the force induced in each bolt.

▌ See Fig. 5-22*b*. Assuming the bottom half is properly supported, only the top half affects the bolt force. $p_1 = (9.79)(1.5 + 1) = 24.48\ \text{kN/m}^2$; $\sum F_y = p_1A_1 - 2F_{\text{bolt}} - W_{H_2O} - W_{\text{tank half}} = 0$, $24.48[(3)(\frac{25}{100})] - 2F_{\text{bolt}} - 9.79[(\frac{25}{100})(\pi)(1.5)^2/2] - 3.5/4 = 0$, $F_{\text{bolt}} = 4.42\ \text{kN}$.

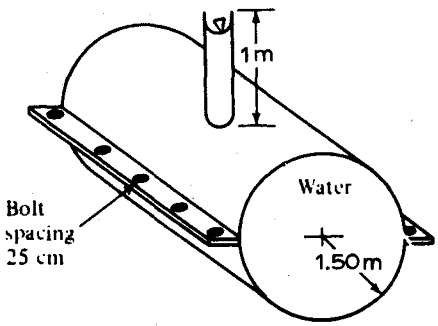



Fig. 5-22(*a*)

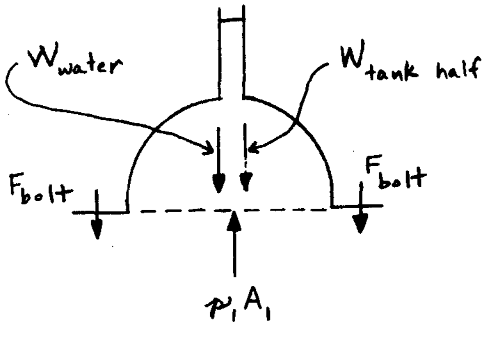



Fig. 5-22(*b*)

**5.23** The cylinder in Fig. 5-23*a* extends 5 ft into the paper. Compute the horizontal and vertical components of the pressure force on the cylinder.

▌ See Fig. 5-23*b*. Note that the net horizontal force is based on the projected vertical area with depth *AB*. $F_H = \gamma h_{cg}A = 62.4[(4 + 2.828)/2][(4 + 2.828)(5)] = 7273\ \text{lb}$; $F_V$ = equivalent weight of fluid in regions 1, 2, 3, and 4 = $(62.4)(5)[(\pi)(4)^2/2 + (2.828)(4) + (2.828)(2.828)/2 + (\pi)(4)^2/8] = 14\,579\ \text{lb}$.

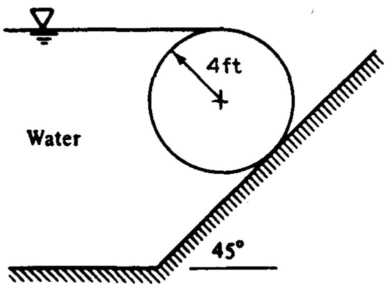



Fig. 5-23(*a*)

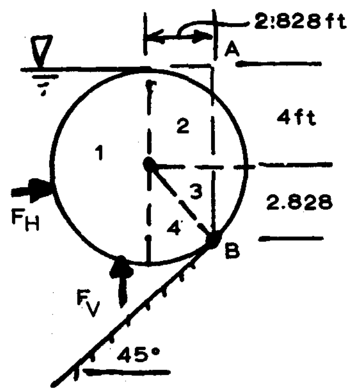



Fig. 5-23(*b*)

**5.24** A 3-ft-diameter log (s.g. = 0.82) divides two shallow ponds as shown in Fig. 5-24*a*. Compute the net vertical and horizontal reactions at point *C*, if the log is 12 ft long.

▌ $F = \gamma hA$. Figure 5-24*b* shows the forces acting on the log.

$$(F_H)_1 = 62.4[(0+3)/2][(1.5+1.5)(12)] = 3370\text{ lb} \qquad (F_H)_2 = 62.4[(0+1.5)/2][(1.5)(12)] = 842\text{ lb}$$

$$(F_V)_1 = 62.4[(12)(\pi)(1.5)^2/2] = 2646\text{ lb} \qquad (F_V)_2 = 62.4[(12)(\pi)(1.5)^2/4] = 1323\text{ lb}$$

$$\sum F_x = 0 \qquad 3370 - 842 - C_x = 0 \qquad C_x = 2528\text{ lb (left)}$$

$$\sum F_y = 0 \qquad 2646 + 1323 - [(0.82)(62.4)][(12)(\pi)(1.5)^2] + C_y = 0 \qquad C_y = 371\text{ lb (up)}$$

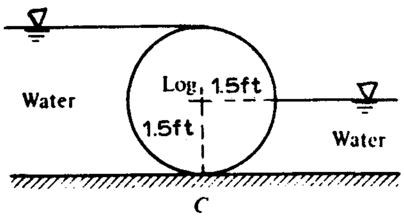



Fig. 5-24(*a*)

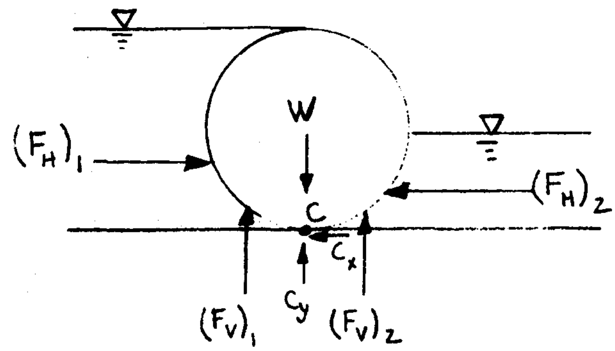



Fig. 5-24(*b*)

**5.25** The 1-m-diameter cylinder in Fig. 5-25*a* is 8 m long into the paper and rests in static equilibrium against a frictionless wall at point *B*. Compute the specific gravity of the cylinder.

▌ See Fig. 5-25*b*. The wall reaction at *B* is purely horizontal. Then the log weight must exactly balance the vertical hydrostatic force, which equals the equivalent weight of water in the shaded area. $W_{\log} = F_V =$ $(9.79)(8)[(\frac{3}{4})(\pi)(0.5)^2 + (0.5)(0.5)] = 65.71$ kN, $\gamma_{\log} = 65.71/[(8)(\pi)(0.5)^2] = 10.46$ kN/m³, s.g. $= 10.46/9.79 =$ 1.07.

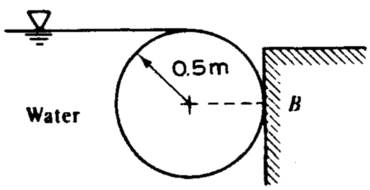



Fig. 5-25(*a*)

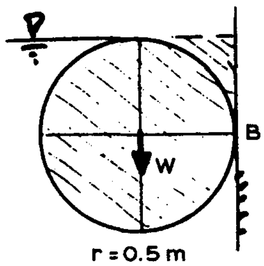



Fig. 5-25(*b*)

**5.26** The tank in Fig. 5-26*a* is 3 m wide into the paper. Neglecting atmospheric pressure, compute the hydrostatic horizontal, vertical, and resultant force on quarter-circle panel *BC*.

▌ $F_H = \gamma h_{cg} A = (9.79)(4 + \frac{5}{2})[(5)(3)] = 954.5$ kN, $F_V$ = weight of water above panel $BC = (9.79)[(3)(5)(4)] +$ $(9.79)[(3)(\pi)(5)^2/4] = 1164$ kN, $F_{\text{resultant}} = \sqrt{954.5^2 + 1164^2} = 1505$ kN. As seen in Fig. 5-26*b*, $F_{\text{resultant}}$ passes through point *O* and acts down and to the right at an angle of arctan (1164/954.5), or 50.6°.

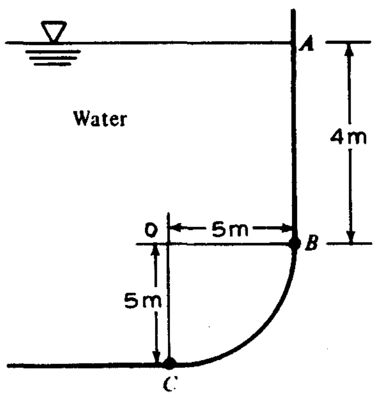



Fig. 5-26(*a*)

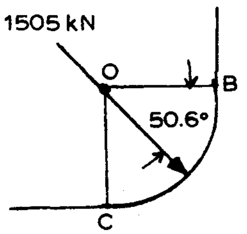



Fig. 5-26(*b*)

**5.27** Gate $AB$ in Fig. 5-27$a$ is a quarter circle 7 ft wide, hinged at $B$ and resting against a smooth wall at $A$. Compute the reaction forces at $A$ and $B$.

▮ $F_H = \gamma h_{cg} A = (64)(11 - \frac{6}{2})[(7)(6)] = 21\,504$ lb $\qquad y_{cp} = \dfrac{-I_{xx}\sin\theta}{h_{cg}A} = \dfrac{-[(7)(6)^3/12](\sin 90°)}{(11-\frac{6}{2})[(7)(6)]} = -0.375$ ft

Thus, $F_H$ acts at $\frac{6}{2} - 0.375$, or 2.625 ft above point $B$. $F_V$ = weight of seawater above gate $AB$ = $(64)(7)[(11)(6)] - (64)(7)[(\pi)(6)^2/4] = 29\,568 - 12\,667 = 16\,901$ lb. The location of $F_V$ can be determined by taking moments about point $A$ in Fig. 5-27$b$. $(29\,568)(\frac{6}{2}) - (12\,667)[(4)(6)/(3\pi)] = 16\,901x$, $x = 3.340$ ft. The forces acting on the gate are shown in Fig. 5-27$c$.

$$\sum M_B = 0 \qquad (21\,504)(2.625) + (16\,901)(6 - 3.340) - 6A_x = 0 \qquad A_x = 16\,901 \text{ lb}$$

$$\sum F_x = 0 \qquad 21\,504 - B_x - 16\,901 = 0 \qquad B_x = 4603 \text{ lb}$$

$$\sum F_y = 0 \qquad B_y - 16\,901 = 0 \qquad B_y = 16\,901 \text{ lb}$$

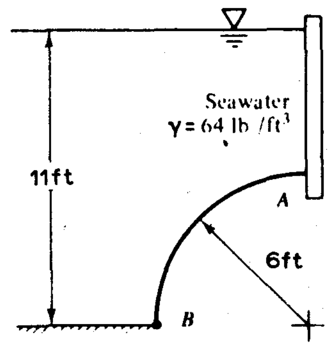



Fig. 5-27(a)

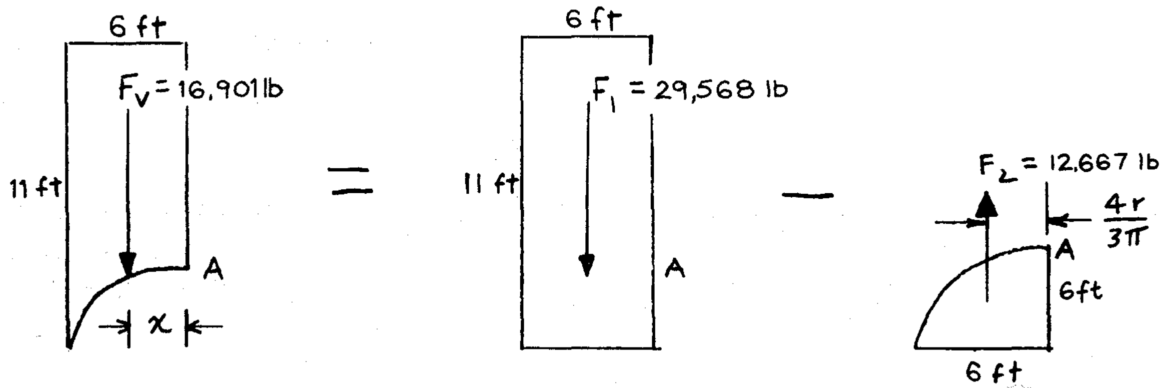



Fig. 5-27(b)

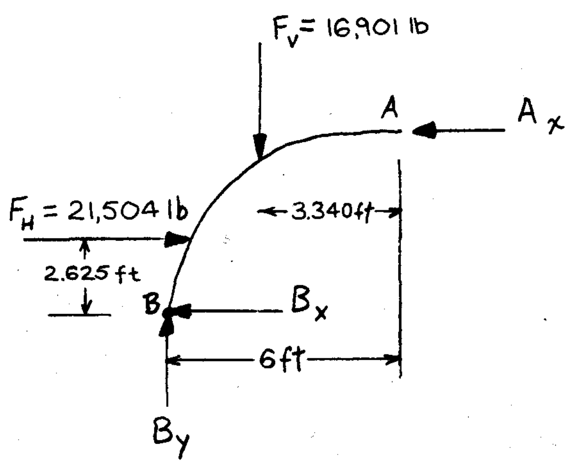
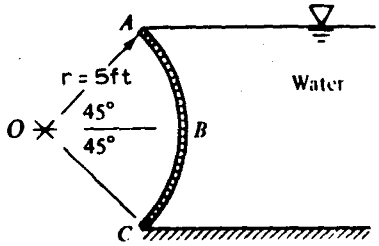



Fig. 5-27(c)

**5.28** Curved wall $ABC$ in Fig. 5-28*a* is a quarter circle 9 ft wide into the paper. Compute the horizontal and vertical hydrostatic forces on the wall and the line of action of the resultant force.

▌ See Fig. 5-28*b*. $F_H = \gamma h_{cg} A = (62.4)(3.536)[(7.072)(9)] = 14\,044$ lb, $F_V$ = weight of (imaginary) water in crosshatched area in Fig. 5-28*b* $= (62.4)(9)[(\pi)(5)^2/4 - (2)(5\sin 45°)(5\cos 45°)/2] = 4007$ lb; $F_{\text{resultant}} = \sqrt{4007^2 + 14\,044^2} = 14\,604$ lb. $F_{\text{resultant}}$ passes through point $O$ and acts at an angle of arctan $\frac{4007}{14\,044}$, or 15.9°, as shown in Fig. 5-28*c*.



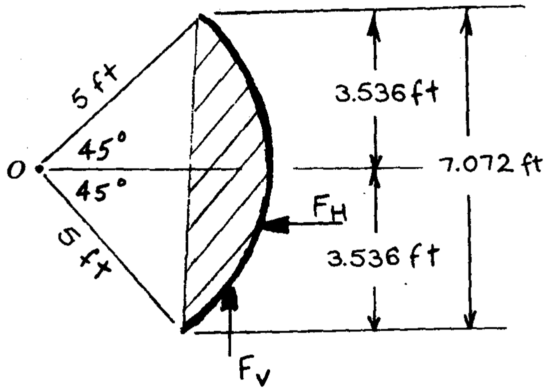




Fig. 5-28(*b*)

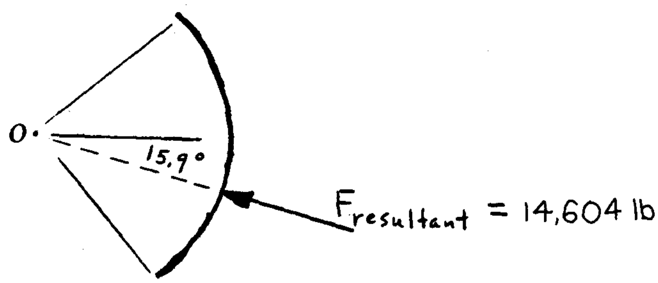



Fig. 5-28(*c*)

**5.29** Pressurized water fills the tank in Fig. 5-29*a*. Compute the net hydrostatic force on conical surface $ABC$.

▌ From symmetry, $F_H = 0$. The gage pressure of 100 kPa corresponds to a fictitious water level at 100/9.79, or 10.215 m above the gage or 10.215 − 7, or 3.215 m above $AC$ (see Fig. 5-29*b*). $F_V$ = weight of fictitious water above cone $ABC = 9.79[(3.215)(\pi)(3)^2/4 + (\frac{1}{3})(6)(\pi)(3)^2/4] = 361$ kN (up).

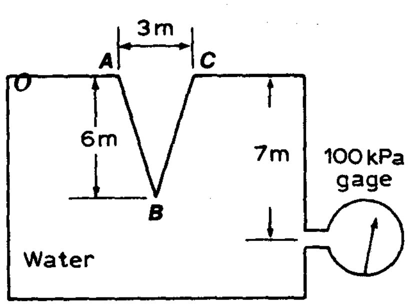



Fig. 5-29(*a*)

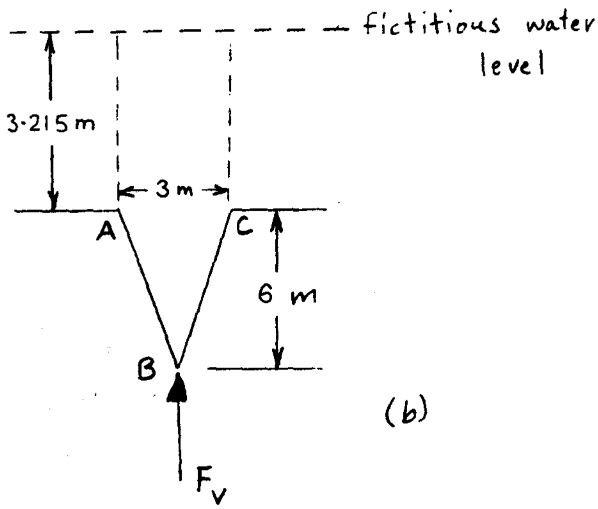



Fig. 5-29(*b*)

**5.30** Gate $AB$ in Fig. 5-30$a$ is 7 m wide into the paper. Compute the force $F$ required to prevent rotation about the hinge at $B$. Neglect atmospheric pressure.

▌ $F_H = \gamma \bar{h} A = 9.79[(9.6 + 0)/2][(9.6)(7)] = 3158$ kN. $F_H$ acts at $\frac{9.6}{3}$, or 3.200 m above $B$ (see Fig. 5-30$b$). $F_V$ = weight of water above the gate $= 9.79[(\frac{2}{3})(6)(9.6)(7)] = 2632$ kN. $F_V$ acts at $\frac{18}{8}$, or 2.250 m right of $B$ (see Fig. 5-30$b$). $\sum M_B = 0$; $(3.200)(3158) + (2.250)(2632) - 9.6F = 0$, $F = 1670$ kN.

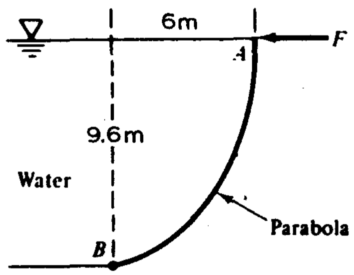


**Fig. 5-30(*a*)**

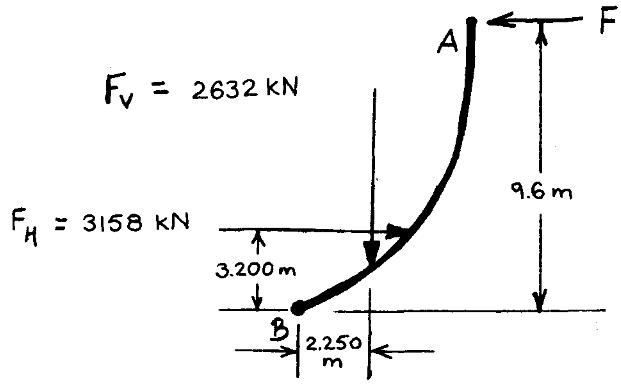


**Fig. 5-30(*b*)**

**5.31** The cylindrical tank in Fig. 5-31 has a hemispherical end cap $ABC$. Compute the total horizontal and vertical forces exerted on $ABC$ by the oil and water.

▌
$$F = \gamma h_{cg} A \qquad (F_H)_1 = [(0.9)(9.79)](3 + \tfrac{2}{2})[(\pi)(2)^2/2] = 221 \text{ kN} \quad \text{(left)}$$
$$(F_H)_2 = \{[(0.9)(9.79)](3 + 2) + (9.79)(\tfrac{2}{2})\}[(\pi)(2)^2]/2 = 338 \text{ kN} \quad \text{(left)}$$
$$(F_H)_{\text{total}} = 221 + 338 = 559 \text{ kN} \quad \text{(left)}$$
$$F_V = \text{weight of fluid within hemisphere} = [(0.9)(9.79)][(\tfrac{1}{4})(\tfrac{4}{3})(\pi)(2)^3] + (9.79)[(\tfrac{1}{4})(\tfrac{4}{3})(\pi)(2)^3] = 156 \text{ kN} \quad \text{(down)}$$

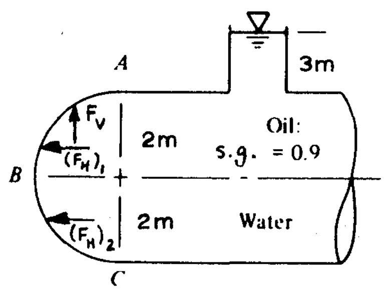


**Fig. 5-31**

**5.32** A cylindrical barrier holds water, as shown in Fig. 5-32. The contact between cylinder and wall is smooth. Consider a 1-m length of cylinder and determine its weight and the force exerted against the wall.

▌
$$(F_V)_{BCD} = (9.79)(1)[(\pi)(2)^2/2 + (2)(2) + (2)(2)] = 139.8 \text{ kN} \quad \text{(up)}$$
$$(F_V)_{AB} = (9.79)(1)[(2)(2) - (\pi)(2)^2/4] = 8.4 \text{ kN} \quad \text{(down)}$$
$$\sum F_y = 0 \qquad 139.8 - W_{\text{cylinder}} - 8.4 = 0 \qquad W_{\text{cylinder}} = 131.4 \text{ kN}$$
$$F_H = \gamma h_{cg} A \qquad (F_H)_{ABC} = (9.79)(2)[(2 + 2)(1)] = 78.3 \text{ kN} \quad \text{(right)}$$
$$(F_H)_{DC} = (9.79)(2 + \tfrac{2}{2})[(2)(1)] = 58.7 \text{ kN} \quad \text{(left)} \qquad (F_H)_{\text{against wall}} = 78.3 - 58.7 = 19.6 \text{ kN} \quad \text{(right)}$$

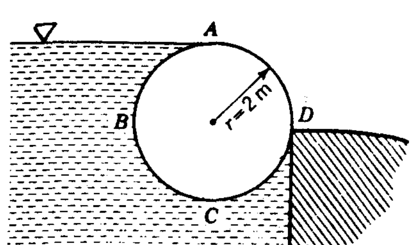


**Fig. 5-32**

**5.33** The revolving gate in Fig. 5-33 is a quarter-cylinder with pivot through $O$. What force $F$ is required to open it? (Treat the gate as weightless.)

▌ At each point of $\widehat{ABC}$ the line of action of the pressure force passes through $O$; hence the pressure has no moment about $O$. It follows that any $F$, no matter how small, suffices to produce a net opening moment.

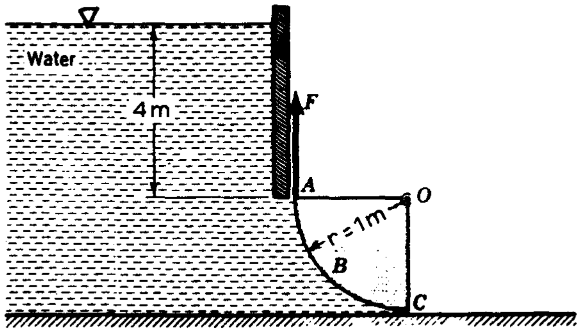



**Fig. 5-33**

**5.34** Find the vertical component of force on the parabolic gate of Fig. 5-34$a$ and its line of action.

▌

$$F_V = \text{weight of imaginary liquid above gate} = \gamma L \int (H-y)\,dx \qquad \text{(see Fig. 5-34}b\text{)}$$

$$= (9.00)(3)\int_0^{0.8} (2-\sqrt{5x})\,dx = (9.00)(3)\left[2x - \frac{\sqrt{5}x^{3/2}}{\frac{3}{2}}\right]_0^{0.8} = 14.40\text{ kN}$$

$$x_{cp} = \frac{\gamma L \int (H-y)x\,dx}{F_V} \qquad \text{(see Fig. 5-34}b\text{)}$$

$$= \frac{(9.00)(3)\int_0^{0.8}(2-\sqrt{5x})x\,dx}{14.40} = \frac{(9.00)(3)\int_0^{0.8}(2x-\sqrt{5}x^{3/2})\,dx}{14.40}$$

$$= (9.00)(3)[x^2 - (\sqrt{5}x^{5/2})/\tfrac{5}{2}]_0^{0.8}/14.40 = 0.240\text{ m}$$

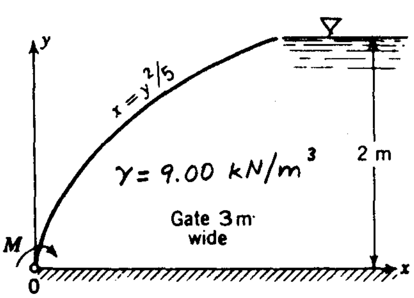



**Fig. 5-34(a)**

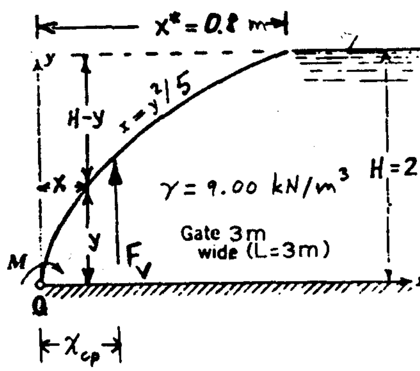



**Fig. 5-34(b)**

**5.35** Determine the moment $M$ needed to hold the gate of Fig. 5-34$a$ shut. Neglect its weight.

▌ $F_H = \gamma \bar{h} A = 9.00[(0+2)/2][(2)(3)] = 54.0$ kN (left). $F_H$ acts at $\frac{2}{3}$, or 0.667 m above point 0. $F_V = 14.40$ kN (up) and $x_{cp} = 0.240$ m (from Prob. 5.34 and Fig. 5-34$b$). $\sum M_0 = 0$; $M - (14.40)(0.240) - (54.0)(0.667) = 0$, $M = 39.5\text{ kN}\cdot\text{m}$.

**5.36** Find the force on the body (part of a parabolic cylinder) of Fig. 5-35. The length normal to the paper is $L = 4.5$ m, and $\gamma$ is 9.20 kN/m³.

▮
$$F_H = \gamma \bar{h} A = (9.20)(\tfrac{1}{2})[(1)(4.5)] = 20.70 \text{ kN}$$

$$F_V = \text{weight of liquid above } OA = \int \gamma L y \, dx = \int_0^{\sqrt{8}} (9.20)(4.5)\left(\frac{x^2}{8}\right) dx = (9.20)(4.5)\left[\frac{x^3}{24}\right]_0^{\sqrt{8}} = 39.03 \text{ kN}$$

$$F_{\text{resultant}} = \sqrt{39.03^2 + 20.70^2} = 44.18 \text{ kN}$$

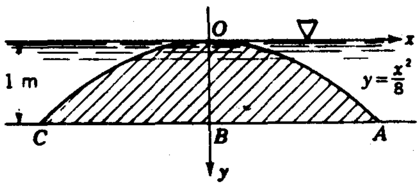



**Fig. 5-35**

**5.37** The curved plate in Fig. 5-36 is an octant of a sphere. Find the resultant force, including its line of action, acting on the outer surface, if the radius of the sphere is 600 mm and its center is 2 m below the water surface.

▮ See Fig. 5-36. $F_H = \gamma \bar{h} A = \gamma[H - 4r/(3\pi)](\pi r^2/4)$, $F_x = F_z = F_H = 9.79[2 - (4)(0.6)/(3\pi)][(\pi)(0.6)^2/4] =$ 4.831 kN (both $F_x$ and $F_z$ act toward 0); $F_y = F_V =$ weight of water above curved surface $= \gamma[(H)(\pi)(r)^2/4 - (\tfrac{4}{3})(\pi)(r)^3/8] = 9.79[(2)(\pi)(0.6)^2/4 - (\tfrac{4}{3})(\pi)(0.6)^3/8] = 4.429$ kN. $F_{\text{resultant}}$ acts on a line through 0 making a 45° angle with the $x$ and $z$ axes because of symmetry; $F_{\text{resultant}} = \sqrt{4.429^2 + 4.831^2 + 4.831^2} = 8.142$ kN. It acts at an angle $\theta = \arccos(4.429/8.142) = 57.0°$.

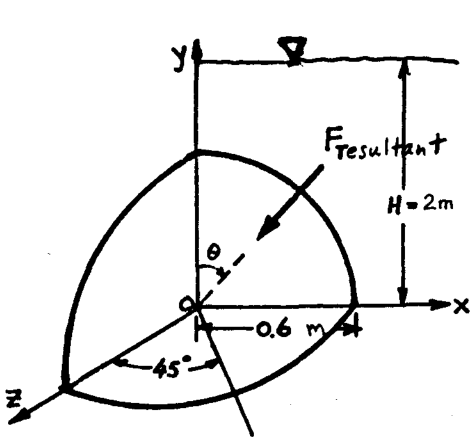



**Fig. 5-36**

**5.38** Find the horizontal and vertical components of the force per unit width exerted by fluids on the horizontal cylinder in Fig. 5-37*a* if the fluid to the left of the cylinder is (*a*) a gas confined in a closed tank at a pressure of 35.0 kN/m² and (*b*) water with a free surface at an elevation coincident with the uppermost part of the cylinder. Assume in both instances that atmospheric pressure occurs to the right of the cylinder.

▮ (*a*) The "net vertical projection" (see Fig. 5-37*a*) of the portion of the cylinder surface under consideration is $4 - (2 - 2\cos 30°)$, or 3.732 m. $F_H = pA = 35.0[(1)(3.732)] = 130.6$ kN (right). Note that the vertical force of the gas on surface *ab* is equal and opposite to that on surface *bc*. Hence, the "net horizontal projection" with regard to the gas is *ae* (see Fig. 5-38*b*), which is $2 \sin 30°$, or 1.000 m. $F_V = 35.0[(1)(1.000)] = 35.0$ kN (up).

(*b*)
$$F_H = \gamma \bar{h} A = (9.79)(3.732/2)[(1)(3.732)] = 68.2 \text{ kN} \quad \text{(right)}$$

$$F_V = \text{weight of crosshatched volume of water} \qquad \text{(Fig. 5-37}b\text{)}$$
$$= (9.79)(1)[(\tfrac{210}{360})(\pi)(4)^2/4 + (\tfrac{1}{2})(1.000)(3.732 - \tfrac{4}{2}) + (1)(\tfrac{4}{2})] = 99.8 \text{ kN} \quad \text{(up)}$$

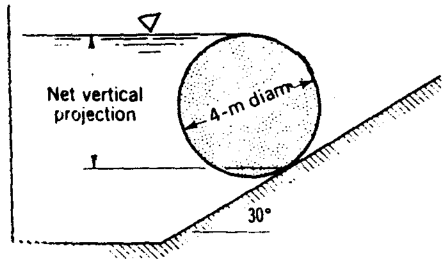



Fig. 5-37(a)

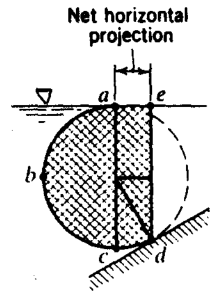



Fig. 5-37(b)

**5.39** A vertical-thrust bearing consists of an 8-in-radius bronze hemisphere mating into a steel hemispherical shell. At what pressure must grease be supplied to the bearing so that an unbroken film is present when the vertical thrust on the bearing is 600 000 lb?

▌ Projected area $= \pi r^2 = (\pi)(8)^2 = 201.1\ \text{in}^2$ $\quad p = F/A = 600\,000/201.1 = 2984\ \text{lb/in}^2$

**5.40** Find horizontal and vertical forces per foot of width on the Tainter gate shown in Fig. 5-38.

▌ $F_H = \gamma \bar{h} A = (62.4)[(0+25)/2][(25)(1)] = 19\,500$ lb. $F_H$ acts at $(\frac{2}{3})(25)$, or 16.67 ft below the water surface. $F_V$ = weight of imaginary water in $ACBA = (62.4)(1)[(\pi)(25)^2/5 - (2)(25\cos 36°)(25\sin 36°)/2] = 5959$ lb. $F_V$ acts through the centroid of segment $ABCA$.

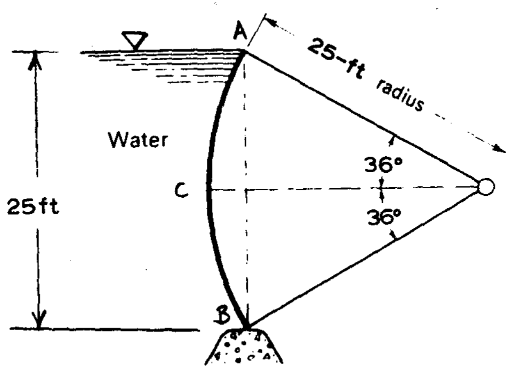



Fig. 5-38

**5.41** The tank indicated in cross section in Fig. 5-39 is 6 m long normal to the paper. Curved panel $MN$ is one-quarter of an ellipse with semiaxes $b$ and $d$. If $b = 5$ m, $d = 7$ m, and $a = 1.0$ m, calculate the horizontal and vertical components of force and the resultant force on the panel.

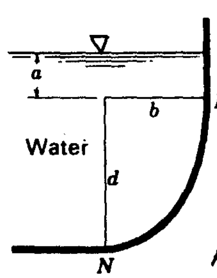
▌
$$F_H = \gamma h_{cg} A = 9.79(1.0 + \tfrac{7}{2})[(6)(7)] = 1850\ \text{kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = (1.0 + \tfrac{7}{2}) + \frac{(6)(7)^3/12}{(1.0+\tfrac{7}{2})[(6)(7)]} = 5.407\ \text{m below water surface}$$

$$F_V = \text{weight of water above surface } MN = (9.79)(6)[(\pi)(5)(7)/4 + (1.0)(5)] = 1908\ \text{kN}$$

$$x_{cp} = 4b/(3\pi) = (4)(5)/(3\pi) = 2.122\ \text{m to the right of } N \qquad F_{\text{resultant}} = \sqrt{1908^2 + 1850^2} = 2658\ \text{kN}$$

$F_{\text{resultant}}$ acts through the intersection of $F_H$ and $F_V$ at an angle of arctan (1908/1850), or 45.9°.




$\text{Area}_{\text{one quarter of elipse}} = \dfrac{\pi b d}{4}$

$\text{Area}_{\text{parabola}} = \dfrac{2bd}{3}$

Fig. 5-39

**5.42** Solve Prob. 5.41 if $a = 1.0$ ft, $b = 5$ ft, $d = 7$ ft, the tank is 6 ft long, and $MN$ represents a parabola with vertex at $N$.

▌

$$F_H = \gamma h_{cg} A = (62.4)(1.0 + \tfrac{7}{2})[(6)(7)] = 11\,794 \text{ lb}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = (1.0 + \tfrac{7}{2}) + \frac{(6)(7)^3/12}{(1.0 + \tfrac{7}{2})[(6)(7)]} = 5.407 \text{ ft below water surface}$$

$$F_V = \text{weight of water above surface } MN = (62.4)(6)[(\tfrac{2}{3})(7)(5) + (1.0)(5)] = 10\,608 \text{ lb}$$

$$x_{cp} = (\tfrac{3}{8})(b) = (\tfrac{3}{8})(5) = 1.88 \text{ ft to the right of } N \qquad F_{\text{resultant}} = \sqrt{10\,608^2 + 11\,794^2} = 15\,863 \text{ lb}$$

$F_{\text{resultant}}$ acts through the intersection of $F_H$ and $F_V$ at an angle of arctan (10 608/11 794), or 42.0°.

**5.43** In the cross section shown in Fig. 5-40, $BC$ is a quarter-circle. If the tank contains water to a depth of 6 ft, determine the magnitude and location of the horizontal and vertical components on wall $ABC$ per 1 ft width.

▌

$$F_H = \gamma \bar{h} A = (62.4)[(0 + 6)/2][(1)(6)] = 1123 \text{ lb} \qquad h_{cp} = (\tfrac{2}{3})(6) = 4.00 \text{ ft}$$

$$F_V = \text{weight of water above surface } BC = (62.4)(1)[(6)(5)] - (62.4)(1)[(\pi)(5)^2/4] = 1872 - 1225 = 647 \text{ lb}$$

The location of $F_V$ can be determined by taking moments about point $B$. $(1872)(\tfrac{5}{2}) - (1225)[(4)(5)/(3\pi)] = 647x_{cp}$, $x_{cp} = 3.22$ ft.

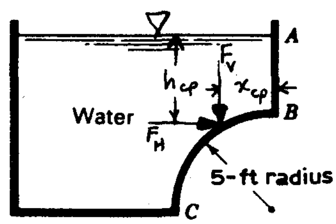



**Fig. 5-40**

**5.44** Rework Prob. 5.43 where the tank is closed and contains gas at a pressure of 10 psi.

▌

$$F_H = pA_V = [(10)(144)][(1)(6)] = 8640 \text{ lb} \qquad h_{cp} = \tfrac{6}{2} = 3.00 \text{ ft}$$

$$F_V = pA_H = [(10)(144)][(1)(5)] = 7200 \text{ lb} \qquad x_{cp} = \tfrac{5}{2} = 2.50 \text{ ft}$$

**5.45** A spherical steel tank of 22 m diameter contains gas under a pressure of 300 kPa. The tank consists of two half-spheres joined together with a weld. What will be the tensile force across the weld? If the steel is 25.0 mm thick, what is the tensile stress in the steel?

▌

$$F = pA = 300[(\pi)(22)^2/4] = 114\,040 \text{ kN} \qquad \sigma = \frac{\text{force/length}}{\text{thickness}} = \frac{114\,040/(22\pi)}{25.0/1000} = 66\,000 \text{ kPa}$$

**5.46** Determine the force required to hold the cone shown in Fig. 5-41*a* in position.

▌ Figure 5-41*b* shows the vertical projection above the opening. $p_{\text{air}} = 0.6 - [(0.83)(62.4)](5.1)/144 = -1.23$ psi, $F_{\text{air}} = [(1.23)(144)][(\pi)(0.804)^2] = 360$ lb, $F_{\text{cylinder}} = (62.4)(0.83)[(\pi)(0.804)^2(5.1 + 3)] = 852$ lb, $F_{\text{cone}} = (62.4)(0.83)[(3)(\pi)(0.804)^2/3] = 105$ lb; $\sum F_y = 0$, $360 - 852 + 105 + F = 0$, $F = 387$ lb.

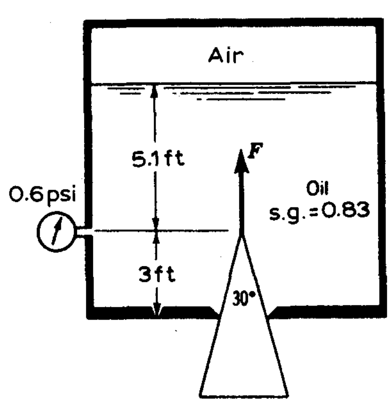



**Fig. 5-41(*a*)**

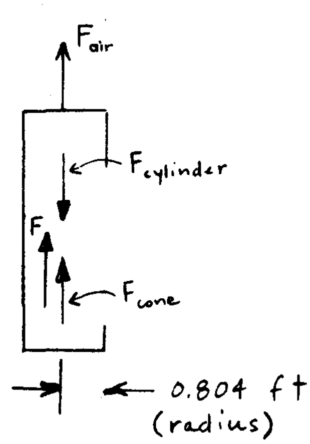



**Fig. 5-41(*b*)**

**5.47** The cross section of the gate in Fig. 5-42 is given by $10x = 3y^2$; its dimension normal to the plane of the paper is 7 m. The gate is pivoted about $O$. Find the horizontal and vertical forces and the clockwise moment acting on the gate if the water depth is 1.8 m.

▌

$$F_H = \gamma \bar{h} A = 9.79[(0+1.8)/2][(7)(1.8)] = 111.0\ \text{kN}$$

$$F_V = \text{weight of water above the gate} = \int_0^{1.8} (9.79)(7)(x\,dy) = (9.79)(7)\int_0^{1.8} 0.3y^2\,dy = (9.79)(7)\left[\frac{0.3y^3}{3}\right]_0^{1.8} = 40.0\ \text{kN}$$

$$M_O = (111.0)(\tfrac{1.8}{3}) + \int_0^{1.8} (9.79)(7)\left(\frac{x}{2}\right)(x\,dy) = 66.6 + (9.79)(7)\int_0^{1.8} \frac{(0.3y^2)^2}{2}\,dy$$

$$= 66.6 + (9.79)(7)\left[\frac{0.09y^5}{10}\right]_0^{1.8} = 78.3\ \text{kN}\cdot\text{m}$$

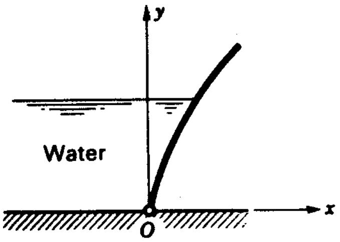



**Fig. 5-42**

**5.48** Find the wall thickness of steel pipe needed to resist the static pressure in a 36-in-diameter steel pipe carrying water under a head of 750 ft of water. Use an allowable working stress for steel pipe of 16 000 psi.

▌

$$p = \gamma h = (62.4)(750) = 46\,800\ \text{lb/ft}^2 \quad \text{or} \quad 325\ \text{lb/in}^2$$

$$T = pd/2 = (325)(36)/2 = 5850\ \text{lb/in of pipe length} \qquad t = 5850/16\,000 = 0.366\ \text{in}$$

**5.49** A vertical cylindrical tank is 6 ft in diameter and 10 ft high. Its sides are held in position by means of two steel hoops, one at the top and one at the bottom. The tank is filled with water up to 9 ft high. Determine the tensile stress in each hoop.

▌ See Fig. 5-43. $F = \gamma \bar{h} A = 62.4[(0+9)/2][(9)(6)] = 15\,163$ lb, $T = F/2 = 15\,163/2 = 7582$ lb; stress in top hoop $= (7582)(\tfrac{3}{10}) = 2275$ lb, stress in bottom hoop $= (7582)[(10-3)/10] = 5307$ lb.

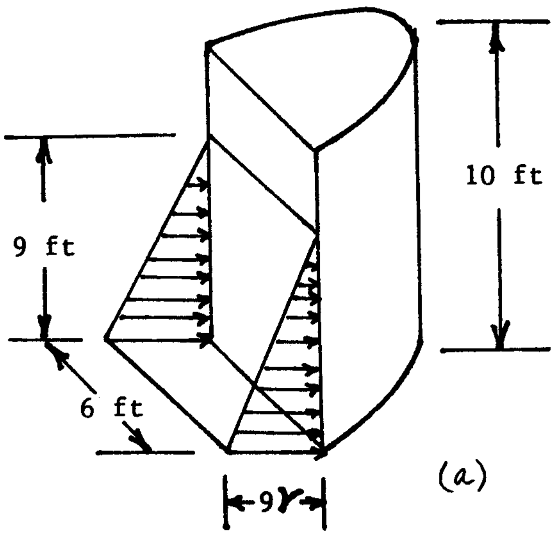



**Fig. 5-43(a)**

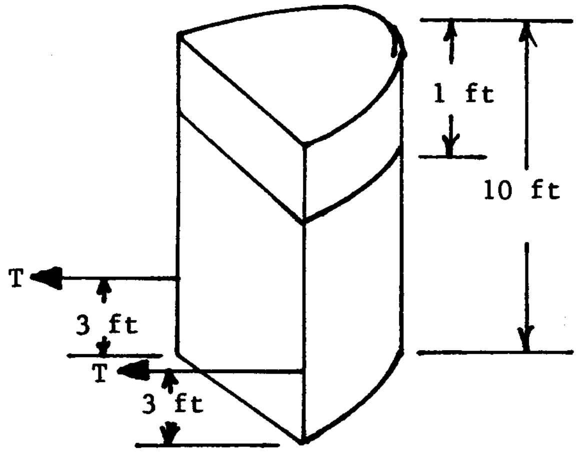



**Fig. 5-43(b)**

**5.50** A 48-in-diameter steel pipe, $\frac{1}{4}$ in thick, carries oil of s.g. = 0.822 under a head of 400 ft of oil. Compute the **(a)** stress in the steel and **(b)** thickness of steel required to carry a pressure of 250 psi with an allowable stress of 18 000 psi.

▮ $$p = \gamma h = [(0.822)(62.4)](400) = 20\,517\ \text{lb/ft}^2 \quad \text{or} \quad 142.5\ \text{lb/in}^2 \qquad \sigma = \frac{pr}{t}$$

**(a)** $$\sigma = \frac{(142.5)(48/2)}{\frac{1}{4}} = 13\,680\ \text{psi}$$

**(b)** $$18\,000 = \frac{(250)(48/2)}{t} \qquad t = 0.333\ \text{in}$$

**5.51** A wooden storage vat, 20 ft in outside diameter, is filled with 24 ft of brine, s.g. = 1.06. The wood staves are bound by flat steel bands, 2 in wide by $\frac{1}{4}$ in thick, whose allowable stress is 16 000 psi. What is the spacing of the bands near the bottom of the vat, neglecting any initial stress? Refer to Fig. 5-44.

▮ Force $P$ represents the sum of all the horizontal components of small forces $dP$ acting on length $y$ of the vat, and forces $T$ represent the total tension carried in a band loaded by the same length $y$.

$$\sum F_x = 0 \qquad 2T - P = 0 \qquad T = A_{\text{steel}}\sigma_{\text{steel}} = [(2)(\tfrac{1}{4})](16\,000) = 8000\ \text{lb}$$

$$p = \gamma hA = [(1.06)(62.4)](24)(20y) = 31\,749y \qquad (2)(8000) - 31\,749y = 0 \qquad y = 0.504\ \text{ft} \quad \text{or} \quad 6.05\ \text{in}$$

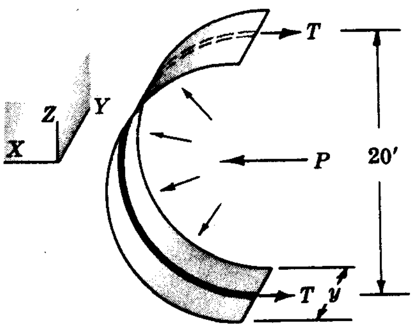



Fig. 5-44

**5.52** A 4.0-in-ID steel pipe has a $\frac{1}{4}$-in wall thickness. For an allowable tensile stress of 10 000 psi, what is the maximum pressure?

▮ $$\sigma = \frac{pr}{t} \qquad 10\,000 = \frac{(p)(4.0/2)}{\frac{1}{4}} \qquad p = 1250\ \text{lb/in}^2$$

**5.53** A thin-walled hollow sphere 3.5 m in diameter holds gas at 1700 kPa. For an allowable stress of 50 000 kPa, determine the minimum wall thickness.

▮ Considering half a sphere of diameter $d$ (3.5 m) and thickness $t$, $(\pi dt)(\sigma) = (p)(\pi d^2/4)$, $[(\pi)(3.5)(t)](50\,000) = 1700[(\pi)(3.5)^2/4]$, $t = 0.02975$ m, or 29.75 mm.

**5.54** A cylindrical container 8 ft high and 3 ft in diameter is reinforced with two hoops a foot from each end. When it is filled with water, what is the tension in each hoop due to the water?

▮ See Fig. 5-45. $F = \gamma \bar{h} A = 62.4[(0+8)/2][(8)(3)] = 5990$ lb. $F$ acts at $(\frac{2}{3})(8)$, or 5.333 ft from the top of the container.

$$\sum F_x = 0$$

$$2T_1 + 2T_2 - 5990 = 0 \tag{1}$$

$$\sum M_A = 0 \qquad (2T_2)(1.667) - (2T_1)(4.333) = 0$$

$$T_2 = 2.60T_1 \tag{2}$$

Solve simultaneous equations (1) and (2). $2T_1 + (2)(2.60T_1) - 5990 = 0$, $T_1 = 832$ lb, $T_2 = (2.60)(832) = 2163$ lb.

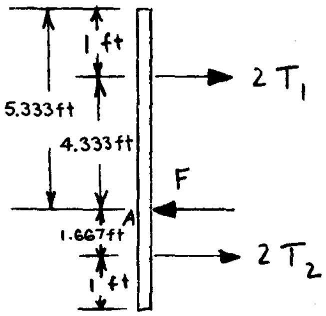



Fig. 5-45

**5.55** A 30-mm-diameter steel ($\gamma = 77.0\ \text{kN/m}^3$) ball covers a 15-mm-diameter hole in the bottom of a chamber in which gas pressure is 27 000 kPa. What force is required to push the ball up into the chamber?

$$F = pA + \text{weight of ball} = 27\,000[(\pi)(\tfrac{15}{1000})^2/4] + [(\tfrac{4}{3})(\pi)(\tfrac{30}{1000})^3](77.0) = 4.780\ \text{kN}$$