CHAPTER 14
Flow in Open Channels

14.1 Water flows in a rectangular, concrete, open channel that is 12.0 m wide at a depth of 2.5 m. The channel slope is 0.0028. Find the water velocity and the flow rate.

$$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad n = 0.013 \qquad \text{(from Table A-13)}$$
$$R = A/p_w = (12.0)(2.5)/(2.5 + 12.0 + 2.5) = 1.765 \text{ m}$$
$$v = (1.0/0.013)(1.765)^{2/3}(0.0028)^{1/2} = 5.945 \text{ m/s} \qquad Q = Av = [(12.0)(2.5)](5.945) = 178 \text{ m}^3/\text{s}$$

14.2 Water flows in the symmetrical trapezoidal channel lined with asphalt shown in Fig. 14-1. The channel bottom drops 0.1 ft vertically for every 100 ft of length. What are the water velocity and flow rate?

$$v = (1.486/n)(R^{2/3})(s^{1/2}) \qquad n = 0.015 \qquad \text{(from Table A-13)}$$
$$R = A/p_w \qquad A = (16.0)(4.5) + (2)\{(4.5)[(3)(4.5)]/2\} = 132.8 \text{ ft}^2$$
$$p_w = 16.0 + 2\sqrt{(4.5)^2 + [(3)(4.5)]^2} = 44.46 \text{ ft} \qquad R = 132.8/44.46 = 2.987 \text{ ft}$$
$$s = 0.1/100 = 0.00100 \qquad v = (1.486/0.015)(2.987)^{2/3}(0.00100)^{1/2} = 6.498 \text{ ft/s}$$
$$Q = Av = (132.8)(6.498) = 863 \text{ ft}^3/\text{s}$$

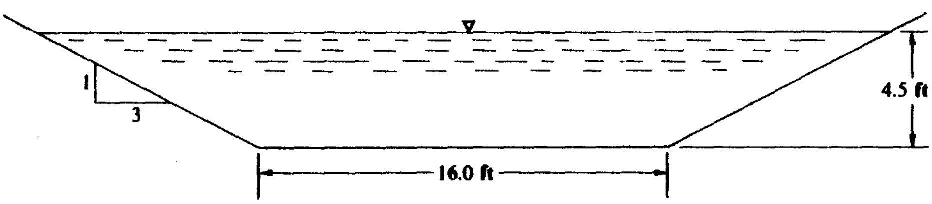

Fig. 14-1

14.3 Water is to flow at a rate of 30 m³/s in the concrete channel shown in Fig. 14-2. Find the required vertical drop of the channel bottom per kilometer of length.

$$A = (3.6)(2.0) + (4.0 - 2.0)[(1.6 + 3.6)/2] = 12.40 \text{ m}^2 \qquad v = (1.0/n)(R^{2/3})(s^{1/2}) = Q/A = 30/12.40 = 2.419 \text{ m/s}$$
$$p_w = 3.6 + 2.0 + \sqrt{(4.0 - 2.0)^2 + (3.6 - 1.6)^2} + 1.6 = 10.03 \text{ m} \qquad R = A/p_w = 12.40/10.03 = 1.236 \text{ m}$$
$$2.419 = (1.0/0.013)(1.236)^{2/3}(s)^{1/2} \qquad s = 0.000746$$

This slope represents a drop of the channel bottom of 0.000746 m per meter of length, or 0.746 m per kilometer of length.

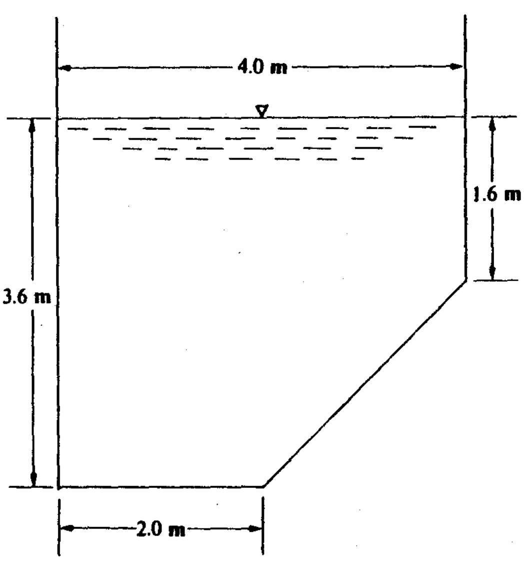

Fig. 14-2

14.4 Water flows in the triangular steel channel shown in Fig. 14-3 at a velocity of 2.9 ft/s. Find the depth of flow if the channel slope is 0.0015.

▌ $v = (1.486/n)(R^{2/3})(s^{1/2}) \qquad R = A/p_w = 2\{[(d)(d \tan 27.5°)/2]/(2d/\cos 27.5°)\} = 0.2309d$

$$2.9 = (1.486/0.014)(0.2309d)^{2/3}(0.0015)^{1/2} \qquad d = 2.57 \text{ ft}$$

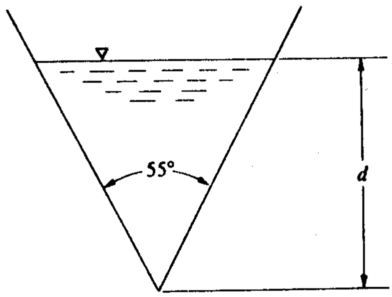

Fig. 14-3

14.5 After a flood had passed an observation station on a river, an engineer visited the site and, by locating flood marks, performing appropriate surveying, and doing necessary computations, determined that the cross-sectional area, wetted perimeter, and water-surface slope at the time of the peak flooding were 2960 m^2, 341 m, and 0.00076, respectively. The engineer also noted that the channel bottom was "earth with grass and weeds" ($n = 0.030$). Estimate the peak flood discharge.

▌ $$v = (1.0/n)(R^{2/3})(s^{1/2}) = (1.0/0.030)(2960/341)^{2/3}(0.00076)^{1/2} = 3.881 \text{ m/s}$$

$$Q = Av = (2960)(3.881) = 11\,490 \text{ m}^3/\text{s}$$

14.6 A rectangular, concrete channel 50 ft wide is to carry water at a flow rate of 800 cfs. The channel slope is 0.00025. Find the depth of flow.

▌ $v = (1.486/n)(R^{2/3})(s^{1/2}) = Q/A = 800/50d = 16.00/d \qquad R = A/p_w = 50d/(50 + 2d)$

$$16.00/d = (1.486/0.013)[50d/(50 + 2d)]^{2/3}(0.00025)^{1/2}$$

This equation is not readily solvable, but a trial-and-error solution (not shown here) reveals that $d = 3.92$ ft.

14.7 Prepare a computer program that will determine the depth of flow of water in a rectangular channel (as in Prob. 14.6).

▌
```
C     THIS PROGRAM DETERMINES THE DEPTH OF FLOW FOR OPEN CHANNEL FLOW
C     IN RECTANGULAR SECTIONS.  IT CAN BE USED FOR PROBLEMS IN BOTH THE
C     ENGLISH SYSTEM OF UNITS AND THE INTERNATIONAL SYSTEM OF UNITS.
C
C     INPUT DATA MUST BE SET UP AS FOLLOWS.
C
C     CARD 1   COLUMN 1        ENTER 0 (ZERO) OR BLANK IF ENGLISH SYSTEM
C                              OF UNITS IS TO BE USED.  ENTER 1 (ONE) IF
C                              INTERNATIONAL SYSTEM OF UNITS IS TO BE
C                              USED.
C              COLUMNS 2-79    ENTER TITLE, DATE, AND OTHER INFORMATION,
C                              IF DESIRED.
C     CARD 2   COLUMNS 1-10    ENTER NUMBER INCLUDING DECIMAL GIVING
C                              WIDTH OF CHANNEL (IN FEET OR METERS).
C              COLUMNS 11-20   ENTER NUMBER INCLUDING DECIMAL GIVING
C                              FLOW RATE (IN CUBIC FEET PER SECOND OR
C                              CUBIC METERS PER SECOND).
C              COLUMNS 21-30   ENTER NUMBER INCLUDING DECIMAL GIVING
C                              SLOPE.
C              COLUMNS 31-40   ENTER NUMBER INCLUDING DECIMAL GIVING
C                              MANNING N-VALUE.
C
C     MULTIPLE DATA SETS FOR SOLVING ANY NUMBER OF PROBLEMS MAY BE
C     INCLUDED FOR PROCESSING.
C
      DIMENSION TITLE(13)
      REAL N
      INTEGER UNITS
1     READ(5,100,END=2)UNITS,TITLE
```

```
100 FORMAT(I1,13A6)
    WRITE(6,105)TITLE
105 FORMAT('1',13A6,////)
    COEFF=1.486
    IF(UNITS.EQ.1)COEFF=1.0
    READ(5,101)W,Q,S,N
101 FORMAT(4F10.0)
    D=0.001
    TRY1=COEFF/N*(W*D/(W+2.0*D))**(2.0/3.0)*SQRT(S)-Q/W/D
104 D=D+0.001
    TRY2=COEFF/N*(W*D/(W+2.0*D))**(2.0/3.0)*SQRT(S)-Q/W/D
    IF(TRY1*TRY2)102,102,103
103 TRY1=TRY2
    GO TO 104
102 D=D-0.0005
    IF(UNITS.EQ.0)WRITE(6,106)W,Q,S,N,D
106 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A RECTANGULAR SE
   *CTION',//5X,'WIDTH =',F7.1,' FT',//5X,'FLOW RATE =',F7.1,' CU FT/S
   *',//5X,'SLOPE =',F10.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE D
   *EPTH OF FLOW WILL BE',F7.2,' FT')
    IF(UNITS.EQ.1)WRITE(6,107)W,Q,S,N,D
107 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A RECTANGULAR SE
   *CTION',//5X,'WIDTH =',F7.1,' M ',//5X,'FLOW RATE =',F7.1,' CU M/S
   *',//5X,'SLOPE =',F10.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE D
   *EPTH OF FLOW WILL BE',F7.2,' M')
    GO TO 1
2   STOP
    END
```

14.8 Solve Prob. 14.6 using the computer program of Prob. 14.7.

Input

```
0SAMPLE ANALYSIS ØF ØPEN CHANNEL FLØW IN A RECTANGULAR SECTIØN
50.0      800.0     0.00025   0.013
```

Output

```
SAMPLE ANALYSIS OF OPEN CHANNEL FLOW IN A RECTANGULAR SECTION


GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A RECTANGULAR SECTION

    WIDTH =   50.0 FT

    FLOW RATE =  800.0 CU FT/S

    SLOPE = 0.0002500

    MANNING N-VALUE = 0.013



THE DEPTH OF FLOW WILL BE   3.92 FT
```

14.9 A rectangular channel ($n = 0.016$) 20 m wide is to carry water at a flow rate of 30 m^3/s at a slope of 0.00032. Find the depth of flow using the computer program of Prob. 14.7.

Input

```
1SAMPLE ANALYSIS ØF ØPEN CHANNEL FLØW IN A RECTANGULAR SECTIØN
20.0      30.0      0.00032   0.016
```

Output

```
SAMPLE ANALYSIS OF OPEN CHANNEL FLOW IN A RECTANGULAR SECTION



GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A RECTANGULAR SECTION

     WIDTH =   20.0 M

     FLOW RATE =    30.0 CU M/S

     SLOPE = 0.0003200

     MANNING N-VALUE = 0.016


 THE DEPTH OF FLOW WILL BE  1.25 M
```

14.10 A corrugated metal pipe of 500 mm diameter flows half-full at a slope of 0.0050 (see Fig. 14-4). What is the flow rate for this condition?

$$v = (1.0/n)(R^{2/3})(s^{1/2}) = (1.0/0.024)[(\tfrac{500}{1000})/4]^{2/3}(0.0050)^{1/2} = 0.7366 \text{ m/s}$$
$$Q = Av = \{[(\pi)(\tfrac{500}{1000})^2/4]/2\}(0.7366) = 0.0723 \text{ m}^3/\text{s}$$

(*Note:* The hydraulic radius for both a circular cross section and a semicircular cross section is one-fourth the diameter.)

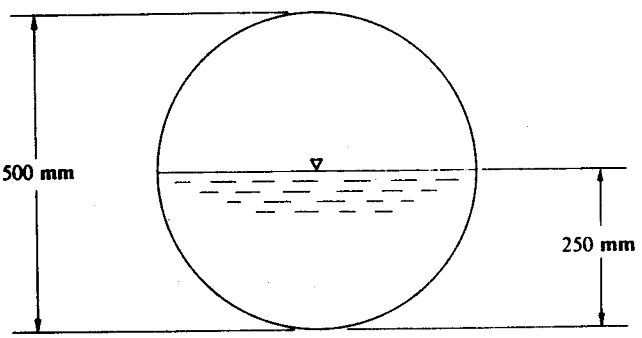

Fig. 14-4

14.11 A 24-in-diameter cast iron pipe on a $\frac{1}{400}$ slope carries water at a depth of 5.6 in, as shown in Fig. 14-5*a*. What is the flow rate?

$v = (1.486/n)(R^{2/3})(s^{1/2})$, $R = A/p_w$. The applicable area in this problem is the shaded area (*AECDA*) in Fig. 14-5*b*: $AB = BC = 12$ in (both are radii), $BE = 12 - 5.6 = 6.4$ in. Therefore, $AE = EC = \sqrt{(12)^2 - (6.4)^2} = 10.15$ in and $\measuredangle ABE = \measuredangle EBC = \arccos(6.4/12) = 57.77°$, $(\text{Area})_{ABCDA} = [(\pi)(24)^2/4][(2)(57.77°)/360°] = 145.19 \text{ in}^2$, $(\text{Area})_{ABEA} = (\text{Area})_{BCEB} = (6.4)(10.15)/2 = 32.48 \text{ in}^2$, $(\text{Area})_{AECDA} = (\text{Area})_{ABCDA} - (2)(\text{Area})_{ABEA} = 145.19 - (2)(32.48) = 80.23 \text{ in}^2$. The applicable wetted perimeter in this problem is the arc distance *ADC* in Fig. 14-5: $p_w = ADC = (\pi)(24)[(2)(57.77°)/360°] = 24.20$ in. $R = 80.23/24.20 = 3.315$ in, or 0.2763 ft, $v = (1.486/0.012)(0.2763)^{2/3}(\frac{1}{400})^{1/2} = 2.627$ ft/s, $Q = Av = (80.23/144)(2.627) = 1.46 \text{ ft}^3/\text{s}$.

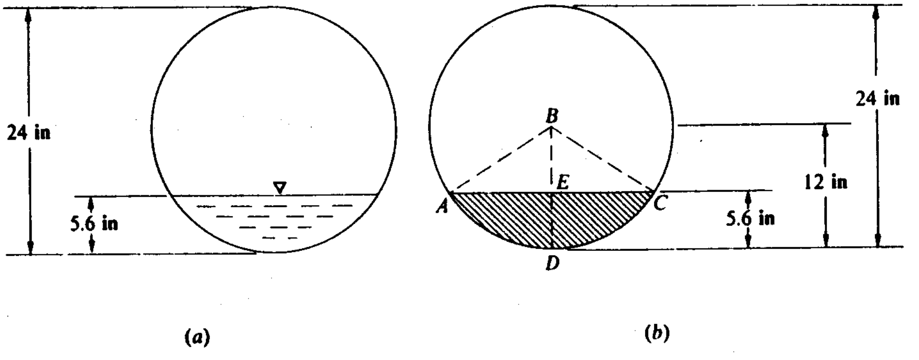

Fig. 14-5

14.12 A 500-mm-diameter concrete pipe on a $\frac{1}{500}$ slope is to carry water at a flow rate of $0.040\ \text{m}^3/\text{s}$. Find the depth of flow. See Fig. 14-6*a*.

▮ $v = (1.0/n)(R^{2/3})(s^{1/2}) = Q/A = 0.040/A$ $\qquad 0.040/A = (1.0/0.013)(R^{2/3})(\frac{1}{500})^{1/2}$ $\qquad AR^{2/3} = 0.01163$

Since $R = A/p_w$,

$$A^{5/3}/p_w^{2/3} = 0.01163 \tag{1}$$

Equation (*1*) contains two unknowns, A and p_w; however, both unknowns can be expressed in terms of the unknown depth of flow, d. The applicable area in this problem is the shaded area ($AECDA$) in Fig. 14-6*b*: $AB = BC = 0.25$ m (both are radii), $BE = 0.25 - d$. Therefore, $AE = CE = \sqrt{(0.25)^2 - (0.25-d)^2}$, $\measuredangle ABE = \measuredangle EBC = \arccos[(0.25-d)/0.25]$,

$$(\text{Area})_{ABCDA} = \left[\frac{(\pi)(0.50)^2}{4}\right]\left\{\frac{(2)\arccos[(0.25-d)/0.25]}{360°}\right\} = (0.001091)\left(\arccos\frac{0.25-d}{0.25}\right)$$

$$(\text{Area})_{ABEA} = (\text{Area})_{BCEB} = \frac{(0.25-d)\sqrt{(0.25)^2-(0.25-d)^2}}{2}$$

$$(\text{Area})_{AECDA} = (\text{Area})_{ABCDA} - 2(\text{Area})_{ABEA} = (0.001091)\left(\arccos\frac{0.25-d}{0.25}\right) - (2)\left[\frac{(0.25-d)\sqrt{(0.25)^2-(0.25-d)^2}}{2}\right]$$

$$p_w = ADC = (\pi)(0.50)\left\{\frac{(2)\arccos[(0.25-d)/0.25]}{360°}\right\} = (0.008727)\left(\arccos\frac{0.25-d}{0.25}\right)$$

Therefore, substituting into Eq. (*1*),

$$\frac{[(0.001091)\{\arccos[(0.25-d)/0.25]\} - (0.25-d)\sqrt{(0.25)^2-(0.25-d)^2}]^{5/3}}{[(0.008727)\{\arccos[(0.25-d)/0.25]\}]^{2/3}} = 0.01163\ \frac{\text{m}^3}{\text{s}}$$

This equation is not readily solvable, but a trial-and-error solution (not shown here) reveals that $d = 0.166$ m or 166 mm.

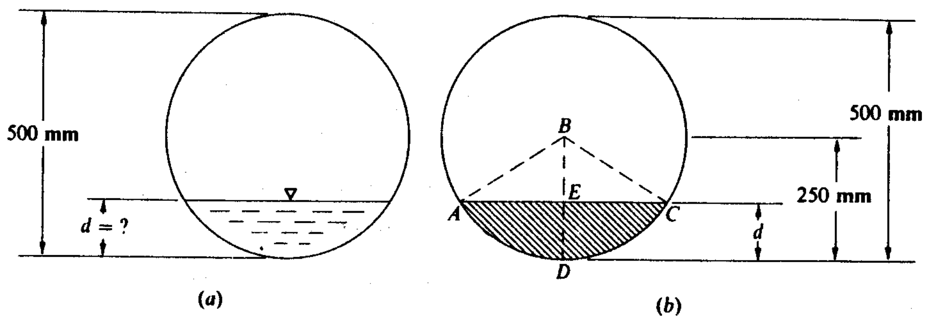

Fig. 14-6

14.13 Solve Prob. 14.11 utilizing Fig. A-18.

▮ $s = \frac{1}{400} = 0.0025$ ft/ft. From Fig. A-15, $Q_{\text{full}} = 11.4\ \text{ft}^3/\text{s}$ and $v_{\text{full}} = 3.6$ ft/s. These values of Q_{full} and v_{full} must be adjusted for an n value of 0.012 for the given cast iron pipe (Fig. A-15 is based on an n value of 0.013): $(Q_{\text{full}})_{n=0.012}/11.4 = 0.013/0.012$, $(Q_{\text{full}})_{n=0.012} = 12.4\ \text{ft}^3/\text{s}$, $(v_{\text{full}})_{n=0.012}/3.6 = 0.013/0.012$, $(v_{\text{full}})_{n=0.012} = 3.9$ ft/s, $d/d_{\text{full}} = 5.6/24 = 0.23$, or 23 percent. Enter the ordinate of Fig. A-18 with a value of d/d_{full} of 23 percent, move horizontally to the line marked "discharge," and then vertically downward to the abscissa to read $Q/Q_{\text{full}} = 12$ percent. In a similar manner using the "velocity" line, read $v/v_{\text{full}} = 63$ percent. Therefore, $Q = (0.12)(12.4) = 1.5\ \text{ft}^3/\text{s}$, $v = (0.63)(3.9) = 2.5$ ft/s.

14.14 Solve Prob. 14.12 utilizing Fig. A-18.

▮ $s = \frac{1}{500} = 0.00200$. From Fig. A-16, $Q_{\text{full}} = 0.169\ \text{m}^3/\text{s}$: $Q/Q_{\text{full}} = 0.040/0.169 = 0.24$, or 24 percent. From Fig. A-18, $d/d_{\text{full}} = 34$ percent; $d = (0.34)(500) = 170$ mm.

14.15 A 30-in-diameter concrete storm sewer pipe must carry a flow rate of 9.0 cfs at a minimum velocity of 2.5 ft/s. Find the required slope and water depth.

▮
$$A = Q/v = 9.0/2.5 = 3.600\ \text{ft}^2 \qquad A_{\text{full}} = [(\pi)(\tfrac{30}{12})^2/4] = 4.909\ \text{ft}^2$$

$$A/A_{\text{full}} = 3.600/4.909 = 0.73 \quad \text{or} \quad 73\ \text{percent}$$

From Fig. A-18, $d/d_{full} = 69$ percent and $R/R_{full} = 116$ percent.

$$d = (0.69)(30) = 20.7 \text{ in} \qquad v = (1.486/n)(R^{2/3})(s^{1/2})$$

$$2.5 = (1.486/0.013)\{(1.16)[(\tfrac{30}{12})/4]\}^{2/3}(s^{1/2}) \qquad s = 0.000734$$

14.16 A concrete pipe must carry water at a slope of 0.0075, at a velocity of 0.76 m/s, and at a depth of flow equal to one-tenth its diameter. What is the required pipe diameter?

$$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad 0.76 = (1.0/0.013)(R^{2/3})(0.0075)^{1/2} \qquad R = 0.03853 \text{ ft}$$

From Fig. A-18 with $d/d_{full} = 10$ percent, $R/R_{full} = 25$ percent.

$$0.03853/R_{full} = 0.25 \qquad R_{full} = 0.1541 \text{ m} \qquad 0.1541 = d/4 \qquad d = 0.616 \text{ m} \quad \text{or} \quad 616 \text{ mm}$$

14.17 Prepare a computer program that will determine either the depth of flow or the flow rate for open channel flow in circular sections.

```
C     THIS PROGRAM DETERMINES EITHER THE DEPTH OF FLOW OR THE FLOW RATE
C     FOR OPEN CHANNEL FLOW IN CIRCULAR SECTIONS.  IT CAN BE USED FOR
C     PROBLEMS IN BOTH THE ENGLISH SYSTEM OF UNITS AND THE INTERNATIONAL
C     SYSTEM OF UNITS.
C
C     INPUT DATA MUST BE SET UP AS FOLLOWS.
C
C     CARD 1   COLUMN 1          ENTER 0 (ZERO) OR BLANK IF ENGLISH SYSTEM
C                                OF UNITS IS TO BE USED.  ENTER 1 (ONE) IF
C                                INTERNATIONAL SYSTEM OF UNITS IS TO BE
C                                USED.
C              COLUMNS 2-79      ENTER TITLE, DATE, AND OTHER INFORMATION,
C                                IF DESIRED.
C     CARD 2   COLUMNS 1-10      ENTER NUMBER INCLUDING DECIMAL GIVING
C                                DIAMETER OF CHANNEL (IN INCHES OR MILLI-
C                                METERS).
C              COLUMNS 11-20     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                DEPTH OF FLOW (IN INCHES OR MILLIMETERS).
C              COLUMNS 21-30     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                SLOPE.
C              COLUMNS 31-40     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                MANNING N-VALUE.
C              COLUMNS 41-50     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                FLOW RATE (IN CUBIC FEET PER SECOND OR
C                                CUBIC METERS PER SECOND).
C
C *********************************************************************
C *                                                                   *
C *   NOTE WELL....EITHER THE DEPTH OF FLOW (COLUMNS 11-20) OR THE    *
C *   FLOW RATE (COLUMNS 41-50), WHICHEVER ONE IS TO BE DETERMINED BY *
C *   THIS PROGRAM, SHOULD BE LEFT BLANK ON CARD 2.                   *
C *                                                                   *
C *********************************************************************
C
C     MULTIPLE DATA SETS FOR SOLVING ANY NUMBER OF PROBLEMS MAY BE
C     INCLUDED FOR PROCESSING.
C
      DIMENSION TITLE(13)
      COMMON D,R,D1,DIAM,PI,FACTOR,AREA,WP
      REAL N
      INTEGER UNITS
      PI=3.14159265
    1 READ(5,100,END=2)UNITS,TITLE
  100 FORMAT(I1,13A6)
      WRITE(6,105)TITLE
  105 FORMAT('1',13A6,////)
      COEFF=1.486
      FACTOR=12.0
      IF(UNITS.EQ.1)COEFF=1.0
      IF(UNITS.EQ.1)FACTOR=1000.0
      READ(5,101)DIAM,D,S,N,Q
  101 FORMAT(5F10.0)
      R=DIAM/2.0
      IF(Q.GT.0.0001)GO TO 102
      D1=D
```

```
      CALL AREAWP
      HR=AREA/WP
      Q=AREA*COEFF/N*HR**(2.0/3.0)*SQRT(S)
      IF(UNITS.EQ.0)WRITE(6,103)DIAM,D,S,N,Q
  103 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTI
     *ON'//5X,'DIAMETER =',F7.2,' IN',//5X,'DEPTH OF FLOW =',F7.2,' IN',
     *//5X,'SLOPE =',F10.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE FLO
     *W RATE WILL BE',F8.3,' CU FT/S')
      IF(UNITS.EQ.1)WRITE(6,104)DIAM,D,S,N,Q
  104 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTI
     *ON'//5X,'DIAMETER =',F7.1,' MM',//5X,'DEPTH OF FLOW =',F7.1,' MM',
     *//5X,'SLOPE =',F10.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE FLO
     *W RATE WILL BE',F8.3,' CU M/S')
      GO TO 1
  102 AWP=Q*N/COEFF/SQRT(S)
      D=0.01
      D1=D
      CALL AREAWP
      TRY1=AREA**(5.0/3.0)/WP**(2.0/3.0)-AWP
  108 D=D+0.01
      IF(D.GT.DIAM)GO TO 112
      D1=D
      CALL AREAWP
      TRY2=AREA**(5.0/3.0)/WP**(2.0/3.0)-AWP
      IF(TRY1*TRY2)106,106,107
  107 TRY1=TRY2
      GO TO 108
  106 D=D-0.005
      IF(UNITS.EQ.0)WRITE(6,109)DIAM,Q,S,N,D
  109 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTI
     *ON',//5X,'DIAMETER =',F7.2,' IN',//5X,'FLOW RATE =',F8.3,' CU FT/S
     *',//5X,'SLOPE =',F13.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE D
     *EPTH OF FLOW WILL BE',F7.2,' IN').
      IF(UNITS.EQ.1)WRITE(6,110)DIAM,Q,S,N,D
  110 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTI
     *ON',//5X,'DIAMETER =',F7.1,' MM',//5X,'FLOW RATE =',F8.3,' CU M/S
     *',//5X,'SLOPE =',F13.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE D
     *EPTH OF FLOW WILL BE',F7.1,' MM')
      GO TO 1
  112 WRITE(6,116)
  116 FORMAT(1X,'THIS CIRCULAR CONDUIT CANNOT CARRY THIS GREAT A FLOW AS
     * OPEN CHANNEL FLOW.')
      GO TO 1
    2 STOP
      END
      SUBROUTINE AREAWP
      COMMON D,R,D1,DIAM,PI,FACTOR,AREA,WP
      IF(D.GT.R)D1=DIAM-D
      ABCDA=DIAM**2/4.0*ARCOS((R-D1)/R)
      ABEA=(R-D1)*SQRT(R**2-(R-D1)**2)/2.0
      AREA=ABCDA-2.0*ABEA
      WP=DIAM*ARCOS((R-D1)/R)
      IF(D.GT.R)AREA=PI*DIAM**2/4.0-AREA
      IF(D.GT.R)WP=PI*DIAM-WP
      AREA=AREA/FACTOR**2
      WP=WP/FACTOR
      RETURN
      END
```

14.18 Solve Prob. 14.11 utilizing the computer program of Prob. 14.17.

▮ Input

```
OSAMPLE ANALYSIS ØF AN ØPEN CHANNEL FLØW IN A CIRCULAR SECTIØN
24.0        5.6         0.0025     0.012
```

Output

```
SAMPLE ANALYSIS OF AN OPEN CHANNEL FLOW IN A CIRCULAR SECTION



GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTION

     DIAMETER =  24.00 IN

     DEPTH OF FLOW =   5.60 IN

     SLOPE = 0.0025000

     MANNING N-VALUE = 0.012



THE FLOW RATE WILL BE   1.463 CU FT/S
```

14.19 Solve Prob. 14.12 utilizing the computer program of Prob. 14.17.

Input

```
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
1SAMPLE ANALYSIS ØF AN ØPEN CHANNEL FLØW IN A CIRCULAR SECTIØN
500.                0.002      0.013      0.040
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Output

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SAMPLE ANALYSIS OF AN OPEN CHANNEL FLOW IN A CIRCULAR SECTION


GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTION

     DIAMETER =  500.0 MM

     FLOW RATE =   0.040 CU M/S

     SLOPE =    0.0020000

     MANNING N-VALUE = 0.013


THE DEPTH OF FLOW WILL BE  165.7 MM
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14.20 An open channel is to be designed to carry 1.0 m^3/s at a slope of 0.0065. The channel material has an n value of 0.011. Find the most efficient cross section for a semicircular section.

$$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad Q/A = (1.0/n)(A/p_w)^{2/3}(s^{1/2})$$
$$A^{5/3}/p_w^{2/3} = Qn/s^{1/2} = (1.0)(0.011)/0.0065^{1/2} = 0.1364$$
$$A = \pi d^2/8 \qquad p_w = \pi d/2 \qquad (\pi d^2/8)^{5/3}/(\pi d/2)^{2/3} = 0.1364 \qquad d = 0.951 \text{ m} \quad \text{or} \quad 951 \text{ mm}$$

(d is the diameter of the semicircular section; the depth of flow would, of course, be half of d.)

14.21 Find the most efficient cross section for Prob. 14.20 for a rectangular section.

$A^{5/3}/p_w^{2/3} = 0.1364$ (from Prob. 14.20). The most efficient rectangular section has a width equal to twice its depth. Letting d = depth, $A = (d)(2d) = 2d^2$, $p_w = d + 2d + d = 4d$, $(2d^2)^{5/3}/(4d)^{2/3} = 0.1364$, $d = 0.434$ m, or 434 mm; width $= 2d = (2)(434) = 868$ mm.

14.22 Find the most efficient cross section for Prob. 14.20 for a triangular section.

$A^{5/3}/p_w^{2/3} = 0.1364$ (from Prob. 14.20). The most efficient triangular section has a 90° angle and 1:1 side slopes (see Fig. 14-7). $A = (\frac{1}{2})(d\sqrt{2})(d\sqrt{2}) = d^2$, $p_w = (2)(d\sqrt{2}) = 2.828d$, $(d^2)^{5/3}/(2.828d)^{2/3} = 0.1364$, $d = 0.614$ m, or 614 mm; sides $= d\sqrt{2} = (614)(\sqrt{2}) = 868$ mm.

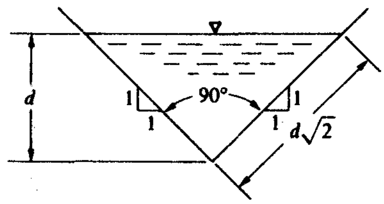


Fig. 14-7

14.23 Find the most efficient cross section for Prob. 14.20 for a trapezoidal section.

▮ $A^{5/3}/p^{2/3} = 0.1364$ (from Prob. 14.20). The most efficient trapezoidal section is half a regular hexagon (see Fig. 14-8). $A = (1.155d)(d) + (2)[(d)(d \tan 30°)/2] = 1.732d^2$, $p_w = (3)(1.155d) = 3.465d$, $(1.732d^2)^{5/3}/(3.465d)^{2/3} = 0.1364$, $d = 0.459$ m; sides and bottom: each $= 1.155d = (1.155)(0.459) = 0.530$ m.

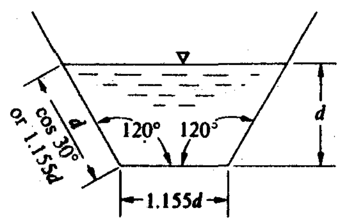


Fig. 14-8

14.24 For the same conditions given in Prob. 14.2, determine the status of flow (i.e., is it critical, subcritical, or supercritical?).

▮ $N_F = v/\sqrt{gd_m}$ $\quad v = 6.498$ ft/s (from Prob. 14.2) $\quad d_m = A/B \quad A = 132.8\ \text{ft}^2$ (from Prob. 14.2)

$$B = (3)(4.5) + 16.0 + (3)(4.5) = 43.0\ \text{ft} \qquad d_m = 132.8/43.0 = 3.088\ \text{ft}$$

$$N_F = 6.498/\sqrt{(32.2)(3.088)} = 0.652$$

Since $N_F < 1.0$, the flow is subcritical.

14.25 The triangular channel ($n = 0.012$) shown in Fig. 14-9 is to carry water at a flow rate of 10 m³/s. Find the critical depth, critical velocity, and critical slope of the channel.

▮ $B/A^3 = g/Q^2 \qquad B = 6d_c \qquad A = 2[(d_c)(3d_c)/2] = 3d_c^2 \qquad 6d_c/(3d_c^2)^3 = 9.807/10^2 \qquad d_c = 1.178$ m

$$v_c = Q/A = 10/[(3)(1.178)^2] = 2.402\ \text{m/s} \qquad s_c = \{nv_c/[(1.0)(R_c^{2/3})]\}^2 \qquad R = A/p_w$$

$$R_c = [(3)(1.178)^2]/[(2)(\sqrt{10})(1.178)] = 0.5588\ \text{m} \qquad s_c = \{(0.012)(2.402)/[(1.0)(0.5588)^{2/3}]\}^2 = 0.00181$$

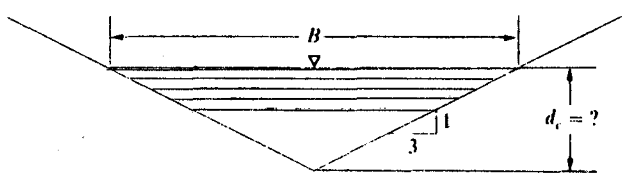


Fig. 14-9

14.26 The semicircular channel ($n = 0.010$) shown in Fig. 14-10 is to carry water at a depth of 1.0 ft. Find the velocity, slope, and discharge at the critical stage.

▮
$$d_m = A/B = [(\tfrac{1}{2})(\pi)(2.0)^2/4]/2.0 = 0.7854\ \text{ft} \qquad v_c = \sqrt{gd_m} = \sqrt{(32.2)(0.7854)} = 5.029\ \text{ft/s}$$

$$s_c = \{nv_c/[(1.486)(R_c^{2/3})]\}^2 = \{(0.010)(5.029)/[(1.486)(2.0/4)^{2/3}]\}^2 = 0.00289$$

$$Q = Av_c = [(\tfrac{1}{2})(\pi)(2.0)^2/4](5.029) = 7.90\ \text{ft}^3/\text{s}$$

14.27 A flow rate of 2.1 m³/s is to be carried in an open channel at a velocity of 1.3 m/s. Determine the dimensions of the channel cross section and required slope if the cross section is rectangular with depth equal to one-half the width. Use $n = 0.020$.

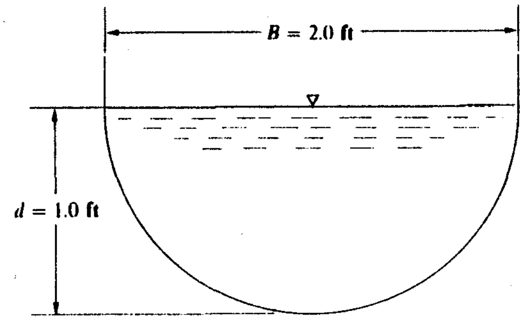


Fig. 14-10

▮ $Q = Av$. Let B = channel width. Then channel depth = $B/2$; $2.1 = [(B)(B/2)](1.3)$, $B = 1.797$ m. Hence, the required width is 1.797 m and depth is 1.797/2, or 0.898 m.

$$R = A/p_w = (1.797)(0.898)/[1.797 + (2)(0.898)] = 0.4491 \text{ m}$$
$$s = \{nv/[(1.0)(R^{2/3})]\}^2 = \{(0.020)(1.3)/[(1.0)(0.4491)^{2/3}]\}^2 = 0.00197$$

14.28 Repeat Prob. 14.27 if the depth must be equal to twice the width. Compare answers with Prob. 14.27.

▮ $Q = Av$. Let B = channel width. Then channel depth = $2B$; $2.1 = [(B)(2B)](1.3)$, $B = 0.899$ m. Hence, the required width is 0.899 m and depth is (2)(0.899), or 1.798 m.

$$R = A/p_w = (1.798)(0.899)/[0.899 + (2)(1.798)] = 0.3596 \text{ m}$$
$$s = \{nv/[(1.0)(R^{2/3})]\}^2 = \{(0.020)(1.3)/[(1.0)(0.3596)^{2/3}]\}^2 = 0.00264$$

The channel area is the same (neglecting round-off errors) but a steeper slope is required for the narrower channel.

14.29 Repeat Prob. 14.27 if the channel cross section is semicircular.

▮ $$Q = Av \qquad 2.1 = [(\tfrac{1}{2})(\pi d^2/4)](1.3) \qquad d = 2.028 \text{ m} \qquad r = 2.028/2 = 1.014 \text{ m}$$
$$s = \{nv/[(1.0)(R^{2/3})]\}^2 = \{(0.020)(1.3)/[(1.0)(2.028/4)^{2/3}]\}^2 = 0.00167$$

14.30 Repeat Prob. 14.27 if the channel cross section is trapezoidal, with depth equal to the width of the channel bottom and side slopes of 1:1.

▮ $Q = Av$. Let depth and channel bottom width = B (see Fig. 14-11). Then surface width = $3B$; $A = (3B)(B) - (2)[(\tfrac{1}{2})(B)(B)] = 2B^2$, $2.1 = (2B^2)(1.3)$, $B = 0.899$ m.

$$R = A/p_w = (2)(0.899)^2/[0.899 + (2)(0.899)\sqrt{2}] = 0.4696 \text{ m}$$
$$s = \{nv/[(1.0)(R^{2/3})]\}^2 = \{(0.020)(1.3)/[(1.0)(0.4696)^{2/3}]\}^2 = 0.00185$$

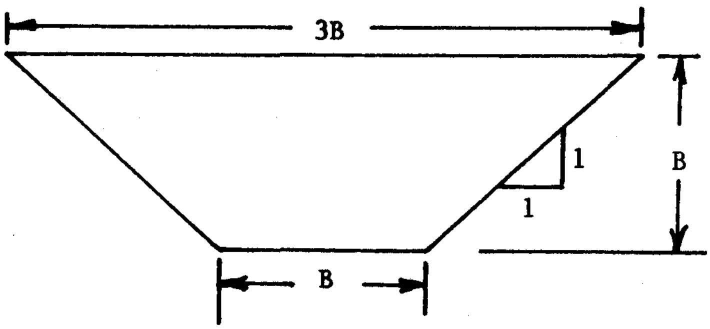


Fig. 14-11

14.31 For each of the channel cross sections shown in Fig. 14-12, compute the area, wetted perimeter, and hydraulic radius.

▮ (*a*) $A = \tfrac{1}{2}[(\pi)(4.0)^2/4] = 6.283 \text{ m}^2$ $\quad p_w = \tfrac{1}{2}[(\pi)(4.0)] = 6.283$ m $\quad R = A/p_w = 6.283/6.283 = 1.000$ m

(*b*) $A = (5.0)(2.5) = 12.50 \text{ m}^2$ $\quad p_w = 2.5 + 5.0 + 2.5 = 10.00$ m $\quad R = 12.50/10.00 = 1.250$ m

(*c*) $A = (5.0)(1.2) + (2)[(\tfrac{1}{2})(1.2)(1.2)] = 7.440 \text{ m}^2$ $\quad p_w = 5.0 + (2)[(1.2)(\sqrt{2})] = 8.394$ m

$$R = 7.440/8.394 = 0.886 \text{ m}$$

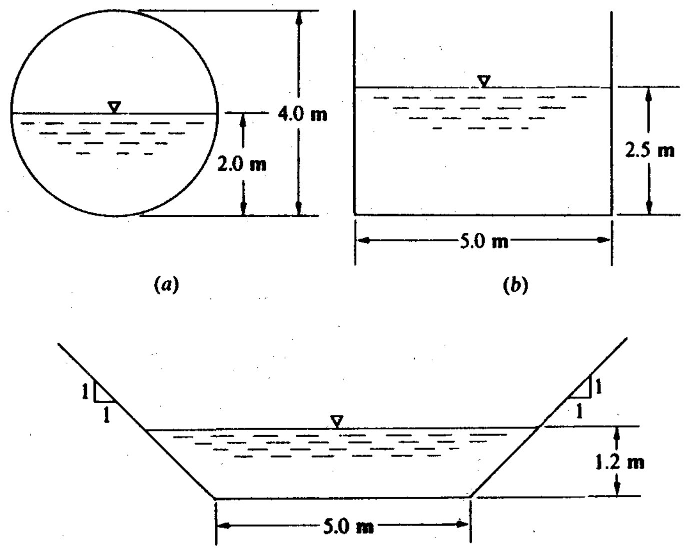


Fig. 14-12

14.32 Water is to flow in a rectangular flume at a rate of 1.42 m^3/s and at a slope of 0.0028. Determine the dimensions of the channel cross section if width must be equal to twice the depth. Use $n = 0.017$.

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$. Let B = channel width and $B/2$ = channel depth; $1.42 = [(B)(B/2)](1.0/0.017)[(B)(B/2)/(B/2 + B + B/2)]^{2/3}(0.0028)^{1/2}$, $B = 1.366$ m. Hence, required channel width = 1.366 m and depth = 1.366/2, or 0.683 m.

14.33 Rework Prob. 14.32, assuming width must be equal to the depth. Note which solution gives the smaller (and therefore more efficient) cross section.

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$. Let B = channel width and depth; $1.42 = [(B)(B)](1.0/0.017)[(B)(B)/(B + B + B)]^{2/3}(0.0028)^{1/2}$, $B = 0.981$ m. Hence, required channel width and depth are each 0.981 m. $A = (0.981)(0.981) = 0.962\ m^2$. For Prob. 14.32, $A = (1.366)(0.683) = 0.933\ m^2$. The cross section of Prob. 14.32 has the smaller cross-sectional area.

14.34 A rectangular channel ($n = 0.011$) 18 m wide is to carry water at a flow rate of 35 cfs. The slope of the channel is 0.00078. Determine the depth of flow.

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 35 = (18d)(1.0/0.011)[18d/(18 + 2d)]^{2/3}(0.00078)^{1/2}$$
$$d = 0.885 \text{ m} \qquad \text{(by trial and error)}$$

14.35 The trapezoidal channel shown in Fig. 14-13 is laid on a slope of 0.00191. The channel must carry 60 cfs. Determine the depth of flow. Use $n = 0.015$.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = 4.0d + (2)[(d)(d)/2] = 4.0d + d^2$$
$$p_w = 4.0 + (2)(d\sqrt{2}) = 4.0 + 2d\sqrt{2} \qquad 60 = (4.0d + d^2)(1.486/0.015)[(4.0d + d^2)/(4.0 + 2d\sqrt{2})]^{2/3}(0.00191)^{1/2}$$
$$d = 2.00 \text{ ft} \qquad \text{(by trial and error)}$$

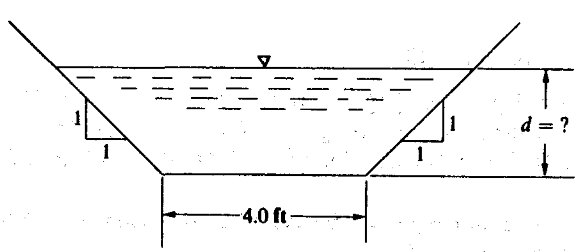


Fig. 14-13

14.36 A 36-in-diameter concrete pipe on a 0.0015 slope carries water at a depth of 26 in. Determine the flow rate for this pipe.

▮ See Fig. 14-14.

$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\qquad OE = 26 - \frac{36}{2} = 8$ in or 0.6667 ft $\qquad \measuredangle COE = \arccos[8/(\frac{36}{2})] = 63.61°$

$$EC = \sqrt{[(\tfrac{36}{2})/12]^2 - 0.6667^2} = 1.344 \text{ ft}$$

$$A = \{[360 - (2)(63.61)]/360\}[(\pi)(\tfrac{36}{12})^2/4] + (2)[(\tfrac{1}{2})(1.344)(0.6667)] = 5.467 \text{ ft}^2$$

$$p_w = \{[360 - (2)(63.61)]/360\}[(\pi)(\tfrac{36}{12})] = 6.094 \text{ ft}$$

$$Q = (5.467)(1.486/0.013)(5.467/6.094)^{2/3}(0.0015)^{1/2} = 22.5 \text{ ft}^3/\text{s}$$

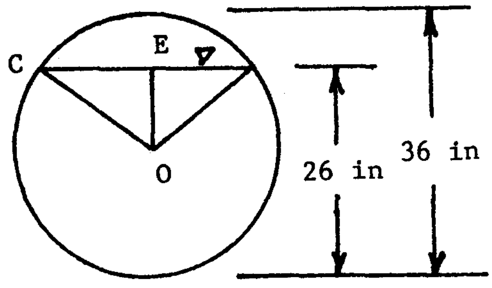


Fig. 14-14

14.37 Rework Prob. 14.36 using Fig. A-18.

▮ From Fig. A-15, $Q_{full} = 25.8 \text{ ft}^3/\text{s}$; $d/d_{full} = \frac{26}{36} = 0.722$, or 72.2 percent. From Fig. A-18, $Q/Q_{full} = 87.5$ percent; $Q = (0.875)(25.8) = 22.6 \text{ ft}^3/\text{s}$.

14.38 A sewer pipe, for which $n = 0.014$, is laid on a slope of 0.00018 and is to carry $2.76 \text{ m}^3/\text{s}$ when the pipe flows at 80 percent of full depth. Determine the required diameter of pipe.

▮ See Fig. 14-15.

$Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\qquad OE = 0.80D - D/2 = 0.3000D$ $\qquad \alpha = \arccos[0.30D/(D/2)] = 53.13°$

$$CE = (0.3000D)(\tan 53.13°) = 0.4000D$$

$$A = \{[360 - (2)(53.13)]/360\}(\pi D^2/4) + 2[(\tfrac{1}{2})(0.3000D)(0.4000D)] = 0.6736D^2$$

$$p_w = \{[360 - (2)(53.13)]/360\}(\pi D) = 2.214D$$

$$2.76 = (0.6736D^2)(1.0/0.014)(0.6736D^2/2.214D)^{2/3}(0.00018)^{1/2} \qquad D = 2.32 \text{ m}$$

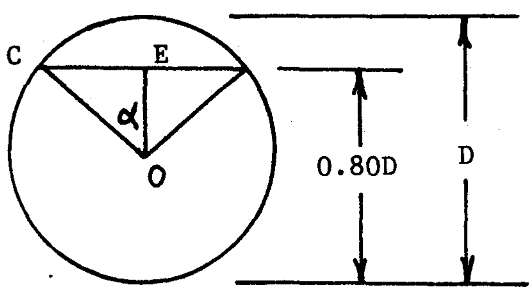


Fig. 14-15

14.39 Rework Prob. 14.38 using Fig. A-18.

▮ $D/D_{full} = 0.80$. From Fig. A-18, $Q/Q_{full} = 0.96$, $A/A_{full} = 0.84$, and $R/R_{full} = 1.21$: $Q_{full} = 2.76/0.96 = 2.88 \text{ m}^3/\text{s}$. Figure A-16 cannot be used for $Q_{full} > 1.0 \text{ m}^3/\text{s}$: $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $A = (0.84)(\pi D^2/4) = 0.6597D^2$, $R = (1.21)(D/4) = 0.3025D$, $2.76 = (0.6597D^2)(1.0/0.014)(0.3025D)^{2/3}(0.00018)^{1/2}$, $D = 2.34$ m.

14.40 A 72-in-diameter vitrified sewer pipe ($n = 0.014$) is laid on a slope of 0.00025 and carries wastewater at a flow rate of 50 cfs. What is the depth of flow?

▮ From Fig. A-15, $Q_{full} = 67 \text{ ft}^3/\text{s}$: $[(Q_{full})_{n=0.014}]/67 = 0.013/0.014$, $(Q_{full})_{n=0.014} = 62.2 \text{ ft}^3/\text{s}$; $Q/Q_{full} = 50/62.2 = 0.804$, or 80.4 percent. From Fig. A-18, $D/D_{full} = 69$ percent, $D = (0.69)(72) = 49.7$ in.

14.41 A 1.0-m-diameter pipe must carry a discharge of 0.40 m^3/s at a velocity of 0.80 m/s. Determine the slope and the depth of water.

$$A = Q/v = 0.40/0.80 = 0.5000\ m^2 \qquad A_{full} = (\pi)(1.0)^2/4 = 0.7854\ m^2$$
$$A/A_{full} = 0.5000/0.7854 = 0.64 \quad \text{or} \quad 64\ \text{percent}$$

From Fig. A-18, $D/D_{full} = 0.63$ and $R/R_{full} = 1.12$.

$$D = (0.63)(1.0) = 0.630\ m \qquad v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad R_{full} = 1.0/4 = 0.2500\ m$$
$$R = (1.12)(0.2500) = 0.2800\ m \qquad 0.80 = (1.0/0.013)(0.2800)^{2/3}(s^{1/2}) \qquad s = 0.000590$$

14.42 The trapezoidal channel of Fig. 14-16 is to carry 500 cfs of water. The maximum allowable velocity of flow is 3.0 fps to avoid scouring. Determine the depth of flow, d, and the width of the channel bottom, B, if the hydraulic radius of the channel is one-half the depth of flow. Also, determine the slope of the channel bottom. Use $n = 0.025$.

$$R = d/2 = A/p_w \qquad A = Bd + 2[(\tfrac{1}{2})(1.5d)(d)] = Bd + 1.5d^2 \qquad p_w = B + 2\sqrt{d^2 + (1.5d)^2} = B + 3.606d$$
$$d/2 = (Bd + 1.5d^2)/(B + 3.606d) \qquad B + 3.606d = 2B + 3.0d \qquad B = 0.606d \qquad A = Q/v$$
$$Bd + 1.5d^2 = 500/3.0$$

Substituting $B = 0.606d$, $(0.606d)(d) + 1.5d^2 = 166.7$, $d = 8.90$ ft; B = (0.606)(8.90) = 5.39 ft.

$$v = (1.486/n)(R^{2/3})(s^{1/2}) \qquad R = 8.90/2 = 4.45\ ft \qquad 3.0 = (1.486/0.025)(4.45)^{2/3}(s^{1/2}) \qquad s = 0.000348$$

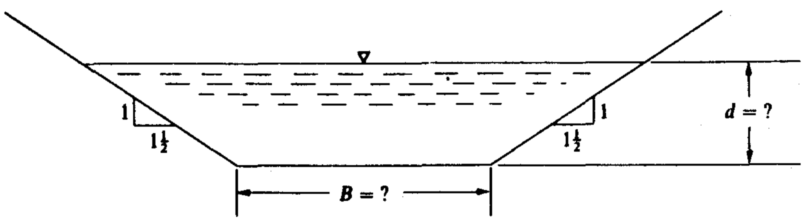


Fig. 14-16

14.43 An open channel to be made of concrete is to be designed to carry 1.5 m^3/s at a slope of 0.00085. Find the most efficient cross section for a semicircular section.

$$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad Q/A = (1.0/n)(A/p_w)^{2/3}(s^{1/2})$$
$$A^{5/3}/p_w^{2/3} = Qn/s^{1/2} = (1.5)(0.013)/0.00085^{1/2} = 0.6688 \qquad A = \pi d^2/8$$
$$p_w = \pi d/2 \qquad (\pi d^2/8)^{5/3}/(\pi d/2)^{2/3} = 0.6688 \qquad d = 1.727\ m$$

(d is the diameter of the semicircular section; the depth of flow would, of course, be half of d.)

14.44 Find the most efficient cross section for Prob. 14.43 for a rectangular section.

$A^{5/3}/p_w^{2/3} = 0.6688$ (from Prob. 14.43). The most efficient rectangular section has a width equal to twice its depth. Letting d = depth, $A = (d)(2d) = 2d^2$, $p_w = d + 2d + d = 4d$, $(2d^2)^{5/3}/(4d)^{2/3} = 0.6688$, $d = 0.789$ m; width $= 2d = (2)(0.789) = 1.578$ m.

14.45 Find the most efficient cross section for Prob. 14.43 for a triangular section.

$A^{5/3}/p_w^{2/3} = 0.6688$ (from Prob. 14.43). The most efficient triangular section has a 90° angle and 1:1 side slopes (see Fig. 14-7): $A = (\tfrac{1}{2})(d\sqrt{2})(d\sqrt{2}) = d^2$, $p_w = (2)(d\sqrt{2}) = 2.828d$, $(d^2)^{5/3}/(2.828d)^{2/3} = 0.6688$, $d =$ 1.115 m; sides $= d\sqrt{2} = (1.115)(\sqrt{2}) = 1.577$ m.

14.46 Find the most efficient cross section for Prob. 14.43 for a trapezoidal section.

$A^{5/3}/p_w^{2/3} = 0.6688$ (from Prob. 14.43). The most efficient trapezoidal section is half a regular hexagon (see Fig. 14-8): $A = (1.155d)(d) + 2[(d)(d \tan 30°)/2] = 1.732d^2$, $p_w = (3)(1.155d) = 3.465d$, $(1.732d^2)^{5/3}/(3.465d)^{2/3} = 0.6688$, $d = 0.832$ m. Sides and bottom: each $= 1.155d = (1.155)(0.832) = 0.961$ m.

14.47 For the conditions given in Prob. 14.32, determine whether the flow is critical, subcritical, or supercritical.

$$v = Q/A = 1.42/[(1.366)(0.683)] = 1.522\ m/s \qquad N_F = v/\sqrt{gd_m} = 1.522/\sqrt{(9.807)(0.683)} = 0.588$$

Since $N_F < 1.0$, the flow is subcritical.

14.48 A rectangular channel with a width of 3.0 m and an n value of 0.014 is to carry water at a flow rate of 13.4 m^3/s. Determine the critical depth, velocity, and channel slope.

▌ $d_c = [(Q/B)^2/g]^{1/3} = [(13.4/3.0)^2/9.807]^{1/3} = 1.267\text{ m}$ $\qquad v_c = Q/A = 13.4/[(1.267)(3.0)] = 3.525\text{ m/s}$

$$R = A/p_w = (1.267)(3.0)/(1.267 + 3.0 + 1.267) = 0.6868\text{ m}$$

$$s_c = \{nv_c/[(1.0)(R^{2/3})]\}^2 = \{(0.014)(3.525)/[(1.0)(0.6868)^{2/3}]\}^2 = 0.00402$$

14.49 The semicircular channel ($n = 0.013$) shown in Fig. 14-17 is to carry water while flowing full (i.e., at a depth of 1.5 ft). Determine the velocity, slope, and discharge when flow is critical.

▌ $d_m = A/B = \frac{1}{2}[(\pi)(3.0)^2/4]/3.0 = 1.178\text{ ft}$ $\qquad v_c = \sqrt{gd_m} = \sqrt{(32.2)(1.178)} = 6.159\text{ ft/s}$

$$s_c = \{nv_c/[(1.486)(R^{2/3})]\}^2 = \{(0.013)(6.159)/[(1.486)(3.0/4)^{2/3}]\}^2 = 0.00426$$

$$Q = Av = \{\tfrac{1}{2}[(\pi)(3.0)^2/4]\}(6.159) = 21.8\text{ ft}^3/\text{s}$$

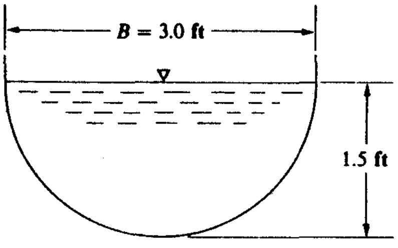


Fig. 14-17

14.50 Determine the dimensions of the most economical trapezoidal brick-lined ($n = 0.016$) channel to carry 200 m^3/s with a slope of 0.0004.

▌ The most economical trapezoidal channel has a cross section as shown in Fig. 14-8 and $R = d/2$ and $A = \sqrt{3}d^2$; $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $200 = (\sqrt{3}d^2)(1.0/0.016)(d/2)^{2/3}(0.0004)^{1/2}$, $d = 6.491$ m. Bottom width = $(1.155)(6.491) = 7.497$ m.

14.51 Determine the discharge for a trapezoidal channel with a bottom width of 8 ft and side slopes 1:1. The depth is 6 ft, and the slope of the bottom is 0.0009. The channel has a finished concrete lining ($n = 0.012$).

▌ $A = (8)(6) + (2)[(\frac{1}{2})(6)(6)] = 84.00\text{ ft}^2$ $\qquad p_w = 8 + (2)[(6)(\sqrt{2})] = 24.97\text{ ft}$

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = (84.00)(1.486/0.012)(84.00/24.97)^{2/3}(0.0009)^{1/2} = 701\text{ ft}^3/\text{s}$$

14.52 What depth is required for 4-m^3/s flow in a rectangular planed-wood ($n = 0.012$) channel 2 m wide with a bottom slope of 0.002?

▌ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\qquad 4 = (2d)(1.0/0.012)[2d/(2+2d)]^{2/3}(0.002)^{1/2}$

$d^{5/3}/(2+2d)^{2/3} = 0.3381$ $\qquad d = 0.888\text{ m}$ $\qquad$ (by trial and error)

14.53 A developer has been required by environmental regulatory authorities to line an open channel to prevent erosion. The channel is trapezoidal in cross section and has a slope of 0.0009. The bottom width is 10 ft and side slopes are 2:1 (horizontal to vertical). If he uses rubble ($\gamma_s = 135\text{ lb/ft}^3$) for the lining, what is the minimum D_{50} of the rubble that can be used? The design flow is 1000 cfs. Assume the shear that rubble can withstand is described by $\tau = (0.040)(\gamma_s - \gamma)(D_{50})$ (lb/ft^2), in which γ_s is the unit weight of rock and D_{50} is the average rock diameter in feet.

▌ A Manning n of 0.03 is appropriate for rubble.

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$1000 = [(d)(10+2d)](1.486/0.03)\{(d)(10+2d)/[10+(2)(\sqrt{5})(d)]\}^{2/3}(0.0009)^{1/2}$$

$[(d)(10+2d)]^{5/3}/[10+(2)(\sqrt{5})(d)]^{2/3} = 672.9$ $\qquad d = 8.63\text{ ft}$ $\qquad$ (by trial and error) $\qquad \tau_0 = \gamma Rs$

$R = 8.63[10+(2)(8.63)]/[10+(2)(\sqrt{5})(8.63)] = 4.841\text{ ft}$ $\qquad \tau_0 = (62.4)(4.841)(0.0009) = 0.2719\text{ lb/ft}^2$

To find the D_{50} size for incipient movement $\tau = \tau_0$ and $0.2719 = (0.040)(135 - 62.4)(D_{50})$, $D_{50} = 0.0936$ ft.

14.54 A metal-lined rectangular sluiceway is to carry 1.0 m^3/s at a slope of 0.010. Determine the minimum area of galvanized iron ($n = 0.011$) needed per meter of length. Neglect freeboard.

For minimum area, $D = B/2$ and $R = B/4$.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 1.0 = [(B)(B/2)](1.0/0.011)(B/4)^{2/3}(0.010)^{1/2} \qquad B = 0.8015\ \text{m}$$

$$D = 0.8015/2 = 0.4008\ \text{m} \qquad A_{\text{metal}}/L = 0.4008 + 0.8015 + 0.4008 = 1.6\ \text{m}^2/\text{m}$$

14.55 The sides of a trapezoidal channel are inclined 63.4° to the vertical; the channel is to carry 18 m^3/s with a bottom slope of 0.0009. Determine the bottom width, depth, and velocity for the best hydraulic section ($n = 0.026$).

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad \text{and} \qquad m = \tan 63.4° = 2.000$$

$$p_w = 4d\sqrt{1+m^2} - 2md = (4)(d)\sqrt{1+(2.000)^2} - (2)(2.000)(d) = 4.944d = B + 2\sqrt{5}d$$

$$B = 0.4719d \qquad A = Bd + 2d^2 = (0.4719d)(d) + 2d^2 = 2.472d^2$$

$$18 = (2.472d^2)(1.0/0.026)(2.472d^2/4.944d)^{2/3}(0.0009)^{1/2} \qquad d = 2.373\ \text{m}$$

$$B = (0.4719)(2.373) = 1.120\ \text{m} \qquad v = Q/A = 18/[(2.472)(2.373)^2] = 1.293\ \text{m/s}$$

14.56 A semicircular corrugated-metal ($n = 0.025$) channel must transport 2.4 m^3/s a distance of 1000 m with a head loss of 2 m. Compute the required radius.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 2.4 = (\pi r^2/2)(1.0/0.025)(r/2)^{2/3}(\tfrac{2}{1000})^{1/2} \qquad r = 1.121\ \text{m}$$

14.57 Determine the best hydraulic trapezoidal section to convey 86 m^3/s with a bottom slope of 0.002. The lining is finished concrete ($n = 0.012$).

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 86 = (A)(1.0/0.012)(A/p_w)^{2/3}(0.002)^{1/2} \qquad A^{5/3}/p_w^{2/3} = 23.08$$

The best trapezoidal section is half a regular hexagon (see Fig. 14-8) for which $A = 1.732d^2$ and $p_w = 3.465d$ (from Prob. 14.23). $(1.732d^2)^{5/3}/(3.465d)^{2/3} = 23.08$, $d = 3.141$ m. Sides and bottom: each $= 1.155d = (1.155)(3.141) = 3.628$ m.

14.58 Calculate the discharge in steady flow through the channel and floodway of Fig. 14-18; take $s = 0.0010$ and $y = 2.438$ m.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$

$$A_1 = (12)(5+2.438) + (2)(5+2.438)(5+2.438)/2 - (2.438)(2.438)/2 = 141.6\ \text{m}^2$$

$$(p_w)_1 = \sqrt{(5+2.438)^2 + (5+2.438)^2} + 12 + \sqrt{5^2+5^2} = 29.59\ \text{m}$$

$$Q_1 = (141.6)(1.0/0.025)(141.6/29.59)^{2/3}(0.0010)^{1/2} = 508.6\ \text{m}^3/\text{s}$$

$$A_2 = (120)(2.438) + (2.438)(2.438)/2 = 295.5\ \text{m}^2$$

$$(p_w)_2 = 120 + \sqrt{2.438^2 + 2.438^2} = 123.4\ \text{m} \qquad Q_2 = (295.5)(1.0/0.040)(295.5/123.4)^{2/3}(0.0010)^{1/2} = 418.1\ \text{m}^3/\text{s}$$

$$Q = Q_1 + Q_2 = 508.6 + 418.1 = 926.7\ \text{m}^3/\text{s}$$

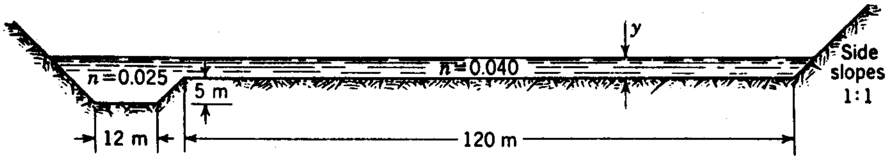


Fig. 14-18

14.59 For 25 000 cfs through the section of Fig. 14-18, find the depth of flow in the floodway (i.e., evaluate y) in feet when the slope of the energy grade line is 0.0004.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A_1 = (12)(5+y) + (2)(5+y)(5+y)/2 - (y)(y)/2 = y^2/2 + 22y + 85$$

$$(p_w)_1 = \sqrt{(5+y)^2 + (5+y)^2} + 12 + \sqrt{y^2+y^2} = (5)(\sqrt{2}) + 12 + (2y)(\sqrt{2})$$

$$A_2 = 120y + (y)(y)/2 = 120y + y^2/2$$

$$(p_w)_2 = 120 + \sqrt{y^2+y^2} = 120 + (y)(\sqrt{2}) \qquad Q = (25\,000)(0.3048)^3 = 707.9\ \text{m}^3/\text{s}$$

$$707.9 = (y^2/2 + 22y + 85)(1.0/0.025)\{(y^2/2 + 22y + 85)/[(5)(\sqrt{2}) + 12 + (2y)(\sqrt{2})]\}^{2/3}(0.0004)^{1/2}$$
$$+ (120y + y^2/2)(1.0/0.040)\{(120y + y^2/12)/[120 + (y)(\sqrt{2})]\}^{2/3}(0.0004)^{1/2}$$

$$y = 2.79\ \text{m} \quad \text{or} \quad 9.15\ \text{ft} \qquad \text{(by trial and error)}$$

14.60 Find the critical depth for flow at 1.6 m^3/s per meter of width.

▮ $$y_c = (q^2/g)^{1/3} = (1.6^2/9.807)^{1/3} = 0.639 \text{ m}$$

14.61 Compute the critical depth for flow at 0.4 m^3/s through the cross section of Fig. 14-19.

▮ $$Q^2T/gA^3 = 1 \qquad T = (2)(y \tan 60°/2) = 1.155y$$
$$A = (y)(y \tan 60°/2) = 0.5774y^2 \qquad (0.4)^2(1.155y)/[(9.807)(0.5774y^2)^3] = 1 \qquad y = 0.628 \text{ m}$$

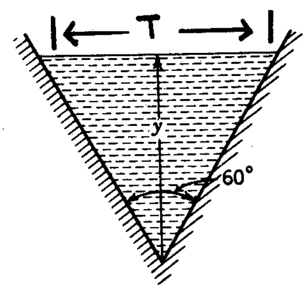


Fig. 14-19

14.62 Determine the critical depth for flow at 8.49 m^3/s through a trapezoidal channel with bottom 2.5 m wide and with 45° sides.

▮ $$Q^2T/gA^3 = 1 \qquad T = 2.5 + 2y \qquad A = By + my^2 = 2.5y + (1)(y)^2$$
$$(8.49)^2(2.5 + 2y)/[(9.807)(2.5y + y^2)^3] = 1 \qquad y = 0.928 \text{ m} \qquad \text{(by trial and error)}$$

14.63 Design a transition from a trapezoidal section, 8 ft bottom width and side slopes 1 on 1, depth 4 ft, to a rectangular section, 6 ft wide and 6 ft deep, for a flow of 250 cfs. The transition is to be 20 ft long, and the loss is one-tenth the difference between velocity heads. Show the bottom profile, and do not make any sudden changes in cross-sectional area.

▮ $A_1 = 8 \times 4 + 4^2 = 48 \text{ ft}^2$, $A_2 = 36 \text{ ft}^2$, loss $= 0.1[(v_2^2/2g) - (v_1^2/2g)]$, and $y_1 + (v_1^2/2g) + z_1 = y_2 + (v_2^2/2g) + z_2 +$ loss. Assume a linear change in area, b, and T: $b = 8 - 2(x/L)$ and $T = 16 - 10(x/L)$. Hence: $A = (b + T)(y/2) = 48 - 12(x/L)$ and $y = 2\{[4 - (x/L)]/[2 - (x/L)]\}$.

x/L	A	$v^2/2g$	loss	EGL	y	z
0	48	0.421		4.421	4.0	0
			0.008			
0.333	44	0.501		4.413	4.4	−0.488
			0.011			
0.667	40	0.607		4.402	5.001	−1.206
			0.014			
1.0	36	0.749		4.388	6.0	−2.361

The profile is shown in Fig. 14-20.

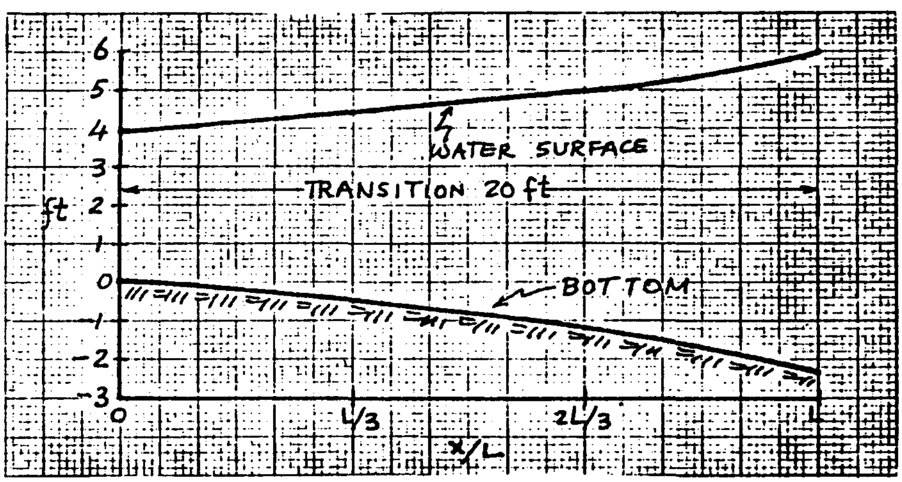


Fig. 14-20

14.64 In a transition from a rectangular channel (2.5 m wide, 2 m deep) to a trapezoidal channel (bottom width 4 m, side slopes 2 on 1, depth 1.3 m) the energy loss is equal to 0.4 times the difference between velocity heads. The discharge is 5.8 m³/s. Determine the difference between elevations of channel bottoms.

▮ $0 + v_1^2/2g + y_1 = \Delta z + v_2^2/2g + y_2 + (0.4)(v_1^2/2g - v_2^2/2g)$ $\quad A_1 = (2.5)(2) = 5.00\ \text{m}^2$

$A_2 = (4)(1.3) + (2)\{[(1.3)(2)](1.3)/2\} = 8.580\ \text{m}^2$ $\quad v_1 = Q/A_1 = 5.8/5.00 = 1.160\ \text{m/s}$

$v_2 = 5.8/8.580 = 0.6760\ \text{m/s}$

$v_1^2/2g = 1.160^2/[(2)(9.807)] = 0.06860\ \text{m}$ $\quad v_2^2/2g = 0.6760^2/[(2)(9.807)] = 0.02330\ \text{m}$

$0 + 0.06860 + 2 = \Delta z + 0.02330 + 1.3 + (0.4)(0.06860 - 0.02330)$ $\quad \Delta z = 0.727\ \text{m}$

14.65 A dam gate (Fig. 14-21) admits water to a horizontal canal. Considering the pressure distribution hydrostatic at section O, compute the depth at O and the discharge per meter of width when $y = 1.0$ m.

▮ $d_0 = C_c y = (0.85)(1.0) = 0.85\ \text{m}$ $\quad p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

$0 + 0 + 6 = 0 + v_t^2/[(2)(9.807)] + 0.85 + 0$ $\quad v_t = 10.05\ \text{m/s}$ $\quad v_a = C_v v_t = (0.95)(10.05) = 9.548\ \text{m/s}$

$Q = Av = [(0.85)(1)](9.548) = 8.12\ \text{m}^3\text{/s}$ per meter of width

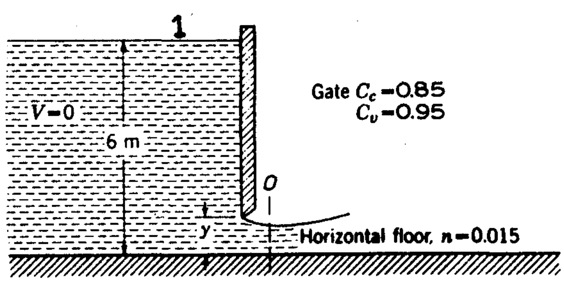


Fig. 14-21

14.66 A discharge of 4.5 m³/s occurs in a rectangular channel 1.83 m wide with $s = 0.002$ and $n = 0.012$. Find the normal depth for uniform flow and determine the critical depth. Is the flow subcritical or supercritical?

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\quad 4.5 = (1.83y_n)(1.0/0.012)[1.83y_n/(y_n + 1.83 + y_n)]^{2/3}(0.002)^{1/2}$

$y_n = 1.060\ \text{m}$ (by trial and error)

$Q^2/g = A^3/B$ $\quad 4.5^2/9.807 = (1.83y_c)^3/1.83$ $\quad y_c = 0.851\ \text{m}$

Since $y_c < y_n$, the flow is subcritical.

14.67 Determine the rate of a 17-in-deep flow in a 48-in circular corrugated-metal ($n = 0.022$) pipe on a slope of 0.003.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$Q_{\text{full}} = [(\pi)(\tfrac{48}{12})^2/4](1.486/0.022)[(\tfrac{48}{12})/4]^{2/3}(0.003)^{1/2} = 46.49\ \text{ft}^3/\text{s} \qquad d/d_{\text{full}} = \tfrac{17}{48} = 0.35 \quad \text{or} \quad 35\ \text{percent}$$

From Fig. A-18, $Q/Q_{\text{full}} = 25$ percent, $Q = (0.25)(46.49) = 11.6\ \text{ft}^3/\text{s}$.

14.68 At what depth will $9.0\ \text{ft}^3/\text{s}$ flow in a 3-ft diameter concrete pipe on a slope of 0.004?

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad Q_{\text{full}} = [(\pi)(3)^2/4](1.486/0.013)[3/4]^{2/3}(0.004)^{1/2} = 42.18\ \text{ft}^3/\text{s}$$

$$Q/Q_{\text{full}} = 9.0/42.18 = 0.21 \quad \text{or} \quad 21\%$$

From Fig. A-18, $d/d_{\text{full}} = 32\%$, $d = (0.32)(3) = 0.96$ ft.

14.69 Find the flow rate in a 400-mm-diameter concrete pipe on a slope of 0.004, if the depth is 85 mm?

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad Q_{\text{full}} = [(\pi)(0.400)^2/4](1.0/0.013)[0.400/4]^{2/3}(0.004)^{1/2} = 0.1317\ \text{m}^3/\text{s}$$

$$d/d_{\text{full}} = 85/400 = 0.21 \quad \text{or} \quad 21\%$$

From Fig. A-18, $Q/Q_{\text{full}} = 10\%$, $Q = (0.10)(0.1317) = 0.0132\ \text{m}^3/\text{s}$.

14.70 Repeat Prob. 14.68 if the pipe is corrugated metal ($n = 0.024$).

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad Q_{\text{full}} = [(\pi)(3)^2/4](1.486/0.024)[3/4]^{2/3}(0.004)^{1/2} = 22.85\ \text{ft}^3/\text{s}$$

$$Q/Q_{\text{full}} = 9.0/22.85 = 0.39 \quad \text{or} \quad 39\%$$

From Fig. A-18, $d/d_{\text{full}} = 43\%$, $d = (0.43)(3) = 1.29$ ft.

14.71 Find the smallest downslope of 3-ft-diameter cast iron piping ($n = 0.015$) that will yield an $18\ \text{ft}^3/\text{s}$, 2.4-ft-deep flow.

▮ $d/d_{\text{full}} = 2.4/30.8$. From Fig. A-18, $Q/Q_{\text{full}} = 96\%$, $Q_{\text{full}} = 18/0.96 = 18.75\ \text{ft}^3/\text{s}$, $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$, $18.75 = [(\pi)(3)^2/4](1.486/0.015)[3/4]^{2/3}(s)^{1/2}$, $s = 0.00105$.

14.72 Find the slope of a 10-ft-wide rectangular channel having $n = 0.012$ that makes critical flow occur at a depth of 4.0 ft.

▮ $$Q = \sqrt{A^3g/B} = \sqrt{[(4)(10)]^3(32.2)/10} = 454.0\ \text{ft}^3/\text{s} = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$454.0 = [(4)(10)](1.486/0.012)[(4)(10)/(4.0 + 10 + 4.0)]^{2/3}(s^{1/2}) \qquad s = 0.00290$$

14.73 Is the flow of Prob. 14.68 subcritical or supercritical?

▮ For critical flow, $Q^2/g = A^3/B$. For a 3-ft pipe transporting at a depth of 0.96 ft, $B \approx 2.8$ ft and $A \approx 1.94\ \text{ft}^2$. Because $9.0^2/32.2 = 2.5 < 1.94^3/2.8 = 2.6$, the flow is barely subcritical.

14.74 Water flows steadily at $16.0\ \text{ft}^3/\text{s}$ in a triangular sluice with side slopes 1 on 1; the bottom has slope 0.0039. At a certain section, the depth of flow is 2.00 ft. Characterize the flow at this section as subcritical or supercritical.

▮ $Q^2/g = A^3/B$. At critical flow, $A = (2)[(y_c)(y_c)/2] = y_c^2$, $16.0^2/32.2 = (y_c^2)^3/(2y_c)$, $y_c = 1.74$ ft. Since $y_c <$ 2.00 ft, the flow at this section is subcritical.

14.75 Water flows at $8.5\ \text{m}^3/\text{s}$ in a 3.0-m-wide open channel of rectangular cross section. The bottom slopes up, rising 2 mm per meter in the direction of flow. If the water depth decreases from 2.10 m to 1.65 m in a 155 m length of channel, determine Manning's n.

$$z_1 + y_1 + v_1^2/2g = z_2 + y_2 + v_2^2/2g + h_L$$
$$v_1 = Q/A_1 = 8.5/[(2.10)(3.0)] = 1.349\text{ m/s} \qquad v_2 = 8.5/[(1.65)(3.0)] = 1.717\text{ m/s}$$
$$0 + 2.10 + 1.349^2/[(2)(9.807)] = (0.002)(155) + 1.65 + 1.717^2/[(2)(9.807)] + h_L \qquad h_L = 0.0825\text{ m}$$
$$v = (1.0/n)(R^{2/3})(s^{1/2}) = v_{avg} = (1.349 + 1.717)/2 = 1.533\text{ m/s} \qquad R = A/p_w$$
$$R_1 = (2.10)(3.0)/(2.10 + 3.0 + 2.10) = 0.8750\text{ m} \qquad R_2 = (1.65)(3.0)/(1.65 + 3.0 + 1.65) = 0.7857\text{ m}$$
$$R_{avg} = (0.8750 + 0.7857)/2 = 0.8304\text{ m} \qquad 1.533 = (1.0/n)(0.8304)^{2/3}(0.0825/155)^{1/2} \qquad n = 0.013$$

14.76 A flow of 1000 ft^3/s occurs in a long 10-ft-wide rectangular channel of constant bottom slope. The Manning equation yields 7.0 ft as the normal depth of flow for this flow rate. Will the depth of flow increase, decrease, or remain the same as one proceeds downstream from a point where the depth is 3.0 ft?

$$y_c = (q^2/g)^{1/3} = [(\tfrac{1000}{10})^2/32.2]^{1/3} = 6.77\text{ ft}$$

which is between 3.0 ft and 7.0 ft. Hence, the depth of flow will increase.

14.77 Determine the cross section of greatest hydraulic efficiency for the trapezoidal channel of Fig. 14-22, if the discharge is 10.0 m^3/s and the channel slope is 0.0005. Use $n = 0.020$.

$R = A/p_w = (1.5y + x)(y)/\{x + (2)[\sqrt{y^2 + (1.5y)^2}]\}$. For greatest hydraulic efficiency, $R = y/2$. Hence, $(1.5y + x)(y)/[x + (2)\sqrt{y^2 + (1.5y)^2}] = y/2$, $x = 0.606y$.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = (1.5y + x)(y) = (1.5y + 0.606y)(y) = 2.106y^2$$
$$10.0 = (2.106y^2)(1.0/0.020)(y/2)^{2/3}(0.0005)^{1/2} \qquad y = 2.05\text{ m} \qquad x = (0.606)(2.05) = 1.24\text{ m}$$

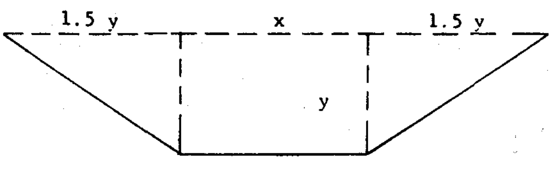


Fig. 14-22

14.78 Specify a canal with $n = 0.0182$ that will carry 500 ft^3/s of water a distance of 1 mile, if the total drop must not exceed 53 ft.

Use a trapezoidal channel with side slopes of 1:1, and assume $v_{max} = 4.0$ ft/s.

$$s_{max} = 53/5280 = 0.0100 \qquad A_{min} = Q/v_{max} = 500/4.0 = 125.0\text{ ft}^2 \qquad Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

Try $b = 8$ ft and $y = 8$ ft: $A = (8)(8 + 8) = 128.0\text{ ft}^2$, $p_w = 8 + (2)(\sqrt{8^2 + 8^2}) = 30.63$ ft, $500 = (128.0)(1.486/0.0182)(128.0/30.63)^{2/3}(s)^{1/2}$, $s = 0.000340$. Because $s < s_{max}$ this design is acceptable.

14.79 Given an open channel with a parabolic cross section ($x = 1.0$ m and $y = 1.0$ m in Fig. 14-23) on a slope of 0.02 with $n = 0.015$, find the normal depth and the critical depth for a flow rate of 2.0 m^3/s.

$Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$. Equation of parabola: $x^2 = y$.

$$A = (2)(\tfrac{2}{3})(xy) = (\tfrac{4}{3})(y^{1/2}y) = 4y^{3/2}/3$$
$$p_w = (2x)[1 + (\tfrac{2}{3})(y/x)^2 - (\tfrac{2}{5})(y/x)^4 + \cdots] = (2y^{1/2})[1 + 2y/3 - 2y^2/5 + \cdots]$$

Try $y_n = 0.5$ m: $A = (4)(0.5)^{3/2}/3 = 0.4714\text{ m}^2$, $p_w = (2)(0.5)^{1/2}[1 + (2)(0.5)/3 - (2)(0.5)^2/5 + \cdots] = 1.744$ m, $Q = (0.4714)(1.0/0.015)(0.4714/1.744)^{2/3}(0.02)^{1/2} = 1.86\text{ m}^3/\text{s}$. Since this value of Q (1.86 m^3/s) is slightly less than the given value (2.0 m^3/s), try a slightly higher value of y_n, say 0.52 m: $A = (4)(0.52)^{3/2}/3 = 0.5000\text{ m}^2$, $p_w = (2)(0.52)^{1/2}[1 + (2)(0.52)/3 - (2)(0.52)^2/5 + \cdots] = 1.786$ m, $Q = (0.5000)(1.0/0.015)(0.5000/1.786)^{2/3}(0.02)^{1/2} = 2.02\text{ m}^3/\text{s}$. An additional iteration (not shown) indicates an appropriate value of y_n of 0.518 m. Critical depth occurs when $Q^2/g = A^3/B$: $2.0^2/9.807 = (4y_c^{3/2}/3)^3/[(2)(\sqrt{y_c})]$, $y_c = 0.766$ m.

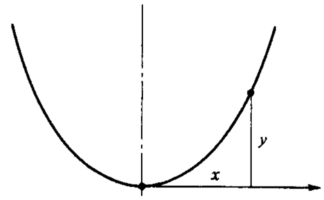


Fig. 14-23

14.80 Consider the frictionless open-channel flow of Fig. 14-24*a*, in which the water depth y_∞ is 2 ft and the volume flow rate per unit width $\dot{Q}/w$ is 5 ft²/s. Discuss what happens as the height of the obstacle on the channel floor is increased.

▌ We first find out whether the flow upstream of the obstacle is subcritical or supercritical by calculating the critical depth y_c, using $y_c = [(\dot{Q}/w)^2/g]^{1/3} = [(5)^2/32.17]^{1/3} = 0.9194$ ft. The upstream Froude number F_∞ is, then, $F_\infty = (y_c/y_\infty)^{3/2} = (0.9194/2)^{3/2} = 0.3117$. F_∞ is less than 1, showing that the upstream flow is subcritical.

Next we calculate the height of an obstacle which results in critical flow. The specific head of the flow upstream of the obstacle is $H_\infty = (\dot{Q}/w)^2/(2gy_\infty^2) + y_\infty = (5)^2/[(2)(32.17)(2)^2] + 2 = 2.097$ ft. The specific head of the flow at the critical point is the minimum specific head for that $\dot{Q}/w$: $H_{\min} = \frac{3}{2}y_c = (\frac{3}{2})(0.9194) = 1.379$ ft. Then the height of an obstacle resulting in critical flow h_c is $h_c = H_\infty - H_{\min} = 2.097 - 1.379 = 0.718$ ft.

Let us calculate the surface profile of a flow over an obstacle which is not high enough to cause critical flow, say an obstacle 0.5 ft high. Since the flow is subcritical, the water surface must dip or depress over the obstacle since dy/dx is opposite in sign to dh/dx. The water surface passes through a minimum depth right over the crest of the obstacle. The specific head of the flow over the crest is $H_{\text{crest}} = H_\infty - h_{\text{crest}} = 2.097 - 0.5 = 1.597$ ft. A dimensionless specific head of $H_{\text{crest}}/y_c = 1.597/0.9194 = 1.737$ corresponds to a dimensionless depth for subcritical flow of $y_{\text{crest}}/y_c = 1.52$. The depth of the flow over the crest is $y_{\text{crest}} = (1.52)(0.9194) = 1.4$ ft. The Froude number of the flow over the crest is $F_{\text{crest}} = (y_c/y_{\text{crest}})^{3/2} = (1/1.52)^{3/2} = 0.534$.

The local surface depth all along the obstacle is calculated in exactly the same way using the local height $h(x)$ of the obstacle. Figure 14-24*a* shows the water surface and Froude-number variation for this flow over a circular obstacle. The flow is symmetrical about the crest of the obstacle. Obstacles of the same crest height but different shape change the shape of the water-surface depression but not its minimum depth.

Suppose the height of the crest of the obstacle is increased to precisely $h_c = 0.718$ ft, the height at which critical flow occurs. The flow decreases in depth from y_∞ upstream of the obstacle to y_c at the crest with a corresponding increase in Froude number from $N_F = 0.3117$ to 1. Downstream of the crest there are two possible surface profiles. The first possibility is that the flow remains subcritical and its depth increases from y_c back to y_∞ as the Froude number decreases from 1 to $N_F = 0.3117$. This is the limiting case of subcritical flow. The flow is symmetrical about the crest. The second possibility is that the flow passes through the critical point and its Froude number continues to increase while its depth continues to decrease. The flow becomes supercritical. Downstream of the obstacle the flow has a new depth corresponding to the supercritical branch for the same specific head as upstream. Although the specific head is the same on either side of the obstacle, there is a different distribution between the kinetic and potential energy. Upstream of the obstacle the flow is subcritical, and most of the energy is potential, whereas downstream of the obstacle the flow is supercritical and a larger proportion of the energy is kinetic. The transition from subcritical to supercritical flow downstream of the critical point depends on whether the conditions downstream are favorable for maintaining supercritical flow.

Let us calculate the depth downstream of the obstacle for supercritical flow $y_{\infty,\text{sup}}$. The specific head is the same as upstream, $H_\infty = 2.097$ ft or $H/y_c = 2.281$. $y/y_c = 0.535$ is on the supercritical branch for this value of H/y_c. The downstream depth is then $y_{\infty,\text{sup}} = 0.535(0.9194) = 0.492$ ft. The Froude number there is $(N)_{\infty,\text{sup}} = (y_c/y_{\infty,\text{sup}})^{3/2} = (N)(1/0.535)^{3/2} = 2.56$.

The surface elevation undergoes its largest changes when the Froude number is in the vicinity of 1. At low subcritical Froude numbers $1 - N_F^2$ is nearly 1, and the decrease in surface elevation is about the same as the increase in obstacle elevation. As the Froude number in supercritical flow increases, $1 - N_F^2$ becomes increasingly more negative, causing the surface elevation to change less and less with changes in obstacle elevation.

Figure 14-24*b* shows the surface profile and Froude-number variation for subcritical and supercritical flow downstream of the crest.

What happens if the obstacle height is increased above h_c, the obstacle height which results in critical flow? The flow can no longer take place with the same values of y_∞ and $\dot{Q}/w$ as before. There must be an adjustment in either y_∞ or $\dot{Q}/w$ or both to raise H_∞ to at least the value which results in critical flow at the crest. This is the minimum specific head required to sustain the flow over the obstacle.

14.81 A large reservoir 5 m deep has a rectangular sluice gate 1 m wide. How does the volume flow rate change as the sluice gate is raised (Fig. 14-25)?

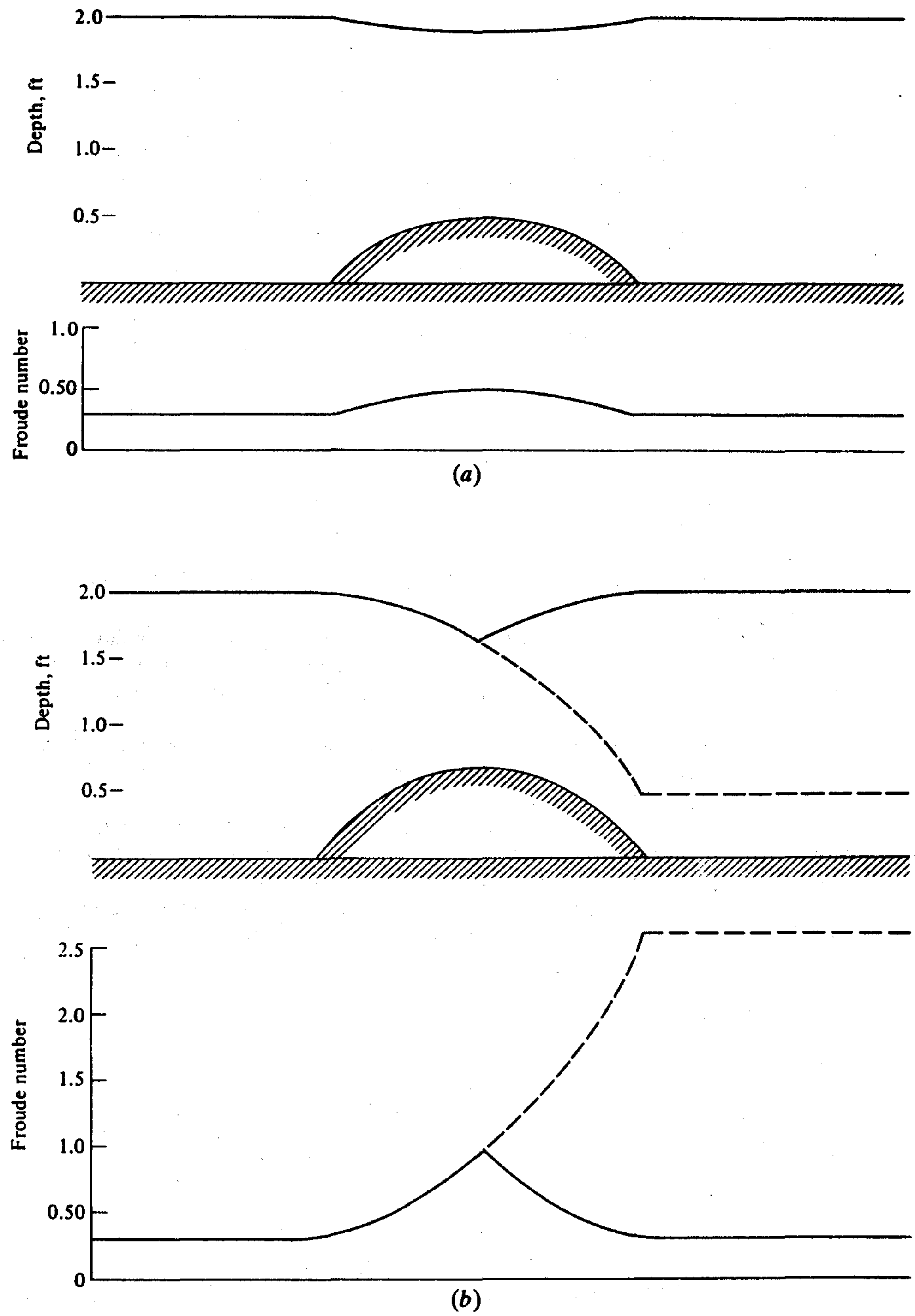


Fig. 14-24

▮ Apply the ideal Bernouilli equation in head form at the water surface and far enough up and downstream of the gate for the streamlines to be parallel: $(p_{atm}/\rho g) + (V_\infty^2/2g) + y_\infty = (p_{atm}/\rho g) + (V^2/2g) + y$. In terms of specific head $H_\infty = H$. This is a constant-specific-head flow. The sluice gate is not an obstacle which changes the specific head of the flow; it simply changes the distribution of energy between kinetic and potential.

Now we examine H_∞, $H_\infty = (V_\infty^2/2g) + y_\infty$. Assume that the reservoir is so large that y_∞ upstream of the gate does not change with changes in sluice-gate opening. Furthermore, V_∞ is very small. Then, $H_\infty \approx y_\infty$ and is the same for all sluice-gate openings. Thus, the outflow has the same specific head regardless of the height of the sluice-gate opening.

The flow rate is then given by $\dot{Q} = wy\sqrt{2g(y_\infty - y)}$. The critical depth y_c changes with the flow rate and consequently with y: $y_c = [(\dot{Q}/w)^2/g]^{1/3} = [2y^2(y_\infty - y)]^{1/3}$. The corresponding Froude number is $N_F = (y_c/y)^{3/2} = \sqrt{2[(y_\infty/y) - 1]}$.

What happens as the sluice gate is raised? First, consider a small opening which results in a y of, say, 0.5 m. The flow rate is $\dot{Q} = (1)(0.5)\sqrt{2(9.8)(5-0.5)} = 4.696\ \text{m}^3/\text{s}$ and the corresponding Froude number is $N_F = \sqrt{2[(5/0.5) - 1]} = 4.243$. The flow is supercritical.

As the gate is raised further, y increases, causing the Froude number to decrease and the flow rate to increase. Finally, at a gate height which gives $y = \frac{2}{3}y_\infty = (\frac{2}{3})(5) = 3.333$ m, The Froude number is 1 and the maximum flow rate is passed: $\dot{Q}_{max} = w\sqrt{\frac{8}{27}gy_\infty^3} = (1)\sqrt{\frac{8}{27}(9.8)(5)^3} = 19.05\ \text{m}^3/\text{s}$.

If the gate is raised farther, the flow becomes subcritical, the Froude number passes through 1 and decreases, and the flow rate starts to decrease.

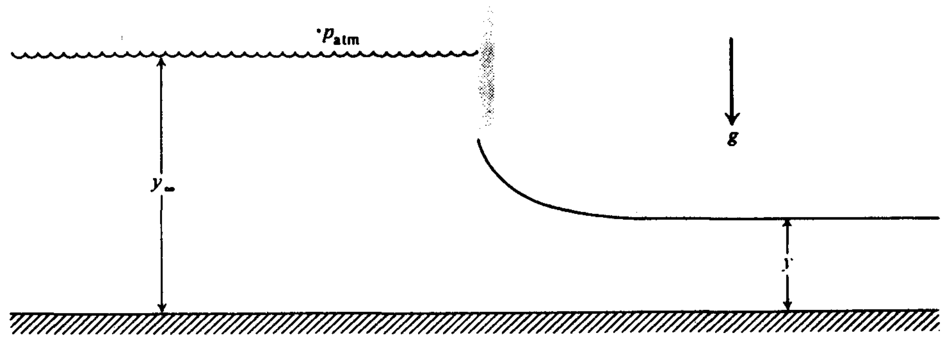


Fig. 14-25

14.82 Reconsider the flow leaving the sluice gate in Prob. 14-81 in which $y_\infty = 5$ m. The flow can be made subcritical by placing a step downstream of the gate. Suppose the flow leaving the sluice gate is $y_1 = 0.5$ m deep. Estimate the height of the step h required to cause a subcritical flow $y_3 = 2$ m deep over the step.

The problem is solved by considering the flow in two parts (Fig. 14-26). First, a hydraulic jump occurs in front of the step, resulting in subcritical flow and a dissipation of mechanical energy. Then the level of the flow coming over the step decreases in a frictionless flow.

First we calculate the characteristics of the hydraulic jump. From Prob. 13.2 $(N_F)_1 = 4.243$. The depth ratio across the hydraulic jump is calculated from $y_2/y_1 = \frac{1}{2}(\sqrt{1 + 8(N_F)_1^2} - 1) = \frac{1}{2}[\sqrt{1 + (8)(4.243)^2} - 1] = 5.521$. The mechanical-energy dissipation across the hydraulic jump is $(h_f)_{1\to2}/y_1 = [(y_2/y_1) - 1]^3/(4y_2/y_1) = (5.521 - 1)^3/(4)(5.521) = 4.184$, $(h_f)_{1\to2} = 2.092$ m. The frictionless flow over the step is described by $H_2 = H_3 + h$. Now, $h_{t_2} = h_{t_1} - (h_f)_{1\to2}$, or $H_2 = H_1 - (h_f)_{1\to2}$. $H_1 = H_\infty = y_\infty = 5$ m. Then $H_2 = H_1 - (h_f)_{1\to2} = 5 - 2.092 = 2.908$ m, $H_3 = [(\dot{Q}/w)^2/2gy_3^2] + y_3 = \{(4.696)^2/[2(9.8)(2)^2]\} + 2 = 2.281$ m. Finally, $h = H_2 - H_3 = 2.908 - 2.281 = 0.627$ m.

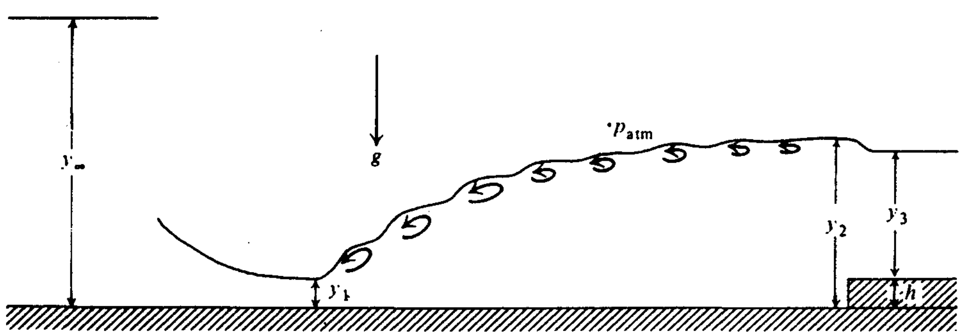


Fig. 14-26

14.83 A long channel with a rectangular cross section and an unfinished concrete surface ($n = 0.017$) is 35 ft wide and has a constant slope of 0.5°. What is the water depth when the channel carries 3500 cfs? Is the channel slope mild or steep?

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 3500 = (35d)(1.486/0.017)[35d/(35+2d)]^{2/3}(\tan 0.5°)^{1/2}$$

$$d = 4.97\text{ ft} \quad \text{(by trial and error)} \qquad d_c = (q^2/g)^{1/3} = [(\tfrac{3500}{35})^2/32.2]^{1/3} = 6.77\text{ ft}$$

Since $d < d_c$, flow is supercritical and the channel slope is steep.

14.84 Find the depth for uniform flow in Fig. 14-27 when the flow rate is 225 cfs if $s = 0.0006$ and n is assumed to be 0.016. Compute the corresponding value of ϵ.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $A = (y_0)(10 + 2y_0)$ $p_w = 10 + (2)(\sqrt{5})(y_0)$

$$225 = [(y_0)(10 + 2y_0)](1.486/0.016)\{[(y_0)(10 + 2y_0)]/[10 + (2)(\sqrt{5})(y_0)]\}^{2/3}(0.0006)^{1/2}$$

$$y_0 = 3.41 \text{ ft} \quad \text{(by trial and error)} \qquad n = 0.093f^{1/2}R^{1/6}$$

$$R = 3.41[10 + (2)(3.41)]/[10 + (2)(\sqrt{5})(3.41)] = 2.272 \text{ ft} \qquad 0.016 = (0.093)(f^{1/2})(2.272)^{1/6} \qquad f = 0.0225$$

$$1/\sqrt{f} = 2 \log (14.8R/\epsilon) \qquad 1/\sqrt{0.0225} = 2 \log [(14.8)(2.272)/\epsilon] \qquad \epsilon = 0.0156 \text{ ft}$$

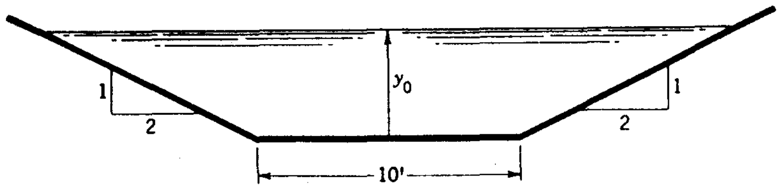


Fig. 14-27

14.85 In Fig. 14-28, water flows uniformly at a steady rate of 14.0 cfs in a very long triangular flume which has side slopes of 1:1. The bottom of this flume is on a slope of 0.006, and $n = 0.012$. ***(a)*** Is the flow subcritical or supercritical? ***(b)*** Find the relation between $v_c^2/2g$ and y_c for this channel.

▮ ***(a)*** $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $A = (y)(2y)/2 = y^2$ $p_w = (2)(\sqrt{2})(y) = 2.828y$

$$14.0 = (y^2)(1.486/0.012)(y^2/2.828y)^{2/3}(0.006)^{1/2} \qquad y = 1.49 \text{ ft} \quad \text{(by trial and error)}$$

$$Q^2/g = A^3/B \qquad 14.0^2/32.2 = (y_c^2)^3/2y_c \qquad y_c = 1.65 \text{ ft}$$

Since $y < y_c$, flow is supercritical.

(b)

$$v_c^2/g = A_c/B_c = y_c^2/2y_c = y_c/2 \qquad y_c^2/2g = y_c/4$$

Consequently, we see that the relation between $v^2/2g$ and y for critical-flow conditions depends on the geometry of the flow section. If the vertex angle of the triangle had been different, the relation would have been different.

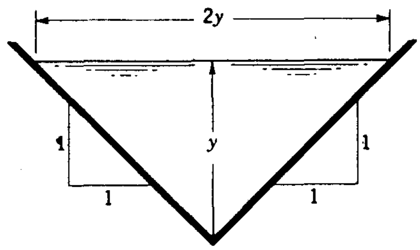


Fig. 14-28

14.86 In Fig. 14-29, uniform flow of water occurs at 27 cfs in a 4-ft-wide rectangular flume at a depth of 2.00 ft. ***(a)*** Is the flow subcritical or supercritical? ***(b)*** If a hump of height $\Delta t = 0.30$ ft is placed in the bottom of the flume, calculate the water depth on the hump. Neglect head loss in flow over the hump. ***(c)*** If the hump height is raised to $\Delta z = 0.60$ ft, what then are the water depths upstream and downstream of the hump? Once again neglect head loss over the hump.

▮ ***(a)*** First find critical depth: $y_c = (q^2/g)^{1/3} = [(\frac{27}{4})^2/32.2]^{1/3} = 1.12$ ft. Since the normal depth (2.00 ft) is greater than the critical depth, the flow is subcritical and the channel slope is mild.

(b) Find the critical hump height. Write the energy equation between sections 1 and 2, assume critical flow on the hump and apply continuity.

$$2.00 + (V_1^2/2g) = (\Delta z)_{crit} + 1.12 + (V_2^2/2g) \qquad (1)$$

$$V_2 = 27/(4 \times 1.12) = 6.03 \text{ fps} \qquad (2)$$

$$V_1 = 27/(4 \times 2) = 3.38 \text{ fps} \qquad (3)$$

Substituging (2) and (3) in (1) gives $(\Delta z)_{crit} = 0.49$ ft. Thus the minimum-height hump that will produce critical depth on the hump is 0.49 ft.

Since the actual hump height, $\Delta z = 0.30$ ft, is less than the critical hump height, 0.49 ft, critical flow does not occur on the hump and there is no damming action.

Let us now find the depth y_2 on the hump:

Energy: $$2.00 + (V_1^2/2g) = 0.30 + y_2 + (V_2^2/2g) \qquad (4)$$

Continuity: $$(4 \times 2)V_1 = 4y_2V_2 = 27 \text{ cfs} \qquad (5)$$

Eliminating V_1 and V_2 from Eqs. (4) and (5) gives three roots for y_2; $y_2 = 1.60$ ft, 0.82 ft, or a negative answer that has no physical meaning. Since the hump height is less than $(\Delta z)_{crit}$, the flow on the hump must be subcritical (that is, $y_2 > y_c$). Hence $y_2 = 1.60$ ft and the drop in the water surface on the hump $= 2.00 - (0.30 + 1.60) = 0.10$ ft.

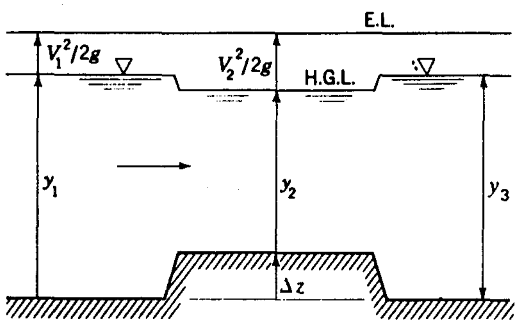
(c) In this case the hump height $\Delta z = 0.60$ ft which is greater than the critical hump height. Hence critical depth ($y_c = 1.12$ ft) will occur on the hump. Writing the energy equation for this case, we have

$$y_1 + (V_1^2/2g) = 0.60 + 1.12 - (V_2^2/2g) \quad (6)$$

From continuity,
$$(4 \times y_1)V_1 = 27 \text{ cfs} \quad (7)$$

and, for critical flow,
$$V_2^2/2g = \tfrac{1}{2}y_2 = 0.56 \text{ ft} \quad (8)$$

Combining Eqs. (6), (7), and (8) gives $y_1 + [(\frac{27}{4})^2/(2g)y_1^2] = 2.28$ from which $y_1 = 2.12$ ft, 0.66 ft, or a negative answer which has no physical meaning. In this case, damming action occurs and the depth y_1 upstream of the hump, is 2.12 ft. On the hump the depth passes through critical depth of 1.12 ft and just downstream of the hump the depth will be 0.66 ft. The depth will then increase in the downstream direction following an M_2 water-surface profile until a hydraulic jump occurs to return the depth to the normal uniform flow depth of 2.00 ft.



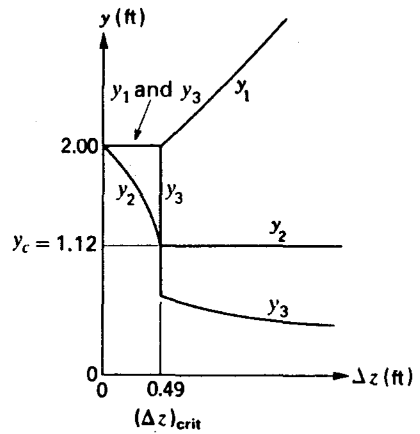


Fig. 14-29

14.87 For the channel of Prob. 14.84, compute the "open-channel Reynolds number" assuming that water at 50 °F is flowing. Refer to Fig. A-5 to verify whether or not the flow is wholly rough. Determine ϵ from Fig. A-5 and compare it with the value computed in Prob. 14.84.

▌ $N_R = Rv/\nu$. From Prob. 14.84, $Q = 225$ cfs, $R = 2.272$ ft, $f = 0.0225$, and $A = 3.41[10 + (2)(3.41)] = 57.36$ ft²; $v = Q/A = 225/57.36 = 3.923$ ft/s.

Open channel: $N_R = (2.272)(3.923)/(1.40 \times 10^{-5}) = 6.37 \times 10^5$

Equivalent pipe: $N_R = (4)(6.37 \times 10^5) = 2.55 \times 10^6$

From Fig. A-5 with $N_R = 2.55 \times 10^6$ and $f = 0.0225$, $\epsilon/D = 0.0018$ and the flow is wholly rough: $\epsilon = 0.0018D = (0.0018)(4R) = 0.0018[(4)(2.272)] = 0.0164$ ft. This value of ϵ (0.0164) is close to the value of 0.0156 computed in Prob. 14.84.

14.88 On the assumption that Fig. A-5 applies also to open channels, find the rate of discharge of water at 60 °F in a 100-in-diameter smooth concrete pipe flowing half full ($R = D/4$), if the pipe is laid on a grade of 1.8 ft/mile.

▌ $\epsilon/D = 0.001/(\frac{100}{12}) = 0.00012$. Try turbulent flow with $f = 0.0135$:

$$v = \sqrt{(8g/f)(Rs)} \qquad R = D/4 = (\tfrac{100}{12})/4 = 2.083 \text{ ft} \qquad s = \tfrac{1.8}{5280} = 0.0003409$$

$$v = \sqrt{[(8)(32.2)/0.0135][(2.083)(0.0003409)]} = 3.681 \text{ ft/s}$$

$$N_R = Dv/\nu = [(4)(2.083)](3.681)/(1.21 \times 10^{-5}) = 2.53 \times 10^6$$

From Fig. A-5, $f = 0.013$. Try $f = 0.013$:

$v = \sqrt{[(8)(32.2)/0.013][(2.083)(0.0003409)]} = 3.751$ ft/s $\qquad N_R = [(4)(2.083)](3.751)/(1.21 \times 10^{-5}) = 2.58 \times 10^6$

$$f = 0.013 \quad \text{(O.K.)} \qquad Q = Av = [(\tfrac{1}{2})(\pi)(\tfrac{100}{12})^2/4](3.751) = 102 \text{ ft}^3/\text{s}$$

14.89 For the channel of Prob. 14.84, compute the flow rate for depth 8 ft.

▮ $Q=(A)(1.486/n)(R^{2/3})(s^{1/2})$. For $y_0=8$ ft, $A=8[10+(2)(8)]=208\ \text{ft}^2$, $p_w=10+(2)(\sqrt{5})(8)=45.78$ ft, $Q=(208)(1.486/0.016)(208/45.78)^{2/3}(0.0006)^{1/2}=1298\ \text{ft}^3/\text{s}$.

14.90 Figure 14-30 shows a cross section of a canal designed to carry 1590 cfs. The canal is lined with concrete ($n=0.014$). Find the grade of the canal and the drop in elevation per mile.

▮

$$Q=(A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A=(10.2)(50.6+20)/2=360.1\ \text{ft}^2$$

$$p_w=18.39+20+18.39=56.78\ \text{ft}$$

$$1590=(360.1)(1.486/0.014)(360.1/56.78)^{2/3}(s)^{1/2} \qquad s=0.000147$$

$$\text{Drop in elevation}=(0.000147)(5280)=0.776\ \text{ft/mile}$$

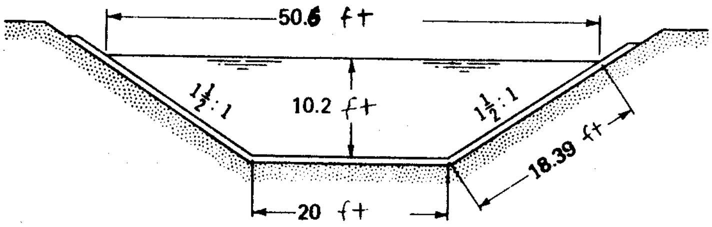


Fig. 14-30

14.91 If the flow in the canal of Prob. 14.90 were halved, all other data, including the slope, being the same, what would be the depth of water?

▮

$$Q=(A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A=y[20+(2)(1.5)(y)+20]/2=20y+1.5y^2$$

$$p_w=20+(2)[\sqrt{y^2+(1.5y)^2}]=20+3.606y$$

$$795=(20y+1.5y^2)(1.486/0.014)[(20y+1.5y^2)/(20+3.606y)]^{2/3}(0.000147)^{1/2}$$

$$y=7.10\ \text{ft} \qquad \text{(by trial and error)}$$

14.92 Evaluate ϵ for Prob. 14.90.

▮ $1/\sqrt{f}=2\log(14.8R/\epsilon)$, $n=0.093f^{1/2}R^{1/6}$. Therefore,

$$2\log(14.8R/\epsilon)=0.093R^{1/6}/n \qquad R=360.1/56.78=6.342\ \text{ft} \qquad \text{(from Prob. 14.90)}$$

$$2\log[(14.8)(6.342)/\epsilon]=(0.093)(6.342)^{1/6}/0.014 \qquad \epsilon=0.00284\ \text{ft}$$

14.93 Find the capacity of the canal of Prob. 14.90, assuming the grade to be 1.5 ft/mile.

▮ $Q=(A)(1.486/n)(R^{2/3})(s^{1/2})$. From Prob. 14.90, $A=360.1\ \text{ft}^2$, $n=0.014$, $R=360.1/56.78=6.342$ ft, $Q=(360.1)(1.486/0.014)(6.342)^{2/3}(1.5/5280)^{1/2}=2207\ \text{ft}^3/\text{s}$.

14.94 Water flows uniformly in a 2-m-wide rectangular channel at a depth of 0.5 m. The channel slope is 0.0025 and $n=0.015$. Find the flow rate.

▮

$$Q=(A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad R=(2)(0.5)/(0.5+2+0.5)=0.3333\ \text{m}$$

$$Q=[(2)(0.5)](1.0/0.015)(0.3333)^{2/3}(0.0025)^{1/2}=1.60\ \text{m}^3/\text{s}$$

14.95 At what depth will $4\ \text{m}^3/\text{s}$ of water flow in a 3-m-wide rectangular channel if $n=0.016$ and $s=0.0009$.

▮

$$Q=(A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 4=(3d)(1.0/0.016)[3d/(d+3+d)]^{2/3}(0.0009)^{1/2}$$

$$d=1.00\ \text{m} \qquad \text{(by trial and error)}$$

14.96 The water cross-sectional area in Fig. 14-31 measures 191 ft², and the wetted perimeter is 39.1 ft. If the flow is 1580 ft³/s and $n = 0.012$, find the slope.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 1580 = (191)(1.486/0.012)(191/39.1)^{2/3}(s)^{1/2}$ $\quad s = 0.000538$

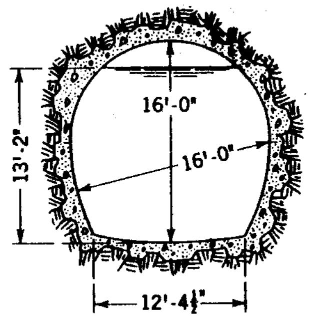


Fig. 14-31

14.97 Rework Prob. 14.96 for a completely filled conduit.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 1580 = [(\pi)(16)^2/4](1.486/0.012)(\frac{16}{4})^{2/3}(s)^{1/2}$ $\quad s = 0.000634$

14.98 A 30-in-diameter pipe is known to have a Manning's n of 0.01800. Calculate Manning's n for a 96-in-diameter pipe that has the same ϵ-value as a 30-in pipe for which $n = 0.01800$.

▮ $1/\sqrt{f} = 2\log(14.8R/\epsilon)$, $n = 0.093 f^{1/2} R^{1/6}$. Therefore, $2\log(14.8R/\epsilon) = 0.093R^{1/6}/n$. For 30-in pipe: $R = (\frac{30}{12})/4 = 0.6250$ ft, $2\log[(14.8)(0.6250)/\epsilon] = (0.093)(0.6250)^{1/6}/0.01800$, $\epsilon = 0.03780$ ft. For 96-in pipe: $R = (\frac{96}{12})/4 = 2.000$ ft, $2\log[(14.8)(2.000)/0.03780] = (0.093)(2.000)^{1/6}/n$, $n = 0.01804$.

14.99 The dimensions indicated in Fig. 14-32 pertain to a flow of 30 cfs with $n = 0.018$. Compute the required slope.

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = (2.5)(5) + (2.5 - 2)[(25 - 5)/2]/2 = 15.00 \text{ ft}^2$$
$$p_w = 2.5 + 5 + 2 + \sqrt{(2.5 - 2)^2 + [(25 - 5)/2]^2} = 19.51 \text{ ft}$$
$$30 = (15.00)(1.0/0.018)(15.00/19.51)^{2/3}(s)^{1/2} \qquad s = 0.00184$$

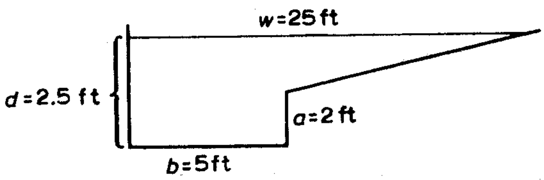


Fig. 14-32

14.100 Water flows at 10 ft/s in a rectangular trough 6 ft wide for which $n = 0.013$. Find the slope needed for a water depth of 3 ft.

▮ $v = (1.486/n)(R^{2/3})(s^{1/2})$ $\quad 10 = (1.486/0.013)[(6)(3)/(3 + 6 + 3)]^{2/3}(s)^{1/2}$ $\quad s = 0.00446$

14.101 In Fig. 14-33, area A_1 is 100 ft by 2 ft, A_2 is 30 ft by 10 ft, and A_3 is 200 ft by 3 ft. Compute the flow rate if $s = 0.0018$, $n_1 = n_3 = 0.03$, and $n_2 = 0.020$.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$
$$Q_1 = [(100)(2)](1.486/0.03)[(100)(2)/(2 + 100)]^{2/3}(0.0018)^{1/2} = 658 \text{ ft}^3/\text{s}$$
$$Q_2 = [(30)(10)](1.486/0.02)\{(30)(10)/[(10 - 2) + 30 + (10 - 3)]\}^{2/3}(0.0018)^{1/2} = 3350 \text{ ft}^3/\text{s}$$
$$Q_3 = [(200)(3)](1.486/0.03)[(200)(3)/(3 + 200)]^{2/3}(0.0018)^{1/2} = 2597 \text{ ft}^3/\text{s}$$
$$Q = Q_1 + Q_2 + Q_3 = 658 + 3350 + 2597 = 6605 \text{ ft}^3/\text{s}$$

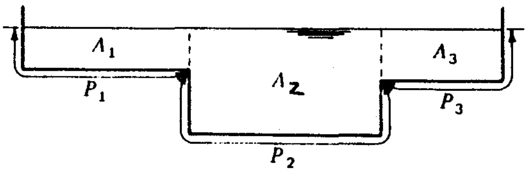


Fig. 14-33

14.102 In Prob. 14.99 the parameters are changed as follows: $a = 3$ ft, $b = 6$ ft, $d = 5$ ft, $w = 36$ ft; $n = 0.020$. Given the slope $s = 0.0015$, determine the rate of flow.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. For depth = 5 ft: $A_1 = (6)(5) = 30.00\ \text{ft}^2$, $A_2 = (5-3)[(36-6)]/2 = 30.00\ \text{ft}^2$, $(p_w)_1 = 5 + 6 + 3 = 14.00$ ft, $(p_w)_2 = \sqrt{(5-3)^2 + [(36-6)]^2} = 30.07$ ft, $Q = (30.00)(1.486/0.020)(30.00/14.00)^{2/3}(0.0015)^{1/2} + (30.00)(1.486/0.020)(30.00/30.07)^{2/3}(0.0015)^{1/2} = 230\ \text{ft}^3/\text{s}$.

14.103 Determine the depth below the surface of clear water at which the velocity (u) as given by the von Karman equation is equal to the mean velocity (v).

▮ $u = v + (1/K)(\sqrt{gy_0s})[1 + 2.3\log(y/y_0)]$. Where $u = v$, $v = v + (1/K)(\sqrt{gy_0s})[1 + 2.3\log(y/y_0)]$, $y/y_0 = 0.367$. Hence, the velocity (u) is equal to the mean velocity (v) when $y/y_0 = 0.367$, or at a depth below the surface of $1 - 0.367$, or 0.633 times the channel depth.

14.104 Figure 14-34 is the longitudinal section of a very wide channel. The fluid is clear water ($K = 0.40$). Given $a = 2.50$ ft and $n = 0.020$, find b.

▮ $v = v + (1/K)(\sqrt{gy_0s})[1 + 2.3\log(y/y_0)]$. Working with a 1-ft width of channel,

$$v = (1.486/0.020)[(2+4+3)(1)/1]^{2/3}(s)^{1/2} \qquad s = 0.000009676v^2 \qquad u_a^2/2g = 2.80\ \text{ft}$$

$$u_a = \sqrt{(2)(32.2)(2.50)} = 12.69\ \text{ft/s} \qquad u = v + (1/K)(\sqrt{gy_0s})[1 + 2.3\log(y/y_0)]$$

$$12.69 = v + (1/0.40)[\sqrt{(32.2)(2+4+3)(0.000009676v^2)}][1 + 2.3\log(2+4)/(2+4+3)] \qquad v = 11.76\ \text{ft/s}$$

$$u_b = 11.76 + (1/0.40)[\sqrt{(32.2)(2+4+3)(0.000009676)(12.45^2)}]\{1 + 2.3\log[2/(4+3+2)]\} = 10.98\ \text{ft/s}$$

$$b = 10.98^2/[(2)(32.2)] = 1.87\ \text{ft}$$

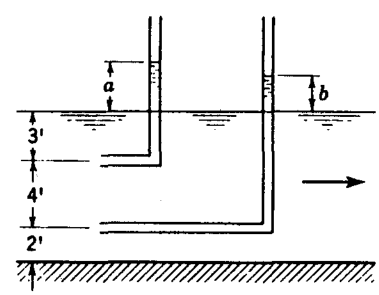


Fig. 14-34

14.105 Water flows uniformly in a very wide rectangular channel at a depth of 1.5 m ($s = 0.006$ and $n = 0.015$). Calculate the velocities at two-thirds and at full depth.

▮ $v = (1.0/n)(R^{2/3})(s^{1/2})$. For a very wide channel, $R = y_0 = 1.5$ m: $v = (1.0/0.015)(1.5)^{2/3}(0.006)^{1/2} = 6.767$ m/s, $u = v + (1/K)(\sqrt{gy_0s})[1 + 2.3\log(y/y_0)]$. At $y = 1.0$ m, $u = 6.767 + (1/0.40)[\sqrt{(9.807)(1.5)(0.006)}][1 + 2.3\log(1.0/1.5)] = 7.21$ m/s. At $y = 1.5$ m, $u = 6.767 + (1/0.40)[\sqrt{(9.807)(1.5)(0.006)}][1 + 2.3\log(1.5/1.5)] = 7.51$ m/s.

14.106 For a rectangular channel cross section of area 5 m^2, tabulate hydraulic radius versus width over the range of widths from 1 m to 5 m.

▮ $R = A/p_w = A/(w + 2d) = Aw(w^2 + 2A)$; see following table.

w, m	R, m
1	0.455
2	0.715
3	0.790
4	0.769
5	0.715

14.107 Set up a general expression for the wetted perimeter p_w of a trapezoidal channel in terms of the cross-sectional area A, depth y, and angle of side slope ϕ. Then differentiate p_w with respect to y with A and ϕ held constant. From this, prove that $R = y/2$ for the section of greatest hydraulic efficiency (i.e., smallest p_w for a given A).

▮ Let B = bottom width.

$$A = By + (y)(y \tan \phi) = By + y^2 \tan \phi \qquad B = A/y - y \tan \phi$$

$$p_w = B + 2y \sec \phi = A/y - y \tan \phi + 2y \sec \phi$$

$$dp_w/dy = -A/y^2 - \tan \phi + 2 \sec \phi = -(By + y^2 \tan \phi)/y^2 - \tan \phi + 2 \sec \phi$$

Setting $dp_w/dy = 0$, $(By + y^2 \tan \phi)/y^2 = 2 \sec \phi - \tan \phi$, $B = 2y \sec \phi - 2y \tan \phi = (2y)(\sec \phi - \tan \phi)$.

$$R = \frac{A}{p_w} = \frac{By + y^2 \tan \phi}{B + 2y \sec \phi} = \frac{(2y)(\sec \phi - \tan \phi)(y) + y^2 \tan \phi}{(2y)(\sec \phi - \tan \phi) + 2y \sec \phi} = \frac{y}{2}$$

14.108 Prove that the most efficient triangular section is the one with a 90° vertex angle.

▮ See Fig. 14-35.

$$A = a^2 \sin \phi \cos \phi \qquad p_w = 2a \qquad R = A/p_w$$

$$R = (a^2 \sin \phi \cos \phi)/2a = (a/2)(\sin \phi \cos \phi) \qquad dR/d\phi = (a/2)(\cos^2 \phi - \sin^2 \phi) = 0$$

Hence, $\cos \phi = \sin \phi$; or $\phi = 45°$ and the vertex angle = (2)(45), or 90°.

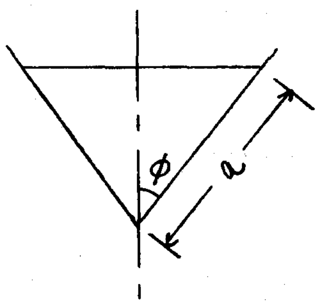


Fig. 14-35

14.109 A canal cut in smooth earth ($n = 0.03123$) must transport 9.0 m^3/s of water at a depth of at most 1.5 m (see Fig. 14-36). If the side slopes are 2:1 and the channel slope is 0.0004, what must be the width at the bottom?

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = 1.5[b + b + (2)(1.5) + (2)(1.5)]/2 = 1.5b + 4.5$$

$$p_w = b + (2)\sqrt{1.5^2 + [(2)(1.5)]^2} = b + 6.708$$

$$9.0 = (1.5b + 4.5)(1.0/0.03123)[(1.5b + 4.5)/(b + 6.708)]^{2/3}(0.0004)^{1/2}$$

$$b = 6.00 \text{ m} \qquad \text{(by trial and error)}$$

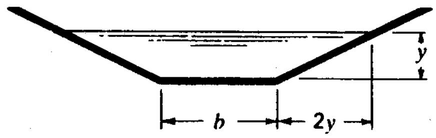


Fig. 14-36

14.110 Refer to Fig. 14-36. If the discharge in the canal ($n = 0.03123$) is to be 6 m³/s at a depth of 1.5 m and if the velocity is not to exceed 1.0 m/s, what must be the width at the bottom and the slope?

$$A = Q/v = 6/(1.0) = 6.00 \text{ m}^2 \qquad (b)(1.5) + (2)[(2)(1.5)(1.5)]/2 = 6.00 \qquad b = 1.0 \text{ m}$$

$$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad p_w = 1.00 + (2)\sqrt{1.5^2 + [(2)(1.5)]^2} = 7.708 \text{ m}$$

$$\tfrac{60}{60} = (1.0/0.03123)(6.00/7.708)^{2/3}(s)^{1/2} \qquad s = 0.001362$$

14.111 A rectangular flume of smooth wood ($n = 0.012$) slopes 1 ft per 1000 ft. ***(a)*** Compute the rate of discharge if the width is 4 ft and the depth of water is 2 ft. ***(b)*** What would be the rate of discharge if the width were 2 ft and the depth of water 4 ft? ***(c)*** Which of the two forms would have the greater capacity and which would require less lumber?

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

(*a*) $$Q = [(4)(2)](1.468/0.012)[(4)(2)/(2 + 4 + 2)]^{2/3}(\tfrac{1}{1000})^{1/2} = 31.3 \text{ ft}^3/\text{s}$$

(*b*) $$Q = [(4)(2)](1.468/0.012)[(4)(2)/(4 + 2 + 4)]^{2/3}(\tfrac{1}{1000})^{1/2} = 27.0 \text{ ft}^3/\text{s}$$

(*c*) Lumber ratio $= (2 + 4 + 2)/(4 + 2 + 4) = 0.80$ Flow ratio $= 31.3/27.0 = 1.16$

Hence, the first design provides 16 percent more flow capacity while requiring only 80 percent as much lumber.

14.112 What diameter of semicircular channel will provide the same capacity as a rectangular channel of width 6 m and depth 3 m? Assume s and n are the same for both channels.

$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. Since $Q_s = Q_r$ and $(1.486/n)(s^{1/2})$ is constant, $A_s R_s^{2/3} = A_r R_r^{2/3}$.

$$[(\pi d^2/4)/2](d/4)^{2/3} = [(6)(3)][(6)(3)/(3 + 6 + 3)]^{2/3} \qquad d = 6.57 \text{ m}$$

14.113 Consider steady flow of water in a circular concrete pipe ($n = 0.016$) of diameter 10 ft at a depth of 4 ft. Using Fig. A-18, determine the flow rate and the average velocity of flow ($s = 0.0004$).

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad Q_{\text{full}} = [(\pi)(10)^2/4](1.486/0.016)(\tfrac{10}{4})^{2/3}(0.0004)^{1/2} = 269 \text{ ft}^3/\text{s}$$

$$v_{\text{full}} = Q_{\text{full}}/A_{\text{full}} = 269/[(\pi)(10)^2/4] = 3.43 \text{ ft/s} \qquad d/d_{\text{full}} = \tfrac{4}{10} = 0.40 \quad \text{or} \quad 40 \text{ percent}$$

From Fig. A-18, $Q/Q_{\text{full}} = 32$ percent and $v/v_{\text{full}} = 88$ percent: $Q = (0.32)(269) = 86.1$ ft³/s, $v = (0.88)(3.43) = 3.02$ ft/s.

14.114 At what depth will water flow at 0.25 m³/s in a 1.0-m-diameter concrete pipe ($n = 0.012$) on a slope of 0.0004?

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$

$$Q_{\text{full}} = [(\pi)(1.0)^2/4](1.0/0.012)[1.0/4]^{2/3}(0.0004)^{1/2} = 0.519 \text{ m}^3/\text{s}$$

$Q/Q_{\text{full}} = 0.25/0.519 = 0.48$, or 48 percent. From Fig. A-18, $d/d_{\text{full}} = 51$ percent: $d = (0.51)(1.0) = 0.51$ m.

14.115 Evaluate the friction factor f for laminar flow in terms of the Reynolds number, and compare with the equation for pipe flow. (*Note:* Recall that for a wide channel the hydraulic radius is approximately equal to the depth.)

▌ $N_R = (4R)(v)/\nu = 4y_0v/\nu$ $\quad q = (g/\nu)(y_0^3/3)(s)$ $\quad v = Q/A = q/y_0 = gsy_0^2/3\nu = [(8g/f)(R)(s)]^{1/2}$

Therefore, $[(8g/f)(R)(s)] = vgsy_0^2/3\nu$. With $R = y_0$, $f = 24\nu/y_0v = 96\nu/4y_0v = 96/N_R$. This compares with $f = 64/N_R$ for pipe flow.

14.116 A viscous fluid ($\nu = 0.0015$ ft²/s) flows down a flat plate 10 ft wide. Find the maximum rate of discharge for laminar flow, assuming a critical Reynolds number of 500.

▌ $N_R = Rv/\nu = y_0q/\nu y_0 = q/\nu$ $\quad q = (500)(0.0015) = 0.7500$ ft³/s per ft $\quad Q = (10)(0.7500) = 7.50$ ft³/s

14.117 At what rate will water at 60 °F flow in a wide rectangular channel on a slope of 0.00018 if the depth is 0.01 ft?

▌ Assuming laminar flow, $q = (g/\nu)(y_0^3/3)(s) = [32.2/(1.21 \times 10^{-5})](0.01^3/3)(0.00018) = 1.60 \times 10^{-4}$ ft³/s/ft, $N_R = Rv/\nu = y_0v/\nu = (y_0)(q/y_0)/\nu = q/\nu = 1.60 \times 10^{-4}/(1.21 \times 10^{-5}) = 13$. Since $N_R < 500$, the assumption of laminar flow is justified.

14.118 At what rate will water at 15 °C flow in a wide, smooth, rectangular channel on a slope of 0.0002, if the depth is 8.0 mm?

▌ Assuming laminar flow, $q = (g/\nu)(y_0^3/3)(s) = [9.807/(1.16 \times 10^{-6})][(0.008)^3/3](0.0002) = 2.89 \times 10^{-4}$ (m³/s)/m. Checking the assumption: $N_R = Rv/\nu = y_0v/\nu = (y_0)(q/y_0)/\nu = q/\nu = 2.89 \times 10^{-4}/(1.16 \times 10^{-6}) = 249$ (laminar).

14.119 Water flows with a velocity of 4 fps and at a depth of 2 ft in a wide rectangular channel. Is the flow subcritical or supercritical? Find the alternate depth for the same discharge and specific energy.

▌ $v^2/2g = 4^2/[(2)(32.2)] = 0.2484$ ft $\quad y/2 = \frac{2}{2} = 1.000$ ft

Since 0.2484 < 1.000, the flow is subcritical.

$$E = y + (1/2g)(q^2/y^2) = 2 + 0.2484 = 2.2484 \text{ ft} \qquad q = (4)(2) = 8.000 \text{ (ft}^2\text{/s)/ft}$$

$$2.2484 = y + \{1/[(2)(32.2)]\}(8.00^2/y^2) \qquad 2.2484y^2 - y^3 - 0.9938 = 0$$

Since $y = 2$ is one known solution, divide by $(y - 2)$ to yield $y^2 - 0.2484y - 0.4968 = 0$, $y = [-(-0.2484) \pm \sqrt{(-0.2484)^2 - (4)(1)(-0.4968)}]/[(2)(1)] = 0.840$ ft.

14.120 Water flows down a wide rectangular channel of concrete ($n = 0.014$) laid on a slope of 2.4 mm/m. Find the depth and rate of flow for critical conditions.

▌ $v = (1.0/n)(R^{2/3})(s^{1/2})$ $\quad v_c = \sqrt{gy_c}$ $\quad \sqrt{(9.807)(y_c)} = (1.0/0.014)(y_c)^{2/3}(0.0024)^{1/2}$

$y_c = 0.514$ m $\quad q = y_cv_c = (y_c)(\sqrt{gy_c}) = (0.514)[\sqrt{(9.807)(0.514)}] = 1.15$ (m³/s)/m

14.121 Water flows at 15 ft/s in a rectangular channel at a depth of 2 ft. Find the critical depth for **(a)** this specific energy, **(b)** this rate of discharge.

▌ **(a)** $E = y + v^2/2g = 2 + 15^2/[(2)(32.2)] = 5.494$ ft $\quad y_c = (\frac{2}{3})(E) = (\frac{2}{3})(5.494) = 3.66$ ft

(b) $q = yv = (2)(15) = 30.00$ ft³/s/ft $\quad y_c = (q^2/g)^{1/3} = (30.00^2/32.2)^{1/3} = 3.03$ ft

14.122 A flow of 120 ft³/s is carried in a rectangular channel 10 ft wide at a depth of 1.5 ft, the channel is made of smooth concrete ($n = 0.013$). Find **(a)** the necessary slope, **(b)** the roughness coefficient needed to produce uniform critical flow for the given rate of discharge on this slope.

▮ (*a*) $Q=(A)(1.486/n)(R^{2/3})(s^{1/2})$ $\qquad 120=[(1.5)(10)](1.486/0.013)[(1.5)(10)/(1.5+10+1.5)]^{2/3}(s)^{1/2}$

$$s=0.00405$$

(*b*) $\qquad y_c=(q^2/g)^{1/3} \qquad y_c=[(\tfrac{120}{10})^2/32.2]^{1/3}=1.648\text{ ft}$

$$v_c=\sqrt{gy_c}=\sqrt{(32.2)(1.648)}=7.285\text{ ft/s}$$

$$7.285=(1.486/n)[(1.648)(10)/(1.648+10+1.648)]^{2/3}(0.00405)^{1/2} \qquad n=0.0150$$

14.123 A rectangular channel 4 m wide shows a wavy surface at depth 2.5 m. Estimate the rate of discharge.

▮ The wavy surface indicates a near-critical depth; hence $y_c \approx 2.5$ m. $q_{\max}=\sqrt{gy_c^3}=\sqrt{(9.807)(2.5)^3}=$ 12.38 m^3/s/m, $Q=(4)(12.38)=49.5$ m^3/s.

14.124 Probing for oil, geologists drive a small pipe vertically down into the bed of a fast-running stream. The upstream "wake" of the pipe has a spread $\theta=120°$. Estimate the stream velocity.

▮ $\qquad \sin(\theta/2)=c/v \qquad c=\sqrt{gy}=\sqrt{(32.2)(2)}=8.025\text{ ft/s} \qquad \sin 60°=8.025/v \qquad v=9.3\text{ ft/s}$

14.125 A speedboat in shallow water lifts a 1-ft wave (height above undisturbed surface), which travels at 9 mph. Find the approximate depth of the water.

▮ $\quad c=\sqrt{(g)(y+\Delta y)[(y+\Delta y/2)/y]}=(9)(5280)/3600=13.2\text{ ft/s} \qquad 13.2=\sqrt{(32.2)(y+1)[(y+\tfrac{1}{2})/y]}$

$$174.2=32.2y+48.30+16.1/y \qquad 32.2y^2-125.9y+16.1=0$$

The larger root is

$$y=[-(-125.9)+\sqrt{(-125.9)^2-(4)(32.2)(16.1)}]/[(2)(32.2)]=3.78\text{ ft}$$

14.126 A rectangular channel 10 ft wide carries a flow of 180 cfs. Find the critical depth and critical velocity for this flow.

▮ $\qquad y_c=(q^2/g)^{1/3}=[(\tfrac{180}{10})^2/32.2]^{1/3}=2.16\text{ ft} \qquad v_c=\sqrt{gy_c}=\sqrt{(32.2)(2.16)}=8.34\text{ ft/s}$

14.127 Water flows at 600 ft^3/min in a isosceles right-triangular flume for which $n=0.012$. Find the critical depth and critical slope.

▮ $Q^2/g=A^3/B$. If y = depth, $A=y^2$ and $B=2y$.

$$(600/60)^2/32.2=(y^2)^3/2y \qquad y=y_c=1.44\text{ ft} \qquad Q=(A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$R=A/p_w=1.44^2/[(2)(\sqrt{1.44^2+1.44^2})]=0.5091\text{ ft} \qquad 10=(1.44^2)(1.486/0.012)(0.5091)^{2/3}(s)^{1/2}$$

$$s=s_c=0.00373$$

14.128 A trapezoidal canal with side slopes of 2:1 has a bottom width of 4 m and carries a flow of 23 m^3/s. Calculate the critical depth.

▮ $Q^2/g=A^3/B$. If y = depth, $A=4y+2y^2$ and $B=4+4y$.

$$23^2/9.807=(4y+2y^2)^3/(4+4y) \qquad y=y_c=1.22\text{ m} \qquad \text{(by trial and error)}$$

14.129 Find the specific energy at depth 3 ft for flow of 100 ft^3/s through a 10-ft-diameter tunnel.

▮ $E=y+v^2/2g$. For $y=3$ ft:

$y/y_{\text{full}}=\tfrac{3}{10}=0.30$ or 30 percent From Fig. A-18 $A/A_{\text{full}}=25$ percent $A=[(\pi)(10)^2/4](0.25)=19.6\text{ ft}^2$

$$v=Q/A=100/19.6=5.10\text{ ft/s} \qquad E=3+5.10^2/[(2)(32.2)]=3.40\text{ ft}$$

14.130 A circular conduit flowing half full carries 500 ft³/s at velocity 10 ft³/s. If $n = 0.13$, will the flow be subcritical or supercritical?

$$A = Q/v = (\pi d^2/4)/2 \qquad \tfrac{500}{10} = (\pi d^2/4)/2 \qquad d = 11.28\text{ ft}$$

$$A = [(\pi)(11.28)^2/4]/2 = 49.97\text{ ft}^2 \qquad N_F = v/\sqrt{gy} = v/\sqrt{(g)(A/B)} = 10/\sqrt{(32.2)(49.97/11.28)} = 0.837$$

Since $N_F < 1.0$, the flow is subcritical.

14.131 Figure 14-37 describes the cross section of an open channel for which $s_0 = 0.02$ and $n = 0.015$. The sketch is drawn to the scale shown. When the flow rate is 100 cfs, find **(a)** the depth for uniform flow and **(b)** the critical depth.

The cross-sectional area is found by planimetry and the wetted perimeter by use of dividers.

(a)

y_0, ft	A, ft	p_w, ft	R, ft	$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$, cfs
1	1.30	3.29	0.395	10
2	4.90	6.12	0.801	59
3	10.48	8.96	1.170	163

A plot of Q versus y_0 (not shown) indicates that $y_0 = 2.50$ ft for $Q = 100$ cfs.

(b)

y_c, ft	A, ft²	$B(x_L + x_R)$, ft	$Q = (gA^3/B)^{1/2}$, cfs
1	1.30	2.60	5
2	4.90	4.60	29
3	10.48	6.55	75
4	18.30	9.10	147

A plot of Q versus y_c (not shown) indicates that $y_c = 3.35$ ft for $Q = 100$ cfs.

14.132 Refer to Fig. 14-37 and replace feet dimensions with meters. Let the slope be 0.007 with $n = 0.015$. When the flow rate is 50 m³/s, find **(a)** the depth for uniform flow and **(b)** the critical depth.

y ft	x_L ft	x_R ft
0	0	0
1	1.5	1.1
2	2.6	2.0
3	3.25	3.3
4	3.6	5.5
5	3.85	8.2

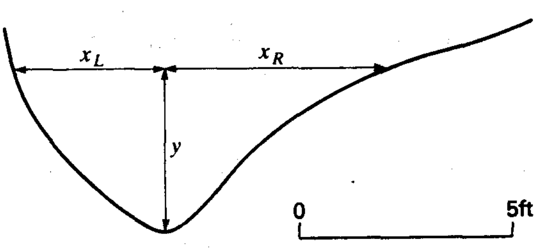


Fig. 14-37

The cross-sectional area is found by planimetry and the wetted perimeter by use of dividers.

(a)

y_0, m	A, m²	p_w, m	R, m	$Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, m³/s
1	1.30	3.29	0.395	4
2	4.90	6.12	0.801	24
3	10.48	8.96	1.170	65

A plot of Q versus y_0 (not shown) indicates that $y_0 = 2.70$ m for $Q = 50\ \text{m}^3/\text{s}$.

(b)

y_c, m	A, m^2	$B(x_L + x_R)$, m	$Q = (gA^3/B)^{1/2}$, m^3/s
1	1.30	2.60	3
2	4.90	4.60	16
3	10.48	6.55	42
4	18.30	9.10	81

A plot of Q versus y_c (not shown) indicates that $y_c = 3.25$ m for $Q = 50\ \text{m}^3/\text{s}$.

14.133 A rectangular channel 2 m wide carries 2.2 m^3/s of water in subcritical uniform flow at a depth of 1.0 m. What is the lowest transverse hump in the bottom such that y_c is attained at the peak?

▮ $$(\Delta z)_c = E_0 - E_{\min} \qquad E = y + v^2/2g \qquad v = Q/A = 2.2/[(2)(1.0)] = 1.10\ \text{m/s}$$

$$E_0 = 1.0 + 1.10^2/[(2)(9.807)] = 1.0617\ \text{m}$$

$$E_{\min} = (\tfrac{3}{2})(y_c) \qquad y_c = (q^2/g)^{1/3} = [(\tfrac{2.2}{2})^2/9.807]^{1/3} = 0.4978\ \text{m}$$

$$E_{\min} = (\tfrac{3}{2})(0.4978) = 0.7467 \qquad (\Delta z)_c = 1.0617 - 0.7467 = 0.315$$ [illegible]

(b) $q = 2.0/1.3 = 1.538$ m $\quad y_c = (1.538^2/9.807)^{1/3} = 0.6225$ m $\quad E_{min} = (\frac{3}{2})(0.6225) = 0.9338$ m

Since $E_{min} = 0.9338 < 1.063 = E_0$, y_c does not occur at the constriction.

(c) $\Delta z_{crit} = E_0 - E_{min}$. With constriction, $E_{min} = 0.9338$ m; hence, $\Delta z_{crit} = 1.063 - 0.9338 = 0.1292$ m. Since $\Delta z = 0.15 > 0.1292 = \Delta z_{crit}$, y_c does occur at the hump with constriction.

14.136 Rework Prob. 14.86 ($y_0 = 2.0$ ft) for the case where the flow rate is 16 cfs.

(a) $q = Q/B = \frac{16}{4} = 4.000\ \text{ft}^2/\text{s/ft} \qquad y_c = (q^2/g)^{1/3} = (4.000^2/32.2)^{1/3} = 0.7921$ ft

Since $y_0 > y_c$, the flow is subcritical (and slope is mild).

(b) Install hump with $\Delta z = 0.30$ ft. First find critical hump height.

$$E = y + (\tfrac{1}{2g})(q^2/y^2) \qquad E_0 = 2.00 + \{1/[(2)(32.2)]\}(4.000^2/2.00^2) = 2.062\ \text{ft}$$

$$E_{min} = (\tfrac{3}{2})(y_c) = (\tfrac{3}{2})(0.7921) = 1.188\ \text{ft} \qquad E_0 = \Delta z_{crit} + E_{min} \qquad 2.062 = \Delta z_{crit} + 1.188 \qquad \Delta z_{crit} = 0.874\ \text{ft}$$

Since $\Delta z < \Delta z_{crit}$, y_c does not occur on the hump and damming action does not result.

$$E_h = E_0 - \Delta z \qquad E = y + (1/2g)(q^2/y^2) = y_h + \{1/[(2)(32.2)]\}(4.000^2/y_h^2)$$

$$y_h + \{1/[(2)(32.2)]\}(4.000^2/y_h^2) = 2.062 - 0.30$$

By trial and error, $y_h = 1.67$ ft (subcritical) and 0.432 ft (supercritical). The flow does not pass through y_c, so it cannot become supercritical. Therefore, the water depth on the 0.30-ft hump is 1.67 ft.

(c) Increase hump height to 0.60 ft. Still, $\Delta z < \Delta z_{crit}$, so y_c does not occur on the hump and damming action does not result. $y_h + \{1/[(2)(32.2)]\}(4.000^2/y_h^2) = 2.062 - 0.60$. By trial and error $y_h = 1.32$ ft (subcritical) and 0.511 ft (supercritical). The flow does not pass through y_c, so it cannot become supercritical. Therefore, the water depth on the 0.60-ft hump is 1.32 ft.

14.137 A 4-ft-wide rectangular ditch carries 40 ft³/s of water at a depth of 2.80 ft. A man standing in the middle of the ditch presents a 1-ft width to the stream. Find the local change in the water-surface elevation.

$v_0 = Q/A = 40/[(2.80)(4)] = 3.571$ ft/s. At the man.

$$v = 40/[(4-1)(y)] = 13.33/y \qquad E = y + v^2/2g = y + (13.33/y)^2/[(2)(32.2)] = y + 2.759/y^2$$

$$E_0 = 2.80 + 3.571^2/[(2)(32.2)] = 2.998\ \text{ft} \qquad y + 2.759/y^2 = 2.998$$

By trial and error, $y = 2.59$ ft or 1.26 ft. Since locally supercritical conditions are impossible, $y = 2.59$ ft. Change in water depth $= 2.80 - 2.59 = 0.21$ ft drop.

14.138 A rectangular channel 10 ft wide carries 20 ft³/s in uniform flow at depth 1.0 ft. Find the local change in water-surface elevation caused by an obstruction 0.20 ft high on the floor of the channel.

$y_c = (q^2/g)^{1/3} = [(\frac{20}{10})^2/32.2]^{1/3} = 0.499$ ft. Since $y_c < 1.0$ ft, the flow is subcritical.

$$E = y + v^2/2g \qquad v = Q/A = 20/[(1.0)(10)] = 2.000\ \text{ft/s} \qquad E_1 = 1.0 + 2.000^2/[(2)(32.2)] = 1.062\ \text{ft}$$

$$E_2 = E_1 - \Delta z = 1.062 - 0.20 = 0.862\ \text{ft} \qquad E = y + (1/2g)(q^2/y^2)$$

$$E_2 = y_2 + \{1/[(2)(32.2)]\}[(\tfrac{20}{10})^2/y_2^2] \qquad 0.862 = y_2 + 0.06211/y_2^2 \qquad y_2 = 0.752\ \text{ft} \quad \text{(by trial and error)}$$

Change in water-surface elevation $= 1.00 - (0.20 + 0.752) = 0.048$ ft (drop).

14.139 Rework Prob. 14.138 for a flow at depth 0.31 ft.

Since $y_c = 0.499\ \text{ft} > 0.31$ ft, the flow is supercritical.

$$E = y + v^2/2g \qquad v = Q/A = 20/[(0.31)(10)] = 6.452\ \text{ft/s} \qquad E_1 = 0.31 + 6.452^2/[(2)(32.2)] = 0.9564\ \text{ft}$$

$$E_2 = E_1 - \Delta z = 0.9564 - 0.20 = 0.7564\ \text{ft} \qquad E = y + (1/2g)(q^2/y^2) \qquad E_2 = y_2 + \{1/[(2)(32.2)]\}[(\tfrac{20}{10})^2/y_2^2]$$

$$0.7564 = y_2 + 0.06211/y_2^2 \qquad y_2 = 0.45\ \text{ft} \quad \text{(by trial and error)}$$

Change in water-surface elevation $= (0.20 + 0.45) - 0.31 = 0.34$ ft (rise).

14.140 Repeat Prob. 14.137 for flow depth 0.90 ft.

▮ $E = y + v^2/2g$ $\quad v_0 = Q/A = 40/[(4)(0.90)] = 11.11$ ft/s $\quad E_0 = 0.90 + 11.11^2/[(2)(32.2)] = 2.817$ ft

As in Prob. 14.137, $v = 13.33/y$.

$$E = y + (13.33/y)^2/[(2)(32.2)] = y + 2.759/y^2 \qquad 2.817 = y + 2.759/y^2$$

$$y = 1.39 \text{ ft} \qquad \text{(by trial and error)}$$

Change in water-surface elevation = 1.39 − 0.90 = 0.49 ft (rise).

14.141 If 1.4 m^3/s of water flows uniformly in a channel of width 1.8 m at a depth of 0.75 m, what is the change in water-surface elevation at a section contracted to a 1.2 m width with a 6-cm depression in the bottom?

▮ $y_c = (q^2/g)^{1/3}$ $\quad (y_c)_2 = [(1.4/1.2)^2/9.807]^{1/3} = 0.5177$ m $\quad E_2 = E_1 + \Delta z$ $\quad E = y + v^2/2g$

$$v_1 = Q/A = 1.4/[(0.75)(1.8)] = 1.037 \text{ m/s} \qquad E_1 = 0.75 + 1.037^2/[(2)(9.807)] = 0.8048 \text{ m}$$

$$v_2 = 1.4/(1.2y_2) = 1.167/y_2$$

$$E_2 = y_2 + (1.167/y_2)^2/[(2)(9.807)] = y_2 + 0.06943/y_2^2 \qquad y_2 + 0.06943/y_2^2 = 0.8048 + \tfrac{6}{100}$$

$y_2 = 0.737$ m (subcritical) or 0.377 m (supercritical) (by trial and error). y_2 cannot be less than $(y_c)_2$; hence, $y_2 = 0.737$ m. Change in water-surface elevation $= 0.75 - (0.737 - \tfrac{6}{100}) = 0.073$ m (drop).

14.142 A finished-concrete channel 8 ft wide has a slope of 0.5° and a water depth of 4 ft. Predict the uniform flow rate by ***(a)*** Manning's formula with $n = 0.012$ and ***(b)*** the friction-factor analysis with $\epsilon = 0.0032$ ft.

▮ ***(a)***

$$R = A/p_w = (8)(4)/(4 + 8 + 4) = 2.000 \text{ ft}$$

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = [(8)(4)](1.486/0.012)(2.000)^{2/3}(\tan 0.5°)^{1/2} = 588 \text{ ft}^3/\text{s}$$

(b)

$$C = (8g/f)^{1/2} \qquad 1/f^{1/2} = 2.0 \log(3.7D_h/\epsilon) \qquad D_h = 4R = (4)(2.000) = 8.000 \text{ ft}$$

$$1/f^{1/2} = (2.0)\log[(3.7)(8.000)/(0.0032)] \qquad f = 0.01589 \qquad C = [(8)(32.2)/0.01589]^{1/2} = 127.3$$

$$Q = CA(Rs)^{1/2} = (127.3)[(4)(8)][(2.000)(\tan 0.5°)]^{1/2} = 538 \text{ ft}^3/\text{s}$$

14.143 The asphalt-lined trapezoidal channel in Fig. 14-38 carries 300 cfs of water under uniform flow conditions when $s = 0.0015$. What is the normal depth? Use $n = 0.016$.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad A = (\tfrac{1}{2})(6 + b_0)(y_n) = 6y_n + y_n^2 \cot 50°$ $\quad p_w = 6 + 2y_n \csc 50°$

$$300 = (6y_n + y_n^2 \cot 50°)(1.486/0.016)\,[(6y_n + y_n^2 \cot 50°)/(6 + 2y_n \csc 50°)]^{2/3}(0.0015)^{1/2}$$

$$y_n = 4.58 \text{ ft} \qquad \text{(by trial and error)}$$

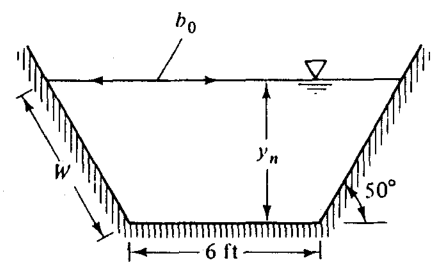


Fig. 14-38

14.144 What are the best dimensions for a rectangular brick ($n = 0.015$) channel designed to carry 5 m^3/s of water in uniform flow with $s = 0.001$?

▮ The best dimensions are for width (b) equal to twice the depth (y): $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $5 = [(y)(2y)](1.0/0.015)[(y)(2y)/(y + 2y + y)]^{2/3}(0.001)^{1/2}$, $y = 1.268$ m, $b = (2)(1.268) = 2.536$ m.

14.145 A wide rectangular clean-earth channel ($n = 0.022$) has a flow rate q of 50 cfs/ft ***(a)*** What is the critical depth? ***(b)*** What type of flow exists if $y = 3$ ft? ***(c)*** What is the critical slope?

▮ ***(a)***

$$y_c = (q^2/g)^{1/3} = (50^2/32.2)^{1/3} = 4.27 \text{ ft}$$

(b) For $y < y_c$, the flow will be supercritical.

(c)

$$s_c = gn^2/(2.208y_c^{1/3}) = (32.2)(0.022)^2/[(2.208)(4.27)^{1/3}] = 0.00435$$

14.146 The 50° triangular channel in Fig. 14-39 has a flow rate Q of 16 m³/s. Compute **(a)** y_c, **(b)** v_c, and **(c)** s_c if $n = 0.018$.

(a)
$$gA^3 = b_0Q^2 \qquad A = y^2 \cot 50° \qquad b_0 = 2y \cot 50°$$
$$(9.807)(y^2 \cot 50°)^3 = (2y \cot 50°)(16^2) \qquad y = y_c = 2.37 \text{ m}$$

(b)
$$v_c = Q/A = 16/[(2.37)^2(\cot 50)] = 3.39 \text{ m/s}$$

(c)
$$R = A/p_w = y^2 \cot 50°/(2y \csc 50°) = (2.37)^2(\cot 50°)/[(2)(2.37)(\csc 50°)] = 0.7617 \text{ m}$$
$$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad 3.39 = (1.0/0.018)(0.7617)^{2/3}(s)^{1/2} \qquad s = s_c = 0.00535$$

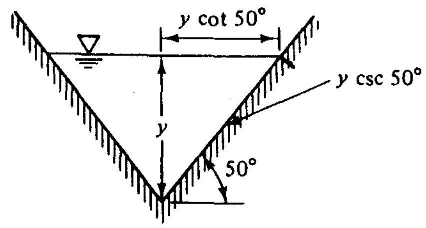


Fig. 14-39

14.147 The formula for shallow-water wave-propagation speed, $c_0 = \sqrt{gy}$, is independent of the physical properties of the liquid, i.e., density, viscosity, or surface tension. Does this mean that waves propagate at the same speed in water, mercury, gasoline, and glycerin? Explain.

$c_0 = \sqrt{gy}$ is correct for any fluid except for viscosity and surface tension (very small wave) effects. It would be accurate for water, mercury, and gasoline but inaccurate for glycerin (too viscous).

14.148 A 10-cm-high wave travels over (shallow) water of depth 1.2 m. Compute the wave speed c and the velocity δv induced by the wave.

$$c = \sqrt{gy(1 + \delta y/y)[1 + (\tfrac{1}{2})(\delta y/y)]} = \sqrt{(9.79)(1.2)(1 + 1/12)[1 + (\tfrac{1}{2})(1/12)]} = 2.60 \text{ m/s}$$
$$\delta v = \frac{(c)(\delta y/y)}{1 + (\delta y/y)} = \frac{(2.60)(1/12)}{13/12} = 0.20 \text{ m/s}$$

14.149 Water flows rapidly in a flat wide channel 0.4 m deep. Pebbles dropped successively in the water at the same spot create two circular ripples which are shown from above in Fig. 14-40. Determine the current speed V.

The centers of the circles move at current V; hence,

$$(x_0)(c_0) = 4V \qquad \text{(smaller circle)}$$
$$(x_0 + 4 + 6 + 9)(c_0) = 9V \qquad \text{(larger circle)}$$

Subtracting these equations, $19c_0 = 5V$, $V = 19c_0/5$, $c_0 = \sqrt{gy} = \sqrt{(9.807)(0.4)} = 1.981$ m/s, $V = (19)(1.981)/5 =$ 7.53 m/s.

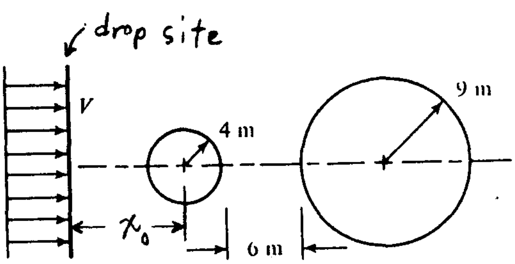


Fig. 14-40

14.150 Rework Prob. 14.149 if the channel depth is 0.6 m and Fig. 14-41 applies.

The centers of the circles move at current V. If the pebble drop site is at distance x_0 ahead of the small circle,

$$(x_0)(c_0) = 3V \qquad \text{(smaller circle)}$$
$$(x_0 + 4)(c_0) = 9V \qquad \text{(larger circle)}$$

Subtracting these equations, $4c_0 = 6V$, $V = 2c_0/3$, $c_0 = \sqrt{gy} = \sqrt{(9.807)(0.6)} = 2.426$ m/s, $V = (2)(2.426)/3 =$ 1.62 m/s.

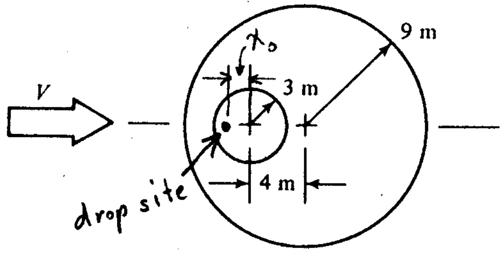


Fig. 14-41

14.151 Consider flow in a wide channel over a bump, as in Fig. 14-42. One can estimate the water-depth change or *transition* with frictionless flow. Use continuity and the Bernoulli equation to show that $dy/dx = -(dh/dx)/(1 - V^2/gy)$. Explain under what conditions the surface might rise above its upstream position y_0.

From the Bernoulli equation, $p_0/\gamma + V_0^2/2g + y_0 = p_1/\gamma + V_1^2/2g + y_1 + h$, $0 + V_0^2/2g + y_0 = 0 + V_1^2/2g + y_1 + h$,

$$V\,dV/g + dy + dh = 0 \qquad (1)$$

From continuity, $V_0 y_0 = V_1 y_1 = \text{constant}$, $V\,dy + y\,dV = 0$, $dV = -V\,dy/y$. Substituting this value of dV into Eq. (1), $(V)(-V\,dy/y)/g + dy + dh = 0$, $dy/dx = -(dh/dx)/(1 - V^2/gy)$. If $dh/dx > 0$ (a bump) and $V^2 < gy$ (subcritical Froude number), dy/dx will be negative and the water level will drop across the bump. If $V^2 > gy$ (supercritical Froude number), the water level will rise.

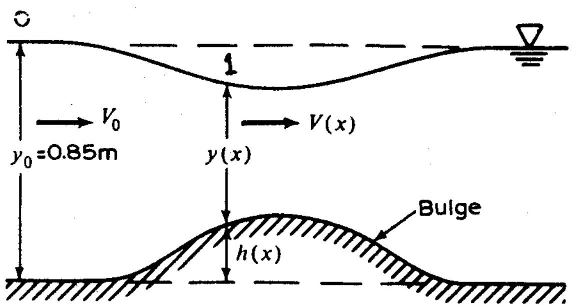


Fig. 14-42

14.152 In Fig. 14-42, the flow rate is 1.2 m³/s per meter of channel width. Compute the velocity and water depth at the top of the bulge ($h_{max} = 0.1$ m). Characterize the flow there.

$$p_0/\gamma + V_0^2/2g + y_0 = p_1/\gamma + V_1^2/2g + y_1 + h \qquad 0 + V_0^2/2g + y_0 = 0 + V_1^2/2g + y_1 + h_{max}$$

$$q = (V_0)(0.85) = V_1 y_1 = 1.2 \qquad V_0 = 1.412\text{ m/s} \qquad y_1 = 1.2/V_1$$

$$1.412^2/[(2)(9.807)] + 0.85 = V_1^2/[(2)(9.807)] + 1.2/V_1 + 0.1 \qquad 0.05098V_1^3 - 0.8516V_1 + 1.2 = 0$$

$$V_1 = 1.7\text{ m/s} \quad \text{(by trial and error)} \qquad y_1 = 1.2/1.7 = 0.706\text{ m}$$

$$(N_F)_1 = V_1/\sqrt{gy_1} = 1.7/\sqrt{(9.807)(0.706)} = 0.646 \quad \text{(subcritical)}.$$

14.153 Rework Prob. 14.152 for a flow rate three times as large.

$$p_0/\gamma + V_0^2/2g + y_0 = p_1/\gamma + V_1^2/2g + y_1 + h \qquad 0 + V_0^2/2g + y_0 = 0 + V_1^2/2g + y_1 + h$$

$$q = (V_0)(0.85) = V_1 y_1 = 3.6 \qquad V_0 = 4.24\text{ m/s} \qquad y_1 = 3.6/V_1$$

$$4.24^2/[(2)(9.807)] + 0.85 = V_1^2/[(2)(9.807)] + 3.6/V_1 + 0.1 \qquad 0.05098V_1^3 - 1.666V_1 + 3.6 = 0$$

$$V_1 = 3.6\text{ m/s} \quad \text{(by trial and error)} \qquad y_1 = 3.6/3.6 = 1.00\text{ m}$$

$$(N_F)_1 = V_1/\sqrt{gy_1} = 3.6/\sqrt{(9.807)(1.00)} = 1.15 \quad \text{(supercritical)}.$$

14.154 Given the flow of a channel of large width b under a sluice gate, as shown in Fig. 14-43 and assuming frictionless steady flow with negligible upstream kinetic energy, derive a formula for the dimensionless ratio $Q^2/y_1^3 b^2 g$ as a function of the ratio y_2/y_1.

▌ From the Bernoulli equation, $y_1 + V_1^2/2g = y_2 + V_2^2/2g$. Assuming $V_1^2/2g$ to be negligible, $y_1 = y_2 + V_2^2/2g$. From continuity: $V_1 y_1 = V_2 y_2 = Q/b$, $V_2 = Q/by_2$, $y_1 = y_2 + (Q/by_2)^2/2g$. Multiplying by y_2^2/y_1^3, $y_2^2/y_1^2 = y_2^3/y_1^3 + Q^2/2gb^2y_1^3$, $Q^2/2gb^2y_1^3 = (y_2/y_1)^2 - (y_2/y_1)^3$.

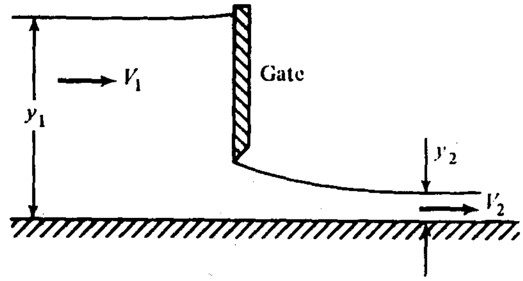


Fig. 14-43

14.155 With reference to Prob. 14.154, take $y_1 = 1.00$ m and $y_2 = 0.60$ m, $b = 10$ m. Compute flow rate Q if the upstream kinetic energy is **(a)** neglected and **(b)** considered.

▌ **(a)** If $V_1^2/2g$ is neglected, the equation developed in Prob. 14.154 applies: $Q^2/2gb^2y_1^3 = (y_2/y_1)^2 - (y_2/y_1)^3$, $Q^2/[(2)(9.807)(10)^2(1.00)^3] = [(0.60)/(1.00)]^2 - [(0.60)/(1.00)]^3$, $Q = 16.8\ \text{m}^3/\text{s}$.
(b) From the Bernoulli equation,

$$y_1 + V_1^2/2g = y_2 + V_2^2/2g \qquad V_1 y_1 = V_2 y_2 \qquad (V_1)(1.00) = (V_2)(0.60) \qquad V_2 = 1.667V_1$$
$$1.00 + V_1^2/[(2)(9.807)] = 0.60 + (1.667V_1)^2/[(2)(9.807)] \qquad V_1 = 2.10\ \text{m/s}$$
$$Q = Av = [(10)(1.00)](2.10) = 21.0\ \text{m}^3/\text{s}$$

14.156 For laminar = sheet draining, the flow may become turbulent if N_R exceeds 500. If $s = 0.0018$, what is the maximum sheet thickness y_0 to ensure laminar flow of water at 20 °C?

▌ $$N_R = gy_0^3 s/3\nu^2 \qquad 500 = (9.807)(y_0^3)(0.0018)/[(3)(1.02\times10^{-6})^2] \qquad y_0 = 4.45\ \text{mm}$$

14.157 A rectangular channel 3 m wide contains water 2 m deep. If the slope is 0.5°, compute the discharge for uniform flow by the Manning formula ($n = 0.014$).

▌ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) = [(3)(2)](1.0/0.014)[(3)(2)/(2+3+2)]^{2/3}(\tan 0.5°)^{1/2} = 36.13\ \text{m}^3/\text{s}$$

14.158 Solve Prob. 14.157 by the Moody formula ($\epsilon = 0.0024$ m).

▌ $$C = \sqrt{8g/f} \qquad 1/f^{1/2} = 2\log(3.7D_h/\epsilon) \qquad D_h = 4R = (4)[(3)(2)/(2+3+2)] = 3.429\ \text{m}$$
$$1/f^{1/2} = 2\log[(3.7)(3.429)/(0.0024)] \qquad f = 0.01804 \qquad C = \sqrt{(8)(9.807)/0.01804} = 65.95$$
$$Q = CA(Rs)^{1/2} = (65.95)[(3)(2)]\{[(3)(2)/(2+3+2)](\tan 0.5°)\}^{1/2} = 34.22\ \text{m}^3/\text{s}$$

14.159 A large metal trough ($n = 0.014$) slopes at 1:500, its section corresponds to Fig. 14-35 with $\phi = 30°$. Find the water depth for uniform flow at 222.2 L/s.

▌ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = (y)[y\tan 30°] = 0.5774y^2 \qquad p_w = 2y/\cos 30° = 2.309y$$
$$R = A/p_w = 0.5774y^2/2.309y = 0.2501y \qquad 0.2222 = (0.5774y^2)(1.0/0.014)(0.2501y)^{2/3}(\tfrac{1}{500})^{1/2} \qquad y = 0.639\ \text{m}$$

14.160 A trapezoidal channel similar to that of Fig. 14-38 has a bottom width of 10 ft, a side angle of 45°, and a depth of 3 ft. Compute the uniform flow discharge for a clean earth channel ($n = 0.021$) with $s = 0.0004$.

▌ $$A = (10)(3) + (3)[3\tan(90° - 45°)] = 39\ \text{ft}^2 \qquad p_w = 10 + (2)[3/\cos(90° - 45°)] = 18.49\ \text{ft}$$
$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = (39)(1.486/0.021)(39/18.49)^{2/3}(0.0004)^{1/2} = 90.8\ \text{ft}^3/\text{s}$$

14.161 For the channel of Prob. 14.160, compute the normal depth if the flow rate is 170 cfs.

▌ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = 10y + (y)[y\tan(90° - 45°)] = 10y + y^2$$
$$p_w = 10 + (2)[y/\cos(90° - 45°)] = 10 + 2.828y$$
$$170 = (10y + y^2)(1.486/0.021)[(10y + y^2)/(10 + 2.828y)]^{2/3}(0.0004)^{1/2}$$
$$y = 5.63\ \text{ft} \qquad \text{(by trial and error)}$$

14.162 Use Moody's formula to express Manning's n as a function of channel size R.

▮ Combining $C = \sqrt{8g/f} = (1.486/n)(R^{1/6})$, $1/f^{1/2} = 2\log(3.7D_h/\epsilon)$, and $D_h = 4R$, we obtain $n = R^{1/6}/(\alpha + \beta \log R)$, for suitable constants α and β.

14.163 A 7-ft-diameter metal ($n = 0.020$) conduit is flowing half-full on a slope of 1:880. Estimate the discharge.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = [(\tfrac{1}{2})(\pi)(7)^2/4](1.486/0.020)(7/4)^{2/3}(1/880)^{1/2} = 70.0\ \text{ft}^3/\text{s}$$

14.164 A trapezoidal conduit similar to that of Fig. 14-38 has a bottom width of 6 m, a side angle of 35°, and carries 60 m³/s of water at a depth of 4 m. If $n = 0.014$, find the slope.

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = (6)(4) + 4[4\tan(90° - 35°)] = 46.85\ \text{m}^2$$

$$p_w = 6 + (2)[4/\cos(90° - 35°)] = 19.95\ \text{m} \qquad 60 = (46.85)(1.0/0.014)(46.85/19.95)^{2/3}(s)^{1/2} \qquad s = 0.000103$$

14.165 For the conduit of Prob. 14.164, determine the water depth given a slope of 0.000512 and a discharge of 40 m³/s.

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = 6y + y[y\tan(90° - 35°)] = 6y + 1.428y^2$$

$$p_w = 6 + 2[y/\cos(90° - 35°)] = 6 + 3.487y$$

$$40 = (6y + 1.428y^2)(1.0/0.014)[(6y + 1.428y^2)/(6 + 3.487y)]^{2/3}(0.000512)^{1/2}$$

$$y = 2.14\ \text{m} \qquad \text{(by trial and error)}$$

14.166 Uniform water flow in a wide brick channel ($n = 0.015$) of slope 0.02° moves over a 10-cm bump as in Fig. 14-44. A slight depression in water surface results. If the minimum water depth over the bump is 50 cm, compute the velocity over the bump and the flow rate per meter of width.

▮ $$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad v_1 = (1.0/0.015)(y_1)^{2/3}(\tan 0.02°)^{1/2} = 1.246y_1^{2/3}$$

From continuity, $v_1 y_1 = v_2 y_2$, $(1.246y_1^{2/3})(y_1) = (v_2)(\tfrac{50}{100})$, $v_2 = 2.492y_1^{5/3}$. Applying the Bernoulli equation and neglecting bottom slope,

$$y_1 + v_1^2/2g = y_2 + v_2^2/2g + h_{max} \qquad y_1 + (1.246y_1^{2/3})^2/[(2)(9.807)] = \tfrac{50}{100} + (2.492y_1^{5/3})^2/[(2)(9.807)] + \tfrac{10}{100}$$

$$y_1 = 0.623\ \text{m} \qquad \text{(by trial and error)} \qquad v_1 = (1.246)(0.623)^{2/3} = 0.9089\ \text{m/s}$$

$$v_2 = (2.492)(0.623)^{5/3} = 1.13\ \text{m/s} \qquad q = v_1 y_1 = (0.9089)(0.623) = 0.566\ (\text{m}^3/\text{s})/\text{m}$$

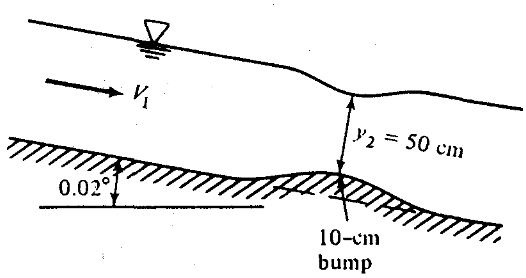


Fig. 14-44

14.167 A tar-coated ($n = 0.014$) triangular channel has 45° sides and a slope of 0.0004. Compute the normal depth for a discharge of 140 ft³/s.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = (y)(y\cot 45°) = 1.000y^2$$

$$p_w = (2y)(\csc 45°) = 2.828y \qquad R = A/p_w = 1.000y^2/2.828y = 0.3536y$$

$$140 = (1.000y^2)(1.486/0.014)(0.3536y)^{2/3}(0.0004)^{1/2} \qquad y = 6.24\ \text{ft}$$

14.168 A brick ($n = 0.015$) rectangular channel with $s = 0.002$ is designed to carry 230 cfs of water in uniform flow. There is an argument over whether the channel width should be 4 ft or 8 ft. Which design needs fewer bricks? By what percentage?

▌ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. For 4-ft width: $230 = (4y)(1.486/0.015)[4y/(y+4+y)]^{2/3}(0.002)^{1/2}$, $y = 9.31$ ft (by trial and error), $p_w = 9.31 + 4 + 9.31 = 22.62$ ft. For 8-ft width: $230 = (8y)(1.486/0.015)[8y/(y+8+y)]^{2/3}(0.002)^{1/2}$, $y = 4.07$ ft (by trial and error), $p_w = 4.07 + 8 + 4.07 = 16.14$ ft. Hence, the 8-ft-width design needs fewer bricks by $(22.62 - 16.14)/22.62 = 0.286$, or 28.6 percent.

14.169 In the T-shaped channel of Fig. 14-45, the two arms have rougher walls than does the trunk. Approximate the discharge, if $y_1 = 20$ ft, $y_2 = 5$ ft, $b_1 = 50$ ft, $b_2 = 100$ ft, $n_1 = 0.018$, $n_2 = 0.036$, with a slope of 0.0004.

▌
$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$
$$Q_1 = [(50)(20+5)](1.486/0.018)[(50)(20+5)/(20+50+20)]^{2/3}(0.0004)^{1/2} = 11\,925 \text{ ft}^3/\text{s}$$
$$Q_2 = [(100)(5)](1.486/0.036)[(100)(5)/(100+5)]^{2/3}(0.0004)^{1/2} = 1168 \text{ ft}^3/\text{s}$$
$$Q = 11\,925 + (2)(1168) = 14\,261 \text{ ft}^3/\text{s}$$

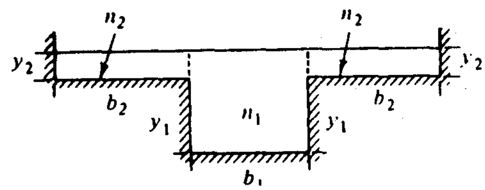


Fig. 14-45

14.170 Repeat Prob. 14.169 for the following parameters: $y_1 = b_1 = 9$ m, $n_1 = 0.018$, $y_2 = 1$ m, $b_2 = 100$ m, $n_2 = 0.036$. The angle of downslope is 0.1°.

▌
$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$
$$Q_1 = [(9)(9+1)](1.0/0.018)[(9)(9+1)/(9+9+9)]^{2/3}(\tan 0.1°)^{1/2} = 466 \text{ m}^3/\text{s}$$
$$Q_2 = [(100)(1)](1.0/0.036)[(100)(1)/(100+1)]^{2/3}(\tan 0.1°)^{1/2} = 115 \text{ m}^3/\text{s}$$
$$Q = 466 + (2)(115) = 696 \text{ m}^3/\text{s}$$

14.171 A 1-m-diameter clay tile ($n = 0.014$) sewer pipe runs half-full on a slope of 0.004. Compute the flow rate by the Manning formula.

▌ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) = [(\tfrac{1}{2})(\pi)(1)^2/4](1.0/0.014)(\tfrac{1}{4})^{2/3}(0.004)^{1/2} = 0.704 \text{ m}^3/\text{s}$$

14.172 Solve Prob. 14.171 by the Moody formula, with $\epsilon = 0.0026$ m.

▌
$$1/f^{1/2} = 2\log(3.7D/\epsilon) = 2\log[(3.7)(1)/0.0026] \qquad f = 0.02514$$
$$C = \sqrt{8g/f} = \sqrt{(8)(9.807)/0.02514} = 55.86$$
$$Q = CA(Rs)^{1/2} = 55.86[(\tfrac{1}{2})(\pi)(1)^2/4][(\tfrac{1}{4})(0.004)]^{1/2} = 0.694 \text{ m}^3/\text{s}$$

14.173 Four of the sewer pipes from Prob. 14.171 empty into a single finished-cement ($n = 0.012$) pipe, also sloping at 0.004°. If the large pipe is also to run at half-full, what should its diameter be?

▌ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\quad (4)(0.704) = [(\tfrac{1}{2})(\pi)(D)^2/4](1.0/0.012)(D/4)^{2/3}(0.004)^{1/2}$ $\quad D = 1.59$ m

14.174 For the circular channel of Fig. 14-46, if $n = 0.016$, $D = 3$ m, and the slope is 0.2°, find the normal depth for a discharge of 20 m³/s.

▌
$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad y = (D/2)(1+\sin\theta) = (\tfrac{3}{2})(1+\sin\theta)$$
$$A = (\tfrac{1}{2})(D/2)^2(\pi + 2\theta + \sin 2\theta) = (\tfrac{1}{2})(\tfrac{3}{2})^2(\pi + 2\theta + \sin 2\theta) = (1.125)(\pi + 2\theta + \sin 2\theta)$$
$$p_w = (D/2)(\pi + 2\theta) = (\tfrac{3}{2})(\pi + 2\theta)$$
$$20 = [(1.125)(\pi + 2\theta + \sin 2\theta)](1.0/0.016)[(1.125)(\pi + 2\theta + \sin 2\theta)/(\tfrac{3}{2})(\pi + 2\theta)]^{2/3}(\tan 0.2°)^{1/2}$$
$$\theta = 31.57° \quad \text{(by trial and error)} \qquad y = (\tfrac{3}{2})(1 + \sin 31.57°) = 2.29 \text{ m}$$

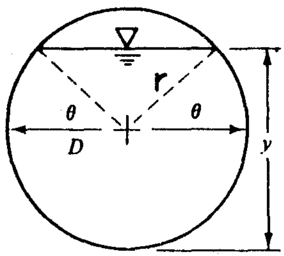


Fig. 14-46

14.175 For fixed n and s, find the diameter of a semicircular channel that will have the same discharge as a rectangular channel with $b = 4$ m and $y = 2$ m.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad Q_1 = Q_2$$

$$[(4)(2)](1.0/n)[(4)(2)/(2+4+2)]^{2/3}(s)^{1/2} = [(\tfrac{1}{2})(\pi)(D)^2/4](1.0/n)(D/4)^{2/3}(s)^{1/2} \qquad D = 4.38\text{ m}$$

14.176 A trapezoidal channel has $n = 0.020$ and $s = 0.0004$ and is made in the shape of a half-hexagon for maximum efficiency (see Fig. 14-47). For what length of side will the channel carry 250 cfs of water?

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = (\tfrac{3}{2})(b^2)(\sin 60°) = 1.299b^2$$

$$250 = (1.299b^2)(1.486/0.020)(1.299b^2/3b)^{2/3}(0.0004)^{1/2} \qquad b = 7.64\text{ ft}$$

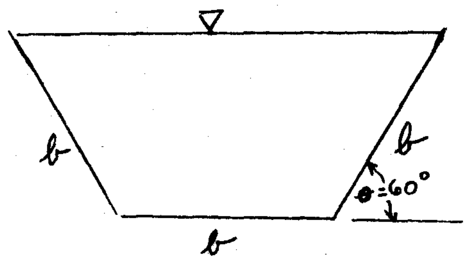


Fig. 14-47

14.177 Compute the discharge of a semicircular channel of the same area, slope, and n-value as the channel of Prob. 14.176.

$$A = (1.299)(7.64)^2 = 75.82\text{ ft}^2 \quad \text{(from Prob. 14.176)} \qquad 75.82 = (\tfrac{1}{2})(\pi)(d)^2/4 \qquad d = 13.90\text{ ft}$$

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = 75.82(1.486/0.020)(13.9/4)^{2/3}(0.0004)^{1/2} = 258\text{ ft}^3/\text{s}$$

14.178 Find the optimal dimensions of a wooden ($n = 0.012$) rectangular channel that will carry 4.0 m^3/s at $s = 0.0008$.

For an optimum, width = twice depth ($w = 2y$): $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $4.0 = [(y)(2y)](1.0/0.012)[(y)(2y)/(y+2y+y)]^{2/3}(0.0008)^{1/2}$, $y = 1.118$ m, $w = (2)(1.118) = 2.236$ m.

14.179 How deep should an asphalt ($n = 0.015$) trapezoidal channel, with sides sloping at 45°, be to carry 5 m^3/s at $s = 0.0006$?

In an optimal design, $R = y/2$. $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $A = (y^2)[(2)(1 + \cot 45°)^{1/2} - \cot 45°] = 1.828y^2$, $5 = (1.828y^2)(1.0/0.015)(y/2)^{2/3}(0.0006)^{1/2}$, $y = 1.44$ m.

14.180 Compare the channel of Prob. 14.179, with a semicircular channel of the same slope and area.

$$A = (1.828)(1.44)^2 = 3.79\text{ m}^2 \quad \text{(from Prob. 14.179)} \qquad 3.79 = (\tfrac{1}{2})(\pi)(D)^2/4 \qquad D = 3.107\text{ m}$$

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) = (3.79)(1.0/0.015)(3.107/4)^{2/3}(0.0006)^{1/2} = 5.23\text{ m}^3/\text{s}$$

The semicircular channel carries $(5.23 - 5)/5 = 0.046$, or 4.6 percent more than the trapezoidal channel.

14.181 If the side angles of the trapezoidal channel of Prob. 14.176 are reduced to $\theta = 20°$ and if the bottom flat width is 8 ft, find the normal depth of the new channel.

$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad A = by + y^2 \cot 20° = 8y + 2.747y^2$ $\quad p_w = b + 2y \csc 20° = 8 + 5.848y$

$$250 = (8y + 2.747y^2)(1.486/0.020)[(8y + 2.747y^2)/(8 + 5.848y)]^{2/3}(0.0004)^{1/2}$$

$$y = 4.40 \text{ ft} \qquad \text{(by trial and error)}$$

14.182 A 16-ft-deep, clean-earth river ($n = 0.030$) has a flow rate $q = 140$ cfs/ft. Calculate the Froude number of the river.

$$y_c = (q^2/g)^{1/3} = (140^2/32.2)^{1/3} = 8.47 \text{ ft} \qquad v = 140/16 = 8.750 \text{ ft/s}$$

$$N_F = v/\sqrt{gy_c} = 8.75/\sqrt{(32.2)(8.47)} = 0.530$$

14.183 Find the critical slope in Prob. 14.182 by the Moody method, using $\epsilon = 0.78$ ft.

$$s_c = f/8 \qquad 1/f^{1/2} = 2 \log (3.7D/\epsilon)$$

Using data from Prob. 14.182, $D = 4R = 4y_c = (4)(8.47) = 33.88$ ft, $1/f^{1/2} = 2 \log [(3.7)(33.88)/0.78]$, $f = 0.05137$, $s_c = 0.05137/8 = 0.00642$.

14.184 Are the normal flows corresponding to the two widths in Prob. 14.168 subcritical or supercritical?

$y_c = (Q^2/b^2g)^{1/3}$. For $w = 4$ ft, $y_c = \{230^2/[(4)^2(32.2)]\}^{1/3} = 4.68 \text{ ft} < 9.31$ ft; so the flow is subcritical. For $w = 8$ ft, $y_c = \{230^2/[(8)^2(32.2)]\}^{1/3} = 2.95 \text{ ft} < 4.07$ ft; so the flow is subcritical.

14.185 In a 250-mm-deep test channel, a (stationary) ship model sets up the bow wave shown in Fig. 14-48. Determine the flow velocity in the channel.

$$\sin \theta = 1/N_F = \sqrt{gh}/v \qquad \sin 30° = \sqrt{(9.807)(0.250)}/v \qquad v = 3.13 \text{ m/s}$$

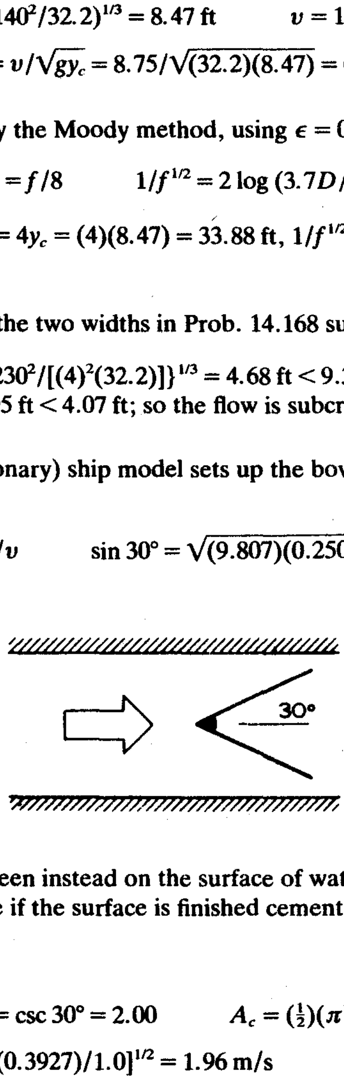


Fig. 14-48

14.186 Suppose that the wave of Fig. 14-48 is seen instead on the surface of water flowing half-full in a circular channel of diameter 1.0 m. What is the flow rate if the surface is finished cement?

Flow is supercritical.

$$v = N_F v_c \qquad N_F = \csc 30° = 2.00 \qquad A_c = (\tfrac{1}{2})(\pi)(1.0)^2/4 = 0.3927 \text{ m}^2$$

$$v_c = (gA_c/b_0)^{1/2} = [(9.807)(0.3927)/1.0]^{1/2} = 1.96 \text{ m/s} \qquad v = (2.00)(1.96) = 3.92 \text{ m/s}$$

$$Q = Av = (0.3927)(3.92) = 1.539 \text{ m}^3/\text{s}$$

14.187 Suppose that the wave of Fig. 14-48 is seen instead on the surface of water flowing full in a half-hexagon of side length 300 mm. What is the flow rate if the sides are of planed wood?

Flow is supercritical.

$$v = N_F v_c \qquad N_F = \csc 30° = 2.00 \qquad v_c = (gA_c/b_0)^{1/2}$$

$$b_0 = 0.300 + (2)(0.300)(\cos 60°) = 0.600 \text{ m} \qquad y = (0.300)(\sin 60°) = 0.260 \text{ m}$$

$$A_c = (\tfrac{1}{2})(0.600 + 0.300)(0.260) = 0.1169 \text{ m}^2 \qquad v_c = [(9.807)(0.1169)/0.600]^{1/2} = 1.38 \text{ m/s}$$

$$v = (2.00)(1.38) = 2.76 \text{ m/s} \qquad Q = Av = (0.1169)(2.76) = 0.323 \text{ m}^3/\text{s}$$

14.188 Find the critical depth and the critical slope for the conduit of Prob. 14.164.

$$A=(b_0Q^2/g)^{1/3} \qquad b_0=6+2y_c\cot 35°=6+2.856y_c \qquad A=(y_c)(6+y_c\cot 35°)=6y_c+1.428y_c^2$$
$$6y_c+1.428y_c^2=[(6+2.856y_c)(50^2)/9.807]^{1/3} \qquad y_c=1.86\text{ m} \qquad \text{(by trial and error)}$$
$$v_c=(1.0/n)(R^{2/3})(s^{1/2}) \qquad A=(6)(1.86)+(1.428)(1.86)^2=16.10\text{ m}^2 \qquad v_c=Q/A=60/16.10=3.727\text{ m/s}$$
$$p_w=6+(2)(1.86)/\sin 35°=12.49\text{ m} \qquad 3.727=(1.0/0.014)(16.10/12.49)^{2/3}(s_c)^{1/2} \qquad s_c=0.00194$$

14.189 For the river of Prob. 14.182, find the depth y_2 at which the specific energy is equal to that at the bottom, $y_1=16$ ft.

$$E=y+v^2/2g \qquad v_1=8.750\text{ ft/s} \qquad \text{(from Prob. 14.182)}$$
$$E_1=16+8.750^2/[(2)(32.2)]=17.19\text{ ft} \qquad E_2=E_1=17.19=y_2+(140/y_2)^2/[(2)(32.2)]$$
$$17.19y_2^2-y_2^3-304.3=0 \qquad y_2=5.00\text{ ft} \qquad \text{(by trial and error)}$$

14.190 Determine the critical slope of a clay-tile ($n=0.014$) triangular channel with sides sloping at 60° and carrying 12 m³/s.

$$gA_c^3=b_0Q^2 \qquad A_c=y_c^2\cot 60°=0.5774y_c^2 \qquad b_0=2y_c\cot 60°=1.155y_c$$
$$(9.807)(0.5774y_c^2)^3=(1.155y_c)(12)^2 \qquad y_c=2.45\text{ m}$$
$$v_c=Q/A_c=12/[(0.5774)(2.45)^2]=3.46\text{ m/s}$$
$$v_c=(1.0/n)(R^{2/3})(s^{1/2})$$
$$3.46=(1.0/0.014)\{(0.5774)(2.45)^2/[(2)(2.45)/\sin 60°]\}^{2/3}(s_c)^{1/2} \qquad s_c=0.00451$$

14.191 A triangular duct flows partly full as shown in Fig. 14-49. If the critical depth is 0.6 m and $n=0.016$, compute (*a*) the critical flow rate and (*b*) the critical slope.

(*a*)
$$b_0=\{[(\sqrt{3}/2)-0.6]/(\sqrt{3}/2)\}(1)=0.3072\text{ m} \qquad A=(\tfrac{1}{2})(0.3072+1)(0.6)=0.3922\text{ m}^2$$
$$Q=A\sqrt{gA/b_0}=(0.3922)\sqrt{(9.807)(0.3922)/0.3072}=1.39\text{ m}^3/\text{s}$$

(*b*)
$$Q=(A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad p_w=1+(2)[0.6/\sin 60°]=2.386\text{ m}$$
$$1.39=(0.3922)(1.0/0.016)(0.3922/2.386)^{2/3}(s)^{1/2} \qquad s=0.0357$$

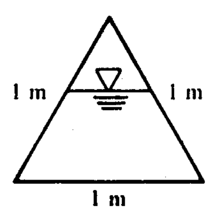


Fig. 14-49

14.192 For the triangular duct of Prob. 14.191, if the critical flow rate is 1.0 m³/s, compute (*a*) the critical depth and (*b*) the critical slope.

(*a*)
$$Q=A\sqrt{gA/b_0} \qquad b_0=[((\sqrt{3}/2)-y_c)/(\sqrt{3}/2)](1)=1-1.155y_c$$
$$A=\tfrac{1}{2}[(1-1.155y_c)+1](y_c)=y_c-0.5775y_c^2$$
$$1.0=(y_c-0.5775y_c^2)\sqrt{(9.807)(y_c-0.5775y_c^2)/(1-1.155y_c)} \qquad y_c=0.493\text{ m} \qquad \text{(by trial and error)}$$

(*b*)
$$Q=(A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad p_w=1+(2)[0.493/\sin 60°]=2.139\text{ m}$$
$$A=0.493-(0.5775)(0.493)^2=0.3526\text{ m}^2 \qquad 1.0=(0.3526)(1.0/0.016)(0.3526/2.139)^{2/3}(s)^{1/2}$$
$$s=0.228$$

14.193 A rectangular channel 2 m wide and 1 m deep has critical slope of 0.009. Estimate the surface roughness height.

$$s_c=gn^2p_w/(1.0b_0R^{1/3}) \qquad R=A/p_w=(2)(1)/(1+2+1)=0.5000\text{ m}$$
$$0.009=(9.807)(n)^2(1+2+1)/[(1.0)(2)(0.5000)^{1/3}] \qquad n=0.0191$$
$$n=0.0382\epsilon^{1/6} \qquad 0.0191=0.0382\epsilon^{1/6} \qquad \epsilon=0.0156\text{ m}=15.6\text{ mm}$$

14.194 For a water flow in a rectangular channel 10 m wide, the critical slope is 0.003491. Find the critical depth and discharge, given $n = 0.014$.

▮ $s_c = gn^2p_w/(1.0b_0R^{1/3})$ $\quad 0.003491 = (9.807)(0.014)^2(10 + 2y_c)/\{(1.0)(10)[10y_c/(10 + 2y_c)]^{1/3}\}$

$y_c = 0.196$ m (by trial and error) $\quad Q_c = (gA^3/b_0)^{1/3} = \{(9.807)[(10)(0.196)]^3/10\}^{1/3} = 1.95\ \text{m}^3/\text{s}$

14.195 A circular aluminium ($n = 0.020$) channel 8 ft in diameter has Froude number 0.5 in uniform half-full flow. Compute the channel slope.

▮ $$v_c = (gA/b_0)^{1/2} = \{(32.2)[(\tfrac{1}{2})(\pi)(8)^2/4]/8\}^{1/2} = 10.06\ \text{ft/s}$$
$$v = (1.486/n)(R^{2/3})(s^{1/2}) = v_c/2 = 10.06/2 = 5.03\ \text{ft/s}$$
$$5.03 = (1.486/0.020)(\tfrac{8}{4})^{2/3}(s)^{1/2} \qquad s = 0.00182$$

14.196 Water is in steady flow through the finished-concrete, semicircular channel shown in Fig. 14-50. If the bed slope is 0.0016, what is the flow rate?

▮ $A = (\tfrac{1}{2})(\pi)(10)^2 + (3)(10 + 10) = 217.1\ \text{ft}^2 \qquad R_h = A/p_w = 217.1/[(\pi)(10) + 3 + 3] = 5.802$ ft

$Q = (A)(1.486/n)(R_h^{2/3})(s_0^{1/2}) = (217.1)(1.486/0.012)(5.802)^{2/3}(0.0016)^{1/2} = 3472\ \text{ft}^3/\text{s}$

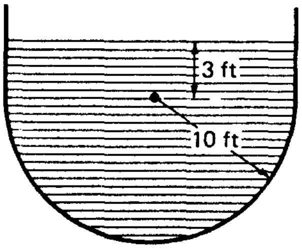


Fig. 14-50

14.197 In a planed-wood ($n = 0.012$) rectangular channel of width 4 m, water is flowing at the rate of 20 m³/s. If the slope of the channel is 0.0012, what is the depth of uniform flow?

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 20 = (4D)(1.0/0.012)[4D/(D + 4 + D)]^{2/3}(0.0012)^{1/2}$

$D = 1.80$ m (by trial and error)

14.198 In a planed-wood rectangular channel of width 4 m, water is flowing at a rate of 5 m³/s. The slope of the channel is 0.0001. The roughness coefficient is 0.5 mm. What is the height h of the cross section of flow for normal, steady flow? The water has a temperature of 10 °C.

▮ $v = Q/A = (8gsR/f)^{1/2} \qquad 5/4y = \{(8)(9.807)(0.0001)[4y/(y + 4 + y)]/f\}^{1/2}$

Like problems in Chap. 9, this requires a trial-and-error solution whereby a value of the friction factor (f) is guessed and subsequent computations and work with Fig. A-5 are done to find the right combination of parameter values to satisfy the problem. Inasmuch as a number of trial-and-error problems of this type were presented in detail in Chap. 9, we shall save time and space in problems of this type in this chapter by "guessing" the correct value of f on the first try! Guess $f = 0.0132$.

$$5/4y = \{(8)(9.807)(0.0001)[4y/(y + 4 + y)]/0.0132\}^{1/2} \qquad y = 1.69\ \text{m} \quad \text{(by trial and error)}$$
$$v = Q/A = 5/[(4)(1.69)] = 0.7396\ \text{m/s} \qquad R = (4)(1.69)/(1.69 + 4 + 1.69) = 0.9160\ \text{m}$$
$$N_R = D_hv/\nu = (4R)(v)/\nu = (4)(0.9160)(0.7396)/(1.30 \times 10^{-6}) = 2.08 \times 10^6$$
$$\epsilon/D_h = \epsilon/4R = (0.5/1000)/[(4)(0.9160)] = 0.00014$$

From Fig. A-5, $f = 0.0132$ (O.K.). Therefore, $y = 1.69$ m.

14.199 Do Prob. 14.196 using friction-factor formulas with $\epsilon = 0.0032$ ft. Assume a temperature of 60 °F.

▮ $v^*\epsilon/\nu = (\sqrt{gRs})(\epsilon)/\nu$. Using values from Prob. 14.196, $v^*\epsilon/\nu = [\sqrt{(32.2)(5.802)(0.0016)}](0.0032)/(1.21 \times 10^{-5}) = 145$. Since $v^*\epsilon/\nu > 100$, we are in the fully rough zone and $1/f^{1/2} = 2.16 - 2\log(\epsilon/R) = 2.16 -$

$2 \log (0.0032/5.802), f = 0.01328, v = (8gsR/f)^{1/2} = \{(8)(32.2)(0.0016)(5.802)/0.01328\}^{1/2} = 13.42$ ft/s, $Q = Av = (217.1)(13.42) = 2913$ ft³/s.

14.200 A wide rectangular section has flow $q = 80$ cfs/ft. Find the critical depth and the critical slope, if $\epsilon = 0.0035$ ft.

$$y_c = (q^2/g)^{1/3} = (80^2/32.2)^{1/3} = 5.84 \text{ ft}$$

$$s_c = f/8 \qquad N_R = D_h v/\nu \qquad D_h = 4R = (4)(5.84) = 23.36 \text{ ft}$$

$$N_R = (23.36)\{80/[(5)(1)]\}/(1.21 \times 10^{-5}) = 3.09 \times 10^7$$

$$\epsilon/D_h = 0.0035/23.36 = 0.000150$$

From Fig. A-5, $f = 0.0128$. $s_c = 0.0128/8 = 0.0016$.

14.201 For steady laminar flow in a thin sheet over a flat surface (see Fig. 14-51), $V_z = [(\gamma \sin\theta)/\mu][(3q\mu/\gamma \sin\theta)^{1/3}y - y^2/2]$, where q is the volumetric flow per unit width. Find the thickness t of such a flow of water at 5 °C for $\theta = 25°$ and $q = 18$ (L/min)/m.

When $y = t$, $dV_z/dy = 0$. $dV_z/dy = 0 = [(\gamma \sin\theta)/\mu][(3q\mu/\gamma \sin\theta)^{1/3} - t]$, $t = (3q\mu/\gamma \sin\theta)^{1/3}$:

$$t = \left\{\frac{(3)[(18 \times 10^{-3})/60](1.52 \times 10^{-3})}{(9.79 \times 10^{-3})(\sin 25°)}\right\}^{1/3} = 0.692 \text{ mm}$$

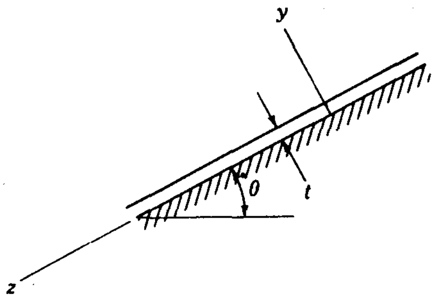


Fig. 14-51

14.202 A film of oil ($\nu = 1.664 \times 10^{-4}$ ft²/s) of thickness 0.0022 ft moves at uniform speed down an inclined surface having an angle $\theta = 25°$ (see Fig. 14-51). Compute the volume flow per unit width.

Assuming laminar flow,

$$t = (3q\mu/\gamma \sin\theta)^{1/3} = (3q\nu/g \sin\theta)^{1/3} \qquad \text{(from Prob. 14.201)}$$

$$0.0022 = \{(3)(q)(1.664 \times 10^{-4})/[(32.17)(\sin 25°)]\}^{1/3} \qquad q = 2.9 \times 10^{-4} \text{ cfs/ft}$$

14.203 Check the assumption of laminar flow in **(a)** Prob. 14.201, **(b)** Prob. 14.202.

(a) $N_R = q/\nu = (3 \times 10^{-4})/(1.52 \times 10^{-6}) \approx 2 < 500$ (O.K.).

(b) $N_R = q/\nu = (2.9 \times 10^{-4})/(1.664 \times 10^{-4}) \approx 2 < 500$ (O.K.).

14.204 Water at 5 °C is flowing 1 m deep in a finished-concrete rectangular channel of width 8 m with a slope of 0.001. Find the volume of flow.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) = [(8)(1)](1.0/0.012)[(8)(1)/(1+8+1)]^{2/3}(0.001)^{1/2} = 18.2 \text{ m}^3/\text{s}$$

14.205 A wide rectangular channel dug from clean earth ($n = 0.024$) is to conduct a flow of 5 m³/s per meter of width. The slope of the bed is 0.0016. What would be the depth of flow for normal flow?

$$s = (n/1.0)^2(q^2/y_N^{10/3}) \qquad 0.0016 = (0.024/1.0)^2(5^2/y_N^{10/3}) \qquad y_N = 1.93 \text{ m}$$

14.206 The channel of Fig. 14-52 has $n = 0.020$ and carries 8 ft³/s of water through a total drop of 10 ft. How long should the channel be for normal flow?

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = (2)(2\tan 45°) = 4.000\ \text{ft}^2$$
$$R = A/p_w = 4.000/[(2)(2/\cos 45°)] = 0.7071\ \text{ft}$$
$$8 = (4.000)(1.486/0.020)(0.7071)^{2/3}[10/L]^{1/2} \qquad L = 8694\ \text{ft}$$

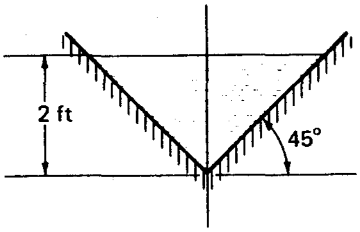


Fig. 14-52

14.207 Find the depth of flow in a 2-m-wide rectangular channel ($n = 0.016$) that carries 5 m³/s of water 1 km with a head loss of 7.5 m.

$$s = 7.5/1000 = 0.0075 \qquad Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$
$$5 = (2y_N)(1.0/0.016)[2y_N/(y_N + 2 + y_N)]^{2/3}(0.0075)^{1/2} \qquad y_N = 0.795\ \text{m} \quad \text{(by trial and error)}$$

14.208 The channel of Fig. 14-53 has a slope of 0.0018 and carries 57 m³/s. Determine the flow depth, if $n = 0.015$.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = 10y_N - (\tfrac{10}{2})[(\tfrac{10}{2})(\tan 10°)] = 10y_N - 4.408$$
$$p_w = 2[y_N - (\tfrac{10}{2})(\tan 10°) + (\tfrac{10}{2})/\cos 10°] = 2y_N + 8.391 \qquad R = A/p_w = (10y_N - 4.408)/(2y_N + 8.391)$$
$$57 = (10y_N - 4.408)(1.0/0.015)[(10y_N - 4.408)/(2y_N + 8.391)]^{2/3}(0.0018)^{1/2}$$
$$y_N = 2.11\ \text{m} \quad \text{(by trial and error)}$$

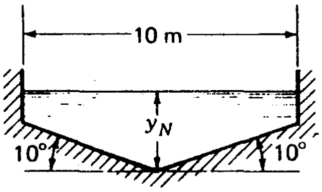


Fig. 14-53

14.209 Shown in Fig. 14-54 is a partially filled pipe. If Manning's n is 0.020, what slope is necessary for a steady flow of 60 cfs?

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad \theta = \arcsin \tfrac{2}{4} = 30.0°$$
$$A = \{[180° + (2)(30.0°)]/360°\}[(\pi)(8)^2/4] + (2)(4\cos 30.0°) = 40.44\ \text{ft}^2$$
$$p_w = \{[180° + (2)(30.0°)]/360°\}[(\pi)(8)] = 16.76\ \text{ft} \qquad R = A/p_w = 40.44/16.76 = 2.413\ \text{ft}$$
$$60 = (40.44)(1.486/0.020)(2.413)^{2/3}(s)^{1/2} \qquad s = 0.0001232$$

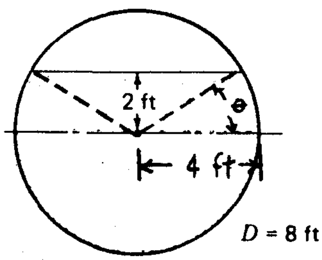


Fig. 14-54

14.210 Find the flow in Fig. 14-55 if the slope of the channel is 0.0004 and $n = 0.024$. Side slopes are all 1:1.

■ $A = (3)(3)/2 + (3)(20) + (5)(3+3+5)/2 + (8)(3+5) + (5)(3+3+5)/2 + (3)(25) + (3)(3)/2 = 263.0\ \text{m}^2$
$p_w = \sqrt{3^2+3^2} + 20 + \sqrt{5^2+5^2} + 8 + \sqrt{5^2+5^2} + 25 + \sqrt{3^2+3^2} = 75.63\ \text{m}$ $\qquad R = A/p_w = 263.0/75.63 = 3.477\ \text{m}$

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) = (263.0)(1.0/0.024)(3.477)^{2/3}(0.0004)^{1/2} = 503\ \text{m}^3/\text{s}$$

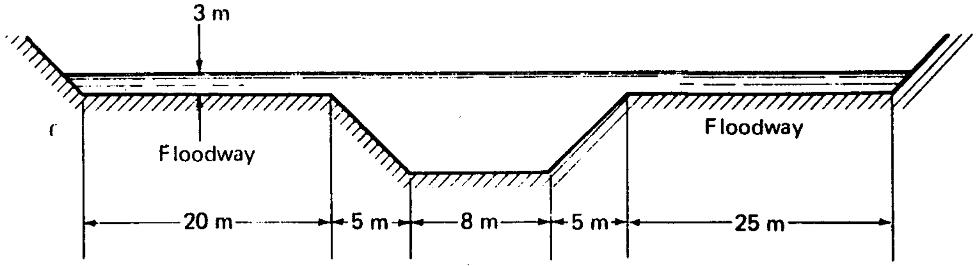


Fig. 14-55

14.211 The channel in Prob. 14.196 is to be replaced by a rectangular channel of width 16 ft. What is the ratio of cost of the concrete allowing 2 ft of freeboard (distance above the free surface) for the walls of the channels?

■ From Prob. 14.196, $Q = 3472\ \text{ft}^3/\text{s}$, $s = 0.0016$, and $n = 0.012$.

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 3472 = (16y)(1.486/0.012)[16y/(y+16+y)]^{2/3}(0.0016)^{1/2}$$
$$y = 14.65\ \text{ft} \qquad \text{(by trial and error)} \qquad (p_w)_1 = 16 + (2)(14.65) + (2)(2) = 49.30\ \text{ft}$$

For the channel of Prob. 14.196, $(p_w)_2 = (\pi)(10) + 3 + 3 + (2)(2) = 41.42$ ft. Ratio of cost $= (p_w)_1/(p_w)_2 = 49.30/41.42 = 1.19$.

14.212 Repeat Prob. 14.204 using the friction-factor approach. Check your assumption of rough-zone flow.

■
$$R = A/p_w = (1)(8)/(1+8+1) = 0.8\ \text{m} \qquad 1/f^{1/2} = 2.16 - 2\log(\epsilon/R) = 2.16 - 2\log(0.001/0.8)$$
$$f = 0.01576 \qquad v = (8gsR/f)^{1/2} = [(8)(9.807)(0.001)(0.8)/0.01576]^{1/2} = 1.996\ \text{m/s}$$
$$Q = Av = [(1)(8)](1.996) = 16.0\ \text{m}^3/\text{s}$$
$$N_R = D_h v/\nu = (4R)(v)/\nu = [(4)(0.8)](1.996)/(1.52\times10^{-6}) = 4.20\times10^6$$

The values $f = 0.01576$ and $N_R = 4.20\times10^6$ fall in in the rough zone of Fig. A-5.

14.213 For a rectangular channel with a flow of $20\ \text{m}^3/\text{s}$ at a velocity of 5 m/s, what should width b and depth y be for the best hydraulic section?

■ $A = Q/v = \frac{20}{5} = 4.000\ \text{m}^2$. For a rectangular section, the best hydraulic section has a width equal to twice the depth. Hence, $(b)(b/2) = 4.000$, $b = 2.828$ m, $y = 2.828/2 = 1.414$ m.

14.214 For steady flow at $6\ \text{m}^3/\text{s}$ on a slope of 0.0018, what is the width of the rectangular channel of least wetted perimeter? Take $n = 0.0015$.

■ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$. The best hydraulic section has a width b equal to twice the depth. Hence, $A = (b)(b/2) = b^2/2$, $R = A/p_w = (b^2/2)/(b/2 + b + b/2) = b/4$, $6 = (b^2/2)(1.0/0.015)(b/4)^{2/3}(0.0018)^{1/2}$, $b = 2.43$ m.

14.215 A stream has a speed of 12 ft/s and is 2 ft deep. Find the angular spread of the wave set up by a thin stationary obstruction.

■
$$\sin\theta/2 = \sqrt{gy}/v = \sqrt{(32.2)(2)}/12 \qquad \theta = 84°$$

14.216 A small boat is moving in shallow still water where the depth is 2 m; its bow wave makes an angle of 60° with the line of motion (see Fig. 14-56). Compute the speed of the boat.

■
$$\sin\alpha = \sqrt{gy}/v \qquad \sin 60° = \sqrt{(9.807)(2)}/v \qquad v = 5.11\ \text{m/s}$$

14.217 A stone is dropped into a pond; the ripples have an amplitude of about 1 in and travel at 4 ft/s. About how deep is the pond?

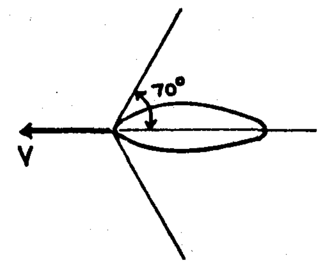


Fig. 14-56

$(g)(y + \Delta y/2)(y + \Delta y) = c^2 y \qquad 32.2[y + (\frac{1}{12})/2](y + \frac{1}{12}) = (4)^2 y \qquad y^2 - 0.3719y + 0.003472 = 0$

Choosing the larger root,

$$y = [-(-0.3719) + \sqrt{(-0.3719)^2 - (4)(1)(0.003472)}]/[(2)(1)] = 0.362 \text{ ft}$$

14.218 Two fishermen are stationed a distance D apart along a straight running stream (depth y, speed v). At a certain moment, a small gravity wave is initiated midway between them. How much later than the downstream man does the upstream man feel the wave?

With $c = \sqrt{gy}$ as the wave velocity,

$$\Delta t = \frac{D/2}{v-c} - \frac{D/2}{v+c} = \frac{Dc}{v^2 - c^2} = \frac{D\sqrt{gy}}{v^2 - gy}$$

14.219 A wide rectangular channel excavated from clean earth has a flow of 4 (m³/s)/m. Determine the minimum specific energy.

$y_c = (q^2/g)^{1/3} = (4^2/9.807)^{1/3} = 1.177 \text{ m} \qquad E_{min} = (\frac{3}{2})(y_c) = (\frac{3}{2})(1.177) = 1.766 \text{ m}$

14.220 Find the critical slope in Prob. 14.219, assuming water at 5 °C ($\epsilon = 0.047$ m).

$s_c = f/8 \qquad D_h = 4R = 4y_c = (4)(1.177) = 4.708 \text{ m} \qquad v = q/y_c = 4/1.177 = 3.398 \text{ m/s}$

$N_R = D_h v/\nu = (4.708)(3.398)/(1.52 \times 10^{-6}) = 1.05 \times 10^7 \qquad \epsilon/D_h = 0.047/4.708 = 0.010$

From Fig. A-5, $f = 0.038$ and so $s_c = 0.038/8 = 0.00475$.

14.221 Find the flow in a wide rectangular channel for which the critical depth is 3 m.

$y_c = (q^2/g)^{1/3} \qquad 3 = (q^2/9.807)^{1/3} \qquad q = 16.27 \text{ (m}^3\text{/s)/m}$

14.222 By what factor does the Froude number of a wide rectangular channel increase when the depth is decreased by a factor of 3/4, keeping the mass flow fixed?

Since $N_F \propto y^{-3/2}$,

$$\text{factor} = \left(\frac{4}{3}\right)^{3/2} = \frac{8}{3\sqrt{3}} = 1.54$$

14.223 At a section in the rectangular channel shown in Fig. 14-57, the average velocity is 28.28 fps. Is the flow tranquil?

$q = (Av)/b = [(10)(50)](28.28)/50 = 282.8 \text{ cfs/ft} \qquad y_c = (q^2/g)^{1/3} = (282.8^2/32.2)^{1/3} = 13.54 \text{ ft}$

Since $y_c > 10$ ft, flow is not tranquil but shooting.

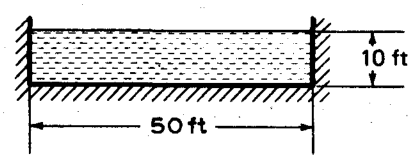


Fig. 14-57

14.224 How deep is a flow of 5 $(m^3/s)/m$ in a wide rectangular channel having a critical slope of 0.002? The fluid is water at 5 °C.

▮ Assume rough-flow zone:

$$v = (8gsR/f)^{1/2} \qquad s_c = f/8 \qquad 0.002 = f/8 \qquad f = 0.016$$
$$5/[(1)(y)] = [(8)(9.807)(0.002)(y)/0.016]^{1/2} \qquad y = 1.366 \text{ m}$$

14.225 Compute the minimum specific energy in an 8-ft-wide channel through which 150 ft^3/s of water is flowing.

▮
$$gA^3 = bQ^2 \qquad (32.2)(8y_c)^3 = (8)(150)^2 \qquad y_c = 2.218 \text{ ft} \qquad E_{min} = A_c/2b + y_c$$
$$E_{min} = (8)(2.218)/[(2)(8)] + 2.218 = 3.327 \text{ ft}$$

14.226 Find the critical slope of a rectangular finished-concrete channel of width 4 m that carries 3 m^3/s of water at 5 °C ($\epsilon = 0001$ m).

▮
$$gA^3 = bQ^2 \qquad (9.807)(4y_c)^3 = (4)(3)^2 \qquad y_c = 0.386 \text{ m} \qquad s_c = fp_w/8b \qquad \epsilon/D_h = \epsilon/4R$$
$$R = A/p_w = (4)(0.386)/(0.386 + 4 + 0.386) = 0.3236 \text{ m}$$
$$\epsilon/D_h = 0.001/[(4)(0.3236)] = 0.00077$$
$$N_R = D_h v/\nu = (4R)(Q/A)/\nu = (4)(0.3236)\{3/[(4)(0.386)]\}/(1.52 \times 10^{-6}) = 1.65 \times 10^6$$

From Fig. A-5, $f = 0.0186$. $s_c = (0.0186)(0.386 + 4 + 0.386)/[(8)(4)] = 0.00277$.

14.227 What is the critical depth for a right triangular cross section for a flow of 4 m^3/s?

▮ See Fig. 14-58.

$$gA^3 = bQ^2 \qquad b = (2)(y_c \tan 45°) = 2.0y_c \qquad A = (y_c)(y_c \tan 45°) = 1.0y_c^2$$
$$(9.807)(1.0y_c^2)^3 = (2.0y_c)(4)^2 \qquad y_c = 1.267 \text{ m}$$

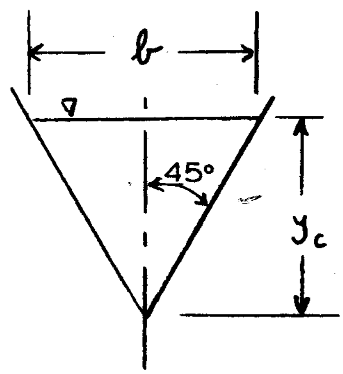


Fig. 14-58

14.228 What is the critical depth of a trapezoidal cross section for a flow of 10 m^3/s? The width at the base is 3 m and angle θ at the sides is 60°.

▮
$$gA^3 = bQ^2 \qquad b = 3 + (2)(y_c/\tan 60°) = 3 + 1.155y_c$$
$$A = 3y_c + (y_c)(y_c/\tan 60°) = 3y_c + 0.5774y_c^2$$
$$(9.807)(3y_c + 0.5774y_c^2)^3 = (3 + 1.155y_c)(10)^2 \qquad y_c = 0.976 \text{ m} \qquad \text{(by trial and error)}$$

14.229 Shown in Fig. 14-59 is a partially filled pipe discharging 450 cfs. What is the critical depth?

▮
$$gA^3 = bQ^2 \qquad b = (2)(4\cos\theta) = 8\cos\theta$$
$$A = [(\pi)(8^2/4)]/2 + [(\pi)(8^2/4)](2\theta/360) + (\tfrac{8}{2})(\cos\theta)(\tfrac{8}{2})(\sin\theta)$$
$$= 25.13 + 0.2793\theta + (16)(\cos\theta)(\sin\theta)$$
$$(32.2)[25.13 + 0.2793\theta + (16)(\cos\theta)(\sin\theta)]^3 = (8\cos\theta)(450)^2$$
$$\theta = 20.5° \qquad \text{(by trial and error)}$$
$$y_c = \tfrac{8}{2} + (\tfrac{8}{2})(\sin 20.5°) = 5.40 \text{ ft}$$

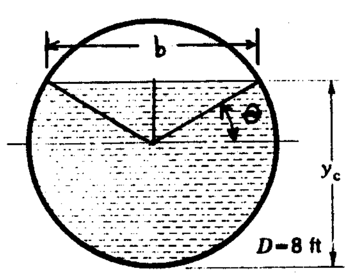


Fig. 14-59

14.230 Calculate the critical depth of the parabolic channel of Fig. 14-60, if the flow is 4 m^3/s.

▌

$$gA^3 = bQ^2$$

$$A_c = (2)\int_0^{y_c} x\,dy = (2)\int_0^{y_c} (y/2)^{1/2}\,dy = 1.414[(\tfrac{2}{3})(y)^{3/2}]_0^{y_c}$$

$$= 1.414[(\tfrac{2}{3})(y_c)^{3/2}] = 0.9427y_c^{3/2}$$

$$b = 2x = (2)(\sqrt{y_c/2}) = 1.414y_c^{1/2}$$

$$(9.807)(0.9427y_c^{3/2})^3 = (1.414y_c^{1/2})(4)^2 \qquad y_c = 1.288\text{ m}$$

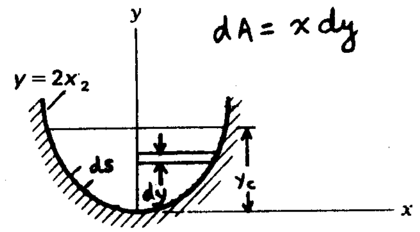


Fig. 14-60

14.231 In Prob. 14.230, what is the critical slope for normal flow? The friction factor is 0.014.

▌ $s_c = (f)(p_w)_c/8b$. To evaluate $(p_w)_c$, consider a differential length along the wetted perimeter (ds in Fig. 14-60): $ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + (dy/dx)^2}\,dx$. From Prob. 14.230, when $y = y_c$, $x = \sqrt{y_c/2} = \sqrt{1.288/2} =$ 0.8025 m.

$$(p_w)_c = \int ds = (2)\int_0^{0.7470} \sqrt{1 + (dy/dx)^2}\,dx \qquad y = 2x^2 \qquad dy/dx = 4x$$

$$(p_w)_c = (2)\int_0^{0.7470} \sqrt{1 + (4x)^2}\,dx = (8)\int_0^{0.8025} \sqrt{(\tfrac{1}{16} + x^2)}\,dx$$

$$= (\tfrac{8}{2})[(x)(\sqrt{\tfrac{1}{16} + x^2}) + \tfrac{1}{16}\{\log(x + \sqrt{\tfrac{1}{16} + x^2})\}]_0^{0.8025} = 2.711\text{ m}$$

$$b = (2)(0.8025) = 1.605\text{ m} \qquad s_c = (0.014)(2.711)/[(8)(1.605)] = 0.00296$$

14.232 At a section in the triangular channel of Fig. 14-61 the average velocity is 8 ft/s. Is the flow tranquil or shooting?

▌

$$\tan(45°/2) = (b/2)/10 \qquad b = 8.284\text{ ft} \qquad A = 10b/2 = (10)(8.282)/2 = 41.41\text{ ft}^2$$

$$Q = Av = (41.41)(8) = 331.3\text{ ft}^3/\text{s} \qquad gA_c^3 = b_cQ^2$$

$$\tan(45°/2) = (b_c/2)/y_c \qquad b_c = 0.8284y_c$$

$$A_c = y_cb_c/2 = (y_c)(0.8284y_c)/2 = 0.4142y_c^2$$

$$(32.2)(0.4142y_c^2)^3 = (0.8284y_c)(331.3)^2 \qquad y_c = 8.31\text{ ft} < 10\text{ ft} = y$$

The flow is tranquil.

14.233 Water is moving in Fig. 14-62 with a velocity of 1 ft/s and a depth of 3 ft. It approaches a smooth rise in the channel bed of 1 ft. What should the estimated depth be after the rise? The channel is rectangular.

▌

$$E = q^2/2y^2g + y \qquad Q = Av = (3b)(1) = 3b \qquad q = Q/b = 3b/b = 3\text{ cfs/ft}$$

$$E_1 = 3^2/[(2)(3)^2(32.2)] + 3 = 3.016\text{ ft}$$

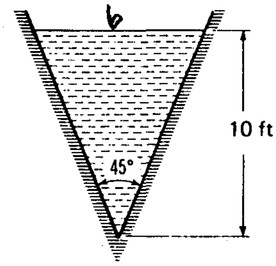


Fig. 14-61

Assuming no losses, $E_1 = E_2 + 1$, $3.016 = E_2 + 1$, $E_2 = 2.016$ ft; $y_c = (q^2/g)^{1/3} = (3^2/32.2)^{1/3} = 0.654$ ft. With surface elevation increasing, E must decrease. We must have one value of y which must be for tranquil flow. Hence, $2.016 = 3^2/[(2)(y_2)^2(32.2)] + y_2$, $y_2 = 1.98$ ft (by trial and error).

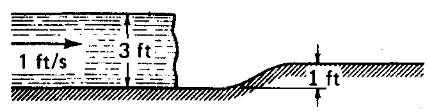


Fig. 14-62

14.234 In Fig. 14-63, a flow of 0.2 cfs flows over the rectangular channel of width 3 ft. If there is a smooth drop of 2 in, what is the elevation of the free surface above the bed of the channel after the drop? The velocity before the drop is 0.3 fps.

$$Q = Av \qquad 0.2 = (3y_1)(0.3) \qquad y_1 = 0.2222 \text{ ft}$$
$$y_c = (q^2/g)^{1/3} = [(0.2/3)^2/32.2]^{1/3} = 0.05168 \text{ ft}$$

Since $y_c < y_1$, we have tranquil flow: $E = q^2/2y^2g + y$, $E_1 = (0.2/3)^2/[(2)(0.2222)^2(32.2)] + 0.2222 = 0.2236$ ft. Assuming no losses, $E_2 = E_1 + \frac{2}{12} = 0.2236 + \frac{2}{12} = 0.3903$ ft. We must have one depth downstream greater than y_1: $0.3903 = (0.2/3)^2/[(2)(y_2)^2(32.2)] + y_2$, $y_2 = 0.390$ ft (by trial and error).

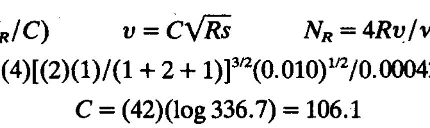


Fig. 14-63

14.235 Using the Powell equation, what quantity of liquid will flow in a smooth rectangular channel 2 ft wide, on a slope of 0.10 if the depth is 1.00 ft? Use $\nu = 0.00042$ ft²/s.

$C = -42 \log (C/N_R + \epsilon/R)$. For smooth channels, ϵ/R is small and can be neglected; hence,

$$C = 42 \log (N_R/C) \qquad v = C\sqrt{Rs} \qquad N_R = 4Rv/\nu = 4RC\sqrt{Rs}/\nu$$
$$N_R/C = (4)[(2)(1)/(1+2+1)]^{3/2}(0.010)^{1/2}/0.00042 = 336.7$$
$$C = (42)(\log 336.7) = 106.1$$
$$Q = CA\sqrt{Rs} = (106.1)[(2)(1)]\sqrt{[(2)(1)/(1+2+1)](0.010)} = 15.0 \text{ ft}^3/\text{s}$$

14.236 Determine C by the Powell equation for a 2 ft by 1 ft rectangular channel, if $v = 5.50$ fps, $\epsilon/R = 0.0020$, and $\nu = 0.00042$ ft²/s.

$$N_R = 4Rv/\nu = (4)[(2)(1)/(1+2+1)](5.50)/0.00042 = 26\,190$$
$$C = -42 \log (C/N_R + \epsilon/R) = (-42)[\log (C/26\,190 + 0.0020)] \qquad C = 95 \qquad \text{(by trial and error)}$$

14.237 Show a correlation between roughness factor f and roughness factor n.

Taking the Manning formula as a basis of correlation, $C = \sqrt{8g/f} = 1.486R^{1/6}/n$, $1/\sqrt{f} = 1.486R^{1/6}/(n\sqrt{8g})$, $f = 8gn^2/(2.208R^{1/3})$.

14.238 What is the average shear stress at the sides and bottom of a rectangular flume 12 ft wide, flowing 4 ft deep, and laid on a slope of 1.60 ft per 1000 ft?

$$\tau_0 = \gamma Rs = 62.4[(4)(12)/(4+12+4)](1.60/1000) = 0.240 \text{ lb/ft}^2$$

14.239 What flow can be expected in a 4-ft-wide rectangular, concrete-lined channel laid on a slope of 4 ft in 10 000 ft, if the water flows 2 ft deep. Solve using the Manning formula with $n = 0.015$.

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = [(4)(2)](1.486/0.015)[(4)(2)/(2+4+2]^{2/3}(4/10\,000)^{1/2} = 15.9\ \text{ft}^3/\text{s}$$

14.240 Solve Prob. 14.239 using Kutter's C.

$$Q = AC\sqrt{Rs} \qquad s = 4/10\,000 = 0.0004 \qquad R = A/p_w = (4)(2)/(2+4+2) = 1.000\ \text{ft}$$

From Table A-15, with $s = 0.0004$, $R = 1.000$ ft, and $n = 0.015$, $C = 98$. $Q = [(4)(2)](98)\sqrt{(1.000)(0.0004)} = 15.7\ \text{ft}^3/\text{s}$.

14.241 In a hydraulics laboratory, a flow of 14.56 cfs was measured from a rectangular channel flowing 4 ft wide and 2 ft deep. If the slope of the channel was 0.00040, what is the roughness factor for the lining of the channel? Solve using Kutter's formula.

$$Q = AC\sqrt{Rs} \qquad 14.56 = [(4)(2)](C)\sqrt{[(4)(2)/(2+4+2)](0.00040)} \qquad C = 91$$

From Table A-15, by interpolation, $n = 0.016$.

14.242 Solve Prob. 14.241 using Manning's formula.

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$
$$14.56 = [(4)(2)](1.486/n)[(4)(2)/(2+4+2)]^{2/3}(0.00040)^{1/2} \qquad n = 0.0163$$

14.243 On what slope should a 24-in vitrified sewer pipe be laid in order that 6.00 cfs will flow when the sewer is half-full? Use $n = 0.013$.

$$Q = (A)(1.486/n)(R^{2/3}) \qquad 6.00 = [(\tfrac{1}{2})(\pi)(\tfrac{24}{12})^2/4](1.486/0.013)[(\tfrac{24}{12})/4]^{2/3}(s)^{1/2} \qquad s = 0.00281$$

14.244 What would the slope be in Prob. 14.243 if the sewer flows full?

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 6.00 = [(\pi)(\tfrac{24}{12})^2/4](1.486/0.013)[(\tfrac{24}{12})/4]^{2/3}(s)^{1/2} \qquad s = 0.000703$$

14.245 A trapezoidal channel, bottom width 20 ft and side slopes 1 to 1, flows 4 ft deep on a slope of 0.0009. For a value of $n = 0.025$, what is the uniform discharge?

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = (20)(4) + (4)(4) = 96.00\ \text{ft}^2$$
$$p_w = 20 + (2)(4)(\sqrt{2}) = 31.31\ \text{ft}$$
$$Q = (96.00)(1.486/0.025)(96.00/31.31)^{2/3}(0.0009)^{1/2} = 361\ \text{ft}^3/\text{s}$$

14.246 Two concrete pipes ($C = 100$) must carry the flow from an open channel of half-square section 6 ft wide and 3 ft deep ($C = 120$). The slope of both structures is 0.00090. ***(a)*** Determine the diameter of the pipes. ***(b)*** Find the depth of water in the rectangular channel after it has become stabilized, if the slope is changed to 0.00160, using $C = 120$.

(a)

$$Q_{\text{channel}} = Q_{\text{pipes}} \qquad (AC\sqrt{Rs})_{\text{channel}} = (2)(AC\sqrt{Rs})_{\text{pipe}}$$
$$[(3)(6)](120)\sqrt{[(3)(6)/(3+6+3)](0.00090)} = (2)[(\pi)(D)^2/4](100)\sqrt{(D/4)(0.00090)} \qquad D = 4.08\ \text{ft}$$

(b)

$$Q = [(3)(6)](120)\sqrt{[(3)(6)/(3+6+3)](0.00090)} = 79.36\ \text{ft}^3/\text{s}$$
$$79.36 = [(y)(6)](120)\sqrt{[(y)(6)/(y+6+y)](0.001600)} \qquad y = 2.39\ \text{ft} \qquad \text{(by trial and error)}$$

14.247 How deep will water flow at the rate of 240 cfs in a rectangular channel 20 ft wide, laid on a slope of 0.00010? Use $n = 0.0149$.

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 240 = (20y)(1.486/0.0149)[20y/(y+20+y)]^{2/3}(0.00010)^{1/2}$$
$$y = 5.27\ \text{ft} \qquad \text{(by trial and error)}$$

14.248 How wide must a rectangular channel be constructed in order to carry 500 cfs at a depth of 6 ft on a slope of 0.00040? Use $n = 0.010$.

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 500 = (6b)(1.486/0.010)[6b/(6+b+6)]^{2/3}(0.00040)^{1/2}$$
$$b = 13.1\ \text{ft} \qquad \text{(by trial and error)}$$

14.249 A rectangular channel carries 200 cfs. Find the critical depth and the critical velocity for **(a)** a width of 12 ft and **(b)** a width of 9 ft. **(c)** What slope will produce the critical velocity in **(a)** if $n = 0.020$?

▮ **(a)** $y_c = (q^2/g)^{1/3} = [(\frac{200}{12})^2/32.2]^{1/3} = 2.05$ ft $\quad v_c = \sqrt{gy_c} = \sqrt{(32.2)(2.05)} = 8.13$ ft/s

(b) $y_c = [(\frac{200}{9})^2/32.2]^{1/3} = 2.48$ ft $\quad v_c = \sqrt{(32.2)(2.48)} = 8.94$ ft/s

(c) $v = (1.486/n)(R^{2/3})(s^{1/2}) \quad 8.13 = (1.486/0.020)[(12)(2.05)/(2.05 + 12 + 2.05)]^{2/3}(s)^{1/2} \quad s = 0.00680$

14.250 A trapezoidal channel with side slopes of 2 horizontal to 1 vertical is to carry a flow of 590 cfs. For a bottom width of 12 ft, calculate the **(a)** critical depth and **(b)** critical velocity.

▮ **(a)** $Q^2/g = A^3/b \quad b = 12 + 4y \quad A = 12y + 2y^2$

$$590^2/32.2 = (12y + 2y^2)^3/(12 + 4y) \quad y = y_c = 3.46 \text{ ft} \quad \text{(by trial and error)}$$

(b) $v_c = \sqrt{gA/b} = \sqrt{(32.2)[(12)(3.46) + (2)(3.46)^2]/[12 + (4)(3.46)]} = 9.03$ ft/s

14.251 A trapezoidal channel has a bottom width of 20 ft, side slopes of 1 to 1, and flows at a depth of 3.00 ft. For $n = 0.015$, and a discharge of 360 cfs, calculate **(a)** the normal slope, **(b)** the critical slope and critical depth for 360 cfs, and **(c)** the critical slope at the normal depth of 3.00 ft.

▮ **(a)** $Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \quad A = (20)(3) + (3)(3) = 69.00 \text{ ft}^2 \quad p_w = 20 + (2)(\sqrt{3^2 + 3^2}) = 28.49$ ft

$$360 = (69.00)(1.486/0.015)(69.00/28.49)^{2/3}(s)^{1/2} \quad s = 0.000853$$

(b) $v_c = \sqrt{gA/b} = \sqrt{(32.2)(20y_c + y_c^2)/(20 + 2y_c)}$

Also,

$$v_c = Q/A_c = 360/(20y_c + y_c^2) \quad \sqrt{(32.2)(20y_c + y_c^2)/(20 + 2y_c)} = 360/(20y_c + y_c^2)$$

$$y_c = 2.08 \text{ ft} \quad \text{(by trial and error)}$$

$$360 = [(20)(2.08) + 2.08^2](1.486/0.015)\{[(20)(2.08) + 2.08^2]/[20 + (2)(\sqrt{2.08^2 + 2.08^2}]\}^{2/3}(s_c)^{1/2} \quad s_c = 0.00291$$

(c) $v_c = \sqrt{(32.2)[(20)(3.00) + 3.00^2]/[20 + (2)(3.00)]} = 9.24$ ft/s $\quad v_c = (1.486/n)(R^{2/3})(s^{1/2})$

$$9.24 = (1.486/0.015)\{[(20)(3.00) + 3.00^2]/[20 + (2)(\sqrt{3.00^2 + 3.00^2}]\}^{2/3}(s_c)^{1/2} \quad s_c = 0.00267$$

14.252 A rectangular channel, 30 ft wide, carries 270 cfs when flowing 3.00 ft deep. **(a)** What is the specific energy? **(b)** Is the flow subcritical or supercritical?

▮ **(a)** $v = Q/A = 270/[(3.00)(30)] = 3.00$ ft/s $\quad E = v^2/2g + y = 3.000^2/[(2)(32.2)] + 3.00 = 3.14$ ft

(b) $y_c = (q^2/g)^{1/3} = [(\frac{270}{30})^2/32.2]^{1/3} = 1.36$ ft

Since $[y_c = 1.36] < [y = 3.00]$, the flow is subcritical.

14.253 A trapezoidal channel has a bottom width of 20 ft and side slopes of 2 horizontal to 1 vertical. When the depth of water is 3.50 ft, the flow is 370 cfs. **(a)** What is the specific energy? **(b)** Is the flow subcritical or supercritical?

▮ **(a)** $A = (20)(3.50) + (3.50)[(2)(3.50)] = 94.50 \text{ ft}^2 \quad v = Q/A = 370/94.50 = 3.915$ ft/s

$$E = v^2/2g + y = 3.915^2/[(2)(32.2)] + 3.50 = 3.74 \text{ ft}$$

(b) $Q^2/g = A_c^3/b \quad 370^2/32.2 = \{20y_c + (y_c)[(2)(y_c)]\}^3/[20 + (2)(2y_c)] \quad y_c = 2.05$ ft (by trial and error)

Since $[y_c = 2.05] < [y = 3.50 \text{ ft}]$, the flow is subcritical.

14.254 The discharge through a rectangular channel ($n = 0.012$) 15 ft wide is 400 cfs when the slope is 1 ft in 100 ft. Is the flow subcritical or supercritical?

▮ $y_c = (q^2/g)^{1/3} = [(\frac{400}{15})^2/32.2]^{1/3} = 2.81$ ft $\quad Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$

$$400 = [(15)(2.81)](1.486/0.012)[(15)(2.81)/(2.81 + 15 + 2.81)]^{2/3}(s_c)^{1/2} \quad s_c = 0.00226$$

Since $[s_c = 0.00226] < [s = \frac{1}{100}]$, the flow is supercritical.

14.255 A rectangular channel, 10 ft wide, carries 400 cfs. **(a)** Tabulate depth of flow against specific energy for depths from 1 ft to 8 ft. **(b)** Determine the minimum specific energy. **(c)** What type of flow exists when the depth is 2 ft and when it is 8 ft? **(d)** For $C = 100$, what slopes are necessary to maintain the depths in **(c)**?

(a) $E = v^2/2g + y = (Q/A)^2/2g + y = (Q/10y)^2/2g + y$. For $y = 1$ ft, $E = \{400/[(10)(1)]\}^2/[(2)(32.2)] + 1 = 25.8$ ft. For succeeding depths,

y, ft	E, ft
1	25.8
2	8.21
3	5.76
4	5.55
5	5.99
6	6.69
7	7.51
8	8.39

(b) $y_c = (q^2/g)^{1/3} = [(\frac{400}{10})^2/32.2]^{1/3} = 3.676$ ft $\qquad E_{min} = \{400/[(10)(3.676)]\}^2/[(2)(32.2)] + 3.676 = 5.51$ ft

(c) Since $[y = 2] < [y_c = 3.676]$, the flow is supercritical for a 2-ft depth. Since $[y = 8] > [y_c = 3.676]$, the flow is subcritical for an 8-ft depth.

(d) $Q = CA\sqrt{Rs}$. For a 2-ft depth: $400 = (100)[(2)(10)]\sqrt{[(2)(10)/(2+10+2)](s)}$, $s = 0.0280$. For an 8-ft depth: $400 = (100)[(8)(10)]\sqrt{[(8)(10)/(8+10+8)](s)}$, $s = 0.000812$.

14.256 A rectangular flume ($n = 0.012$) is laid on a slope of 0.0036 and carries 580 cfs. For critical-flow conditions, what width is required?

$y_c = (q^2/g)^{1/3} = [(Q/b)^2/g]^{1/3}$. Try $b = 8.0$ ft:

$$y_c = [(580/8.0)^2/32.2]^{1/3} = 5.465 \text{ ft}$$

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = [(5.465)(8.0)](1.486/0.012) \times [(5.465)(8.0)/(5.465 + 8.0 + 5.465)]^{2/3}(0.0036)^{1/2} = 568 \text{ ft}^3/\text{s}$$

Try $b = 8.5$ ft:

$$y_c = [(580/8.5)^2/32.2]^{1/3} = 5.249 \text{ ft}$$

$$Q = [(5.249)(8.5)](1.486/0.012)[(5.249)(8.5)/(5.249 + 8.5 + 5.249)]^{2/3}(0.0036)^{1/2} = 586 \text{ ft}^3/\text{s}$$

Try $b = 8.33$ ft:

$$y_c = [(580/8.33)^2/32.2]^{1/3} = 5.320 \text{ ft}$$

$$Q = [(5.320)(8.33)](1.486/0.012)[(5.320)(8.33)/(5.320 + 8.33 + 5.320)]^{2/3}(0.0036)^{1/2} = 580 \text{ ft}^3/\text{s}$$

Hence, $b = 8.33$ ft.

14.257 For a constant specific energy of 6.60 ft, what maximum flow may occur in a rectangular channel 10.0 ft wide?

$$y_c = (\tfrac{2}{3})(E) = (\tfrac{2}{3})(6.60) = 4.40 \text{ ft} \qquad v_c = \sqrt{gy_c} = \sqrt{(32.2)(4.40)} = 11.90 \text{ ft/s}$$

$$Q_{max} = A_c v_c = [(10.0)(4.40)](11.90) = 524 \text{ ft}^3/\text{s}$$

14.258 A rectangular channel, 20 ft wide, $n = 0.025$, flows 5 ft deep on a slope of 14.7 ft in 10 000 ft. A suppressed weir C, 2.45 ft high, is built across the channel ($m = 3.45$). Taking the elevation of the bottom of the channel just upstream from the weir to be 100.00 ft, estimate (using one reach) the elevation of the water surface at a point A, 1000 ft upstream from the weir. See Fig. 14-64.

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = [(20)(5)](1.486/0.025)[(20)(5)/(5 + 20 + 5)]^{2/3}(14.7/10\,000)^{1/2} = 509 \text{ ft}^3/\text{s}$$

Calculate the new elevation of the water surface at B (before dropdown). Note that the flow is nonuniform since the depths, velocities, and areas are not constant after the weir is installed. Estimate a depth of 6 ft just upstream from the weir (i.e., at B). $v_{approach} = Q/A = 509/[(20)(6)] = 4.24$ ft. The applicable weir formula is $Q = mb[(H + v^2/2g)^{3/2} - (v^2/2g)^{3/2}]$.

$$509 = (3.45)(20)\left\{\left[H + \frac{4.24^2}{(2)(32.2)}\right]^{3/2} - \left[\frac{4.24^2}{(2)(32.2)}\right]^{3/2}\right\} \qquad H = 3.56 \text{ ft}$$

$$y_B = 3.56 + 2.45 = 6.01 \text{ ft} \qquad \text{(estimated depth of 6 ft O.K.)}$$

The new elevation at A must lie between $101.47 + 5 = 106.47$ ft and $101.47 + 6 = 107.47$ ft. Try an elevation of 106.90 ft:

$$(A_A)_{new} = (20)(106.90 - 101.47) = 108.6\text{ ft}^2 \qquad (v_A)_{new} = Q/A = 509/108.6 = 4.69\text{ ft/s}$$

$$v_{mean} = (4.24 + 4.69)/2 = 4.46\text{ ft/s} \qquad y_A = (106.90 - 101.47) = 5.43\text{ ft}$$

$$R_{mean} = \frac{\frac{1}{2}[(6)(20) + 108.6]}{\frac{1}{2}[(6 + 20 + 6) + (5.43 + 20 + 5.43)]} = 3.637\text{ ft}$$

$$h_L = (vn/1.486R^{2/3})^2(L) = \{(4.46)(0.025)/[(1.486)(3.637)^{2/3}]\}^2(1000) = 1.01\text{ ft}$$

Check by the Bernoulli equation: $v_A^2/2g + z_A = v_B^2/2g + z_B + h_L$, $4.69^2/[(2)(32.2)] + 106.90 = 4.24^2/[(2)(32.2)] + 106.00 + 1.01$, $107.24 = 107.29$ (approximately). Further refinement is not necessary. Hence, use an elevation of 106.90 ft at A.

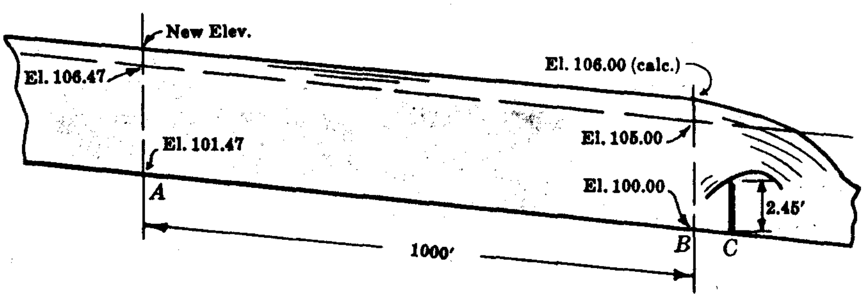


Fig. 14-64

14.259 Develop a formula for the length–energy–slope relationship for nonuniform flow problems similar to Prob. 14.258.

▌ Energy at 1 − head loss = energy at 2, $(z_1 + y_1 + v_1^2/2g) - h_L = (z_2 + y_2 + v_2^2/2g)$. Let s = slope of the energy line and s_0 = slope of the channel bottom: $s = h_L/L$, $h_L = sL$, $s_0 = (z_1 - z_2)/L$, $z_1 - z_2 = s_0L$. Therefore, $s_0L + (y_1 - y_2) + (v_1^2/2g - v_2^2/2g) = sL$, $L = [(y_1 + v_1^2/2g) - (y_2 + v_2^2/2g)]/(s - s_0) = (E_1 - E_2)/(s - s_0)$.

14.260 A rectangular flume ($n = 0.013$) is 6 ft wide and carries 66 cfs of water. At a certain section F, the depth is 3.20 ft. If the slope of the channel bed is constant at 0.000400, determine the distance from F where the depth is 2.70 ft. (Use one reach.)

▌ Assume the depth is upstream from F. Let subscript 2 refer to point F and subscript 1 to the other point.

$$L = [(y_1 + v_1^2/2g) - (y_2 + v_2^2/2g)]/(s - s_0) \qquad A_1 = (6)(2.70) = 16.20\text{ ft}^2$$

$$v_1 = Q/A_1 = 66/16.20 = 4.074\text{ ft/s}$$

$$R = A/p_w \qquad R_1 = 16.20/(2.70 + 6 + 2.70) = 1.421\text{ ft}$$

$$A_2 = (6)(3.20) = 19.20\text{ ft}^2 \qquad v_2 = Q/A_2 = 66/19.20 = 3.438\text{ ft/s}$$

$$R_2 = 19.20/(3.20 + 6 + 3.20) = 1.548\text{ ft} \qquad v_{mean} = (4.074 + 3.438)/2 = 3.756\text{ ft/s}$$

$$R_{mean} = (1.421 + 1.548)/2 = 1.484\text{ ft}$$

$$v = (1.486/n)(R^{2/3})(s^{1/2}) \qquad 3.756 = (1.486/0.013)(1.484)^{2/3}(s)^{1/2} \qquad s = 0.000638$$

$$L = \frac{\{2.70 + 4.074^2/[(2)(32.2)]\} - \{3.20 + 3.438^2/[(2)(32.2)]\}}{0.000638 - 0.000400} = -1789\text{ ft}$$

The minus sign signifies that the section with the 2.70-ft depth is downstream from F, not upstream as assumed.

14.261 A rectangular channel, 40 ft wide, carries 900 cfs of water. The slope of the channel is 0.00283. At section 1 the depth is 4.50 ft and at section 2, 300 ft downstream, the depth is 5.00 ft. What is the average value of roughness factor n?

▌

$$L = [(y_1 + v_1^2/2g) - (y_2 + v_2^2/2g)]/(s - s_0) \qquad A_1 = (40)(4.50) = 180.0\text{ ft}^2$$

$$v_1 = Q/A_1 = 900/180.0 = 5.000\text{ ft/s}$$

$$R = A/p_w \qquad R_1 = 180.0/(4.50 + 40 + 4.50) = 3.673\text{ ft}$$

$$A_2 = (40)(5.00) = 200.0\text{ ft}^2 \qquad v_2 = Q/A = 900/200.0 = 4.500\text{ ft/s}$$

$$R_2 = 200.0/(5.00 + 40 + 5.00) = 4.000\text{ ft} \qquad v_{mean} = (5.000 + 4.500)/2 = 4.750\text{ ft/s}$$

$$R_{mean} = (3.673 + 4.000)/2 = 3.836 \text{ ft}$$

$$300 = \frac{\{4.50 + 5.000^2/[(2)(32.2)]\} - \{5.00 + 4.500^2/[(2)(32.2)]\}}{s - 0.00283} \qquad s = 0.001409$$

$$v = (1.486/n)(R^{2/3})(s^{1/2}) \qquad 4.750 = (1.486/n)(3.836)^{2/3}(0.001409)^{1/2} \qquad n = 0.0288$$

14.262 A rectangular channel, 20 ft wide, has a slope of 1 ft per 1000 ft. The depth at section 1 is 8.50 ft and at section 2, 2000 ft downstream, the depth is 10.25 ft. If $n = 0.011$, determine the probable flow.

▮ $$E = y + v^2/2g + z \qquad E_1 = 8.50 + v_1^2/2g + (2000)(\tfrac{1}{1000}) \qquad E_2 = 10.25 + v_2^2/2g + 0$$

$$s = \text{head loss}/L = [(10.50 - 10.25) + (v_1^2/2g - v_2^2/2g)]/2000$$

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$A_1 = (20)(8.50) = 170.0 \text{ ft}^2 \qquad R = A/p_w$$

$$R_1 = 170.0/(8.50 + 20 + 8.50) = 4.595 \text{ ft} \qquad A_2 = (20)(10.25) = 205.0 \text{ ft}^2$$

$$R_2 = 205.0/(10.25 + 20 + 10.25) = 5.062 \text{ ft} \qquad A_{mean} = (170.0 + 205.0)/2 = 187.5 \text{ ft}^2$$

$$R_{mean} = (4.595 + 5.062)/2 = 4.828 \text{ ft}$$

Assume $s = 0.000144$: $Q = (187.5)(1.486/0.011)(4.828)^{2/3}(0.000144)^{1/2} = 868.3 \text{ ft}^3/\text{s}$. Check on s:

$$v_1 = Q/A_1 = 868.3/170.0 = 5.108 \text{ ft/s} \qquad v_1^2/2g = 5.108^2/[(2)(32.2)] = 0.4052 \text{ ft}$$

$$v_2 = 868.3/205.0 = 4.236 \text{ ft/s}$$

$$v_2^2/2g = 4.236^2/[(2)(32.2)] = 0.2786 \text{ ft} \qquad s = [(10.50 - 10.25) + (0.4052 - 0.2786)]/2000 = 0.000188$$

This value of s (0.000188) does not equal the assumed value (0.000144); hence, try $s = 0.000210$: $Q = (187.5)(1.486/0.011)(4.828)^{2/3}(0.000210)^{1/2} = 1049 \text{ ft}^3/\text{s}$. Check on s:

$$v_1 = Q/A_1 = 1049/170.0 = 6.171 \text{ ft/s} \qquad v_1^2/2g = 6.171^2/[(2)(32.2)] = 0.5913 \text{ ft}$$

$$v_2 = 1049/205.0 = 5.117 \text{ ft/s}$$

$$v_2^2/2g = 5.117^2/[(2)(32.2)] = 0.4066 \text{ ft} \qquad s = [(10.50 - 10.25) + (0.5913 - 0.4066)]/2000 = 0.000217$$

This is close to the assumed value of s of 0.000210; hence, approximate $Q = 1050 \text{ ft}^3/\text{s}$.

14.263 A reservoir feeds a rectangular channel, 15 ft wide, $n = 0.015$, as shown in Fig. 14-65. At the entrance, the depth of water is 6.22 ft above the channel bottom. The flume is 800 ft long and drops 0.72 ft in this length. The depth behind a weir at the discharge end of the channel is 4.12 ft. Determine, using one reach, the capacity of the channel assuming the loss at the entrance to be $0.25v_1^2/2g$.

▮ $$p_A/\gamma + v_A^2/2g + z_A = p_1/\gamma + v_1^2/2g + z_1 + h_m \qquad 0 + 0 + 6.22 = 0 + v_1^2/2g + y_1 + 0.25v_1^2/2g$$

$$L = [(v_2^2/2g + y_2) - (v_1^2/2g + y_1)]/(s_0 - s) \qquad s = (nv/1.486R^{2/3})^2$$

Solve these equations by successive trials until L approximates or equals 800 ft. Try $y_1 = 5.0$ ft:

$$6.22 = v_1^2/[(2)(32.2)] + 5.0 + 0.25v_1^2/[(2)(32.2)] \qquad v_1 = 7.928 \text{ ft/s}$$

$$q = y_1v_1 = (5.0)(7.928) = 39.64 \text{ ft}^3/\text{s/ft} \qquad v_2 = q/y_2 = 39.64/4.12 = 9.621 \text{ ft/s}$$

$$v_{mean} = (7.928 + 9.621)/2 = 8.774 \text{ ft/s}$$

$$R_1 = (15)(5.0)/(5.0 + 15 + 5.0) = 3.000 \text{ ft} \qquad R_2 = (15)(4.12)/(4.12 + 15 + 4.12) = 2.659 \text{ ft}$$

$$R_{mean} = (3.000 + 2.659)/2 = 2.830 \text{ ft}$$

$$s = \{(0.015)(8.774)/[(1.486)(2.830)^{2/3}]\}^2 = 0.001960$$

$$L = \frac{\{(9.621)^2/[(2)(32.2)] + 4.12\} - \{(7.928)^2/[(2)(32.2)] + 5.0\}}{0.72/800 - 0.001960} = 395 \text{ ft}$$

Since L is not equal to 800 ft, try $y_1 = 5.21$ ft:

$$6.22 = v_1^2/[(2)(32.2)] + 5.21 + 0.25v_1^2/[(2)(32.2)] \qquad v_1 = 7.214 \text{ ft/s}$$

$$q = y_1v_1 = (5.21)(7.214) = 37.58 \text{ ft}^3/\text{s/ft}$$

$$v_2 = q/y_2 = 37.58/4.12 = 9.121 \text{ ft/s} \qquad v_{mean} = (7.214 + 9.121)/2 = 8.168 \text{ ft/s}$$

$$R_1 = (15)(5.21)/(5.21 + 15 + 5.21) = 3.074 \text{ ft}$$

$$R_2 = (15)(4.12)/(4.12 + 15 + 4.12) = 2.659\text{ ft} \qquad R_{\text{mean}} = (3.074 + 2.659)/2 = 2.866\text{ ft}$$

$$s = \{(0.015)(8.168)/[(1.486)(2.866)^{2/3}]\}^2 = 0.001670$$

$$L = \frac{\{(9.121)^2/[(2)(32.2)] + 4.12\} - \{(7.214)^2/[(2)(32.2)] + 5.21\}}{0.72/800 - 0.001670} = 787\text{ ft}$$

L is not exactly equal to 800 ft, but additional computations (not shown here) show that 5.21 ft is the best value of y_1 to the nearest hundredth of a foot. Hence, $Q = (37.58)(15) = 564\text{ ft}^3/\text{s}$.

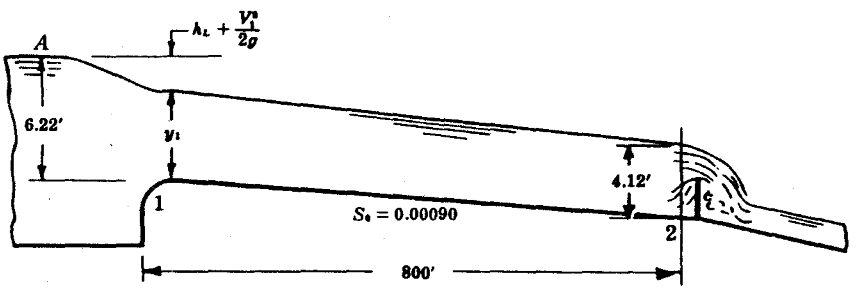


Fig. 14-65

14.264 A rectangular concrete channel 15 ft wide carries water as shown in Fig. 14-66. The channel bed slope is 0.0010. Find the theoretical rate of flow in the channel.

I

$$v_1^2/2g + d_1 + s_0L = v_2^2/2g + d_2 + sL \qquad v_1 = Q/[(15)(5.1)] = 0.01307Q$$

$$v_2 = Q/[(15)(3.9)] = 0.01709Q$$

$$s = (nv_m/1.486R_m^{2/3})^2 \qquad v_m = (0.01307Q + 0.01709Q)/2 = 0.01508Q \qquad R = A/p_w$$

$$R_1 = (15)(5.1)/(5.1 + 15 + 5.1) = 3.036\text{ ft} \qquad R_2 = (15)(3.9)/(3.9 + 15 + 3.9) = 2.566\text{ ft}$$

$$R_m = (3.036 + 2.566)/2 = 2.801\text{ ft}$$

$$s = \{(0.013)(0.01508Q)/[(1.486)(2.801)^{2/3}]\}^2 = 4.408 \times 10^{-9}Q^2$$

$$(0.01307Q)^2/[(2)(32.2)] + 5.1 + (0.0010)(1000)$$

$$= (0.01709Q)^2/[(2)(32.2)] + 3.9 + (4.408 \times 10^{-9}Q^2)(1000) \qquad Q = 591\text{ ft}^3/\text{s}$$

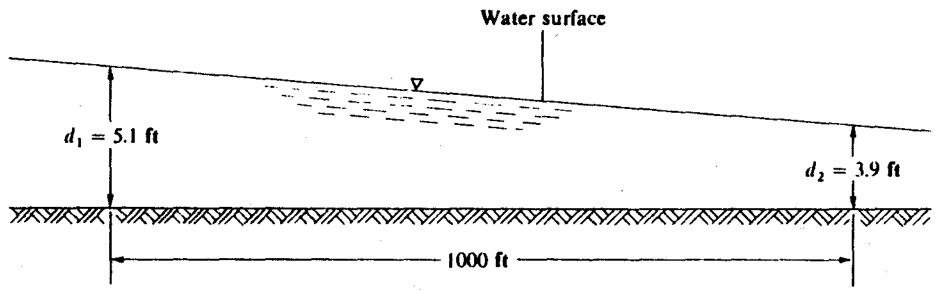


Fig. 14-66

14.265 Water flowing at the normal depth in a rectangular concrete channel that is 12.0 m wide encounters an obstruction, as shown in Fig. 14-67, causing the water level to rise above the normal depth at the obstruction and for some distance upstream. The water discharge is 126 m³/s and the channel bottom slope is 0.00086. The depth of water just upstream from the obstruction (d_0) is 4.55 m. Find the distance upstream to the point where the water surface is at the normal depth.

I

$$d_c = (q^2/g)^{1/3} = [(126/12.0)^2/9.807]^{1/3} = 2.24\text{ m} \qquad Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$

$$126 = (12.0d)(1.0/0.013)[12.0d/(d + 12.0 + d)]^{2/3}(0.00086)^{1/2}$$

$$2.256[12.0d/(d + 12.0 + d)]^{2/3} - 10.5/d = 0$$

$$d = 2.95\text{ m} \qquad \text{(by trial and error)}$$

Since $d > d_c$, flow is subcritical, and computations should proceed upstream. The problem now is to determine the distance from the point where the depth is 4.55 m to the point upstream where the depth is 2.95 m. This will be done in ten equal depth increments of 0.16 m. The computations are given in the table below.

(1) d, m	(2) v, m/s $\frac{126}{12.0\times(1)}$	(3) v_m, m/s	(4) $v^2/2g$, m $\frac{(2)^2}{2\times 9.807}$	(5) R, m $\frac{12.0\times(1)}{12.0+2\times(1)}$	(6) R_m, m	(7) s $\left[\frac{0.013\times(3)^2}{(6)^{2/3}}\right]^2$	(8) L, m $\frac{[(4)+(1)]_2-[(4)+(1)]_1}{0.00086-(7)}$
4.55	2.308		0.2716	2.588			
		2.350			2.562	0.0002662	−236
4.39	2.392		0.2917	2.535			
		2.437			2.508	0.0002946	−243
4.23	2.482		0.3141	2.481			
		2.531			2.453	0.0003272	−253
4.07	2.580		0.3394	2.425			
		2.633			2.396	0.0003654	−266
3.91	2.685		0.3676	2.367			
		2.743			2.338	0.0004098	−284
3.75	2.800		0.3997	2.308			
		2.863			2.277	0.0004626	−311
3.59	2.925		0.4362	2.246			
		2.993			2.214	0.0005246	−353
3.43	3.061		0.4777	2.182			
		3.136			2.150	0.0005989	−429
3.27	3.211		0.5257	2.117			
		3.294			2.083	0.0006893	−613
3.11	3.376		0.5811	2.048			
		3.468			2.013	0.0007997	−1580
2.95	3.559		0.6458	1.978			
							−4568 m

Hence, the answer to the problem is 4568 m.

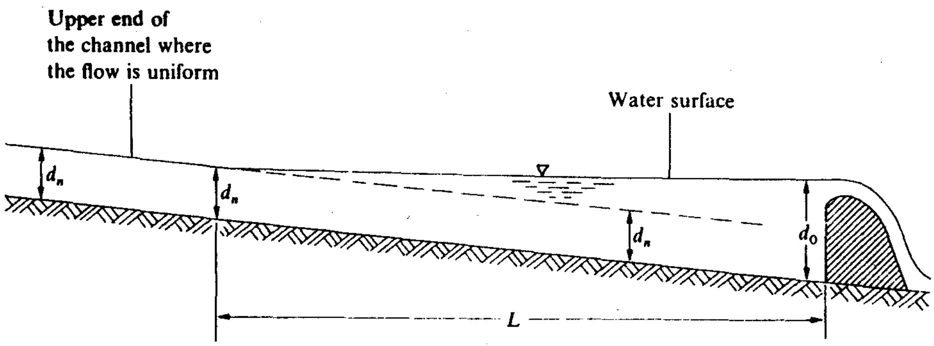


Fig. 14-67

14.266 Water flows in a rectangular concrete channel that is 5.0 ft wide, as shown in Fig. 14-68*a*, at a discharge of 16.5 cfs. Find the water-surface profile through the channel.

▌ $d_c = (q^2/g)^{1/3} = [(16.5/5.0)^2/32.2]^{1/3} = 0.70$ ft. In segment AB,

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$16.5 = (5.0d)(1.486/0.013)[5.0d/(d+5.0+d)]^{2/3}(0.00040)^{1/2}$$

$$2.286[5.0d/(d+5.0+d)]^{2/3} - 3.300/d = 0 \qquad d = 1.50\text{ ft} \qquad \text{(by trial and error)}$$

Since $d > d_c$, the flow in segment AB is subcritical.

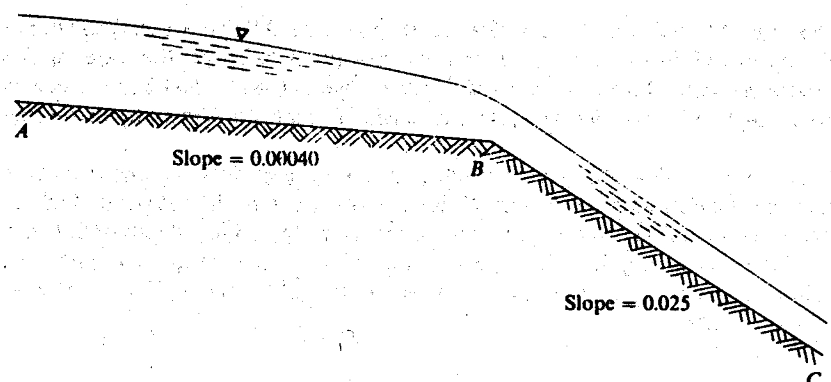
In segment BC,

$$16.5 = (5.0d)(1.486/0.013)[5.0d/(d+5.0+d)]^{2/3}(0.025)^{1/2}$$

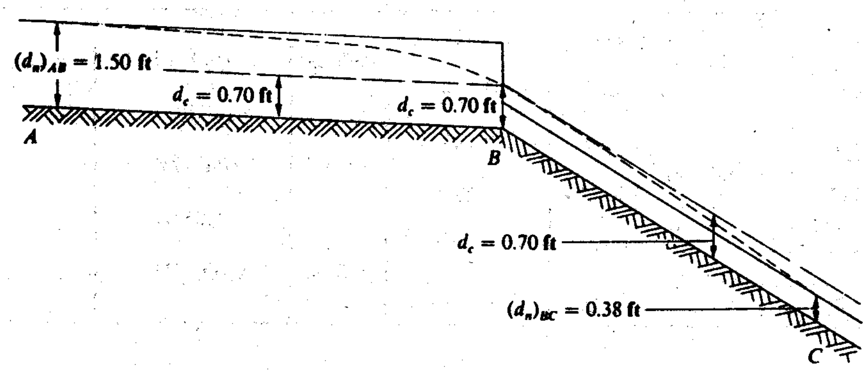
$$18.07[5.0d/(d+5.0+d)]^{2/3} - 3.300/d = 0$$

$$d = 0.38\text{ ft} \qquad \text{(by trial and error)}$$

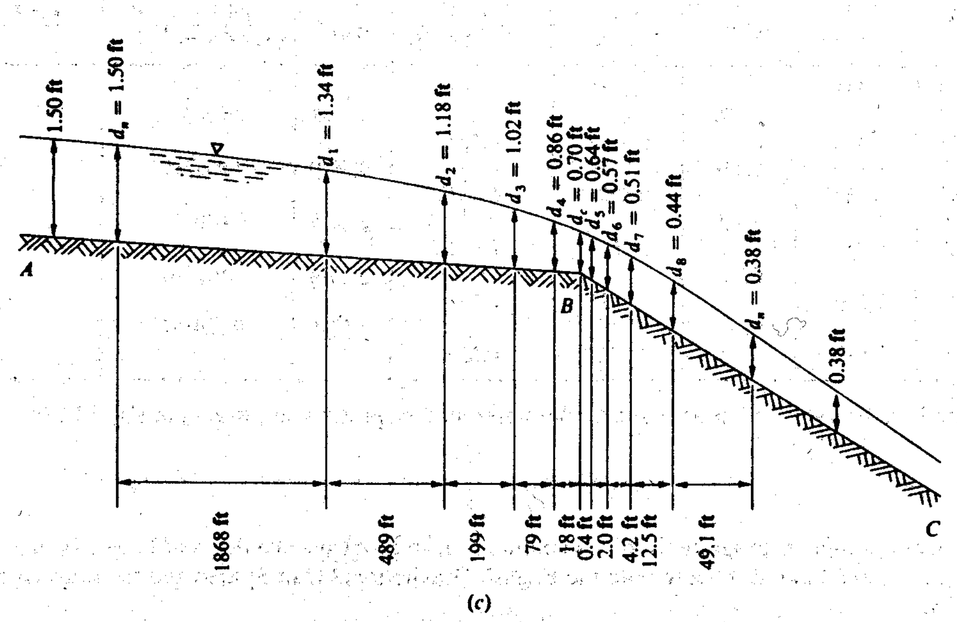
Since $d < d_c$, the flow in segment BC is supercritical.


A
Slope = 0.00040
B
Slope = 0.025
C

(a)


$(d_n)_{AB}$ = 1.50 ft
d_c = 0.70 ft
d_c = 0.70 ft
A
B
d_c = 0.70 ft
$(d_n)_{BC}$ = 0.38 ft
C

(b)


1.50 ft
d_n = 1.50 ft
d_1 = 1.34 ft
d_2 = 1.18 ft
d_3 = 1.02 ft
d_4 = 0.86 ft
d_c = 0.70 ft
d_5 = 0.64 ft
d_6 = 0.57 ft
d_7 = 0.51 ft
d_8 = 0.44 ft
d_n = 0.38 ft
0.38 ft
A
B
C
1868 ft
489 ft
199 ft
79 ft
18 ft
0.4 ft
2.0 ft
4.2 ft
12.5 ft
49.1 ft

(c)

Fig. 14-68

Figure 14-68*b* shows the location of the critical depth (the dashed line), which is constant throughout, along with the normal depths of flow in segment *AB* and segment *BC*. Obviously, the water-surface profile cannot drop instantaneously at point *B* from the normal depth in segment *AB* (1.50 ft) to the normal depth in segment *BC* (0.38 ft); there must be a transition zone on both sides of point *B* as shown by the dashed line in Fig. 14-68*b*.

As a matter of fact, as the flow changes from subcritical to supercritical in going from segment *AB* to segment *BC*, it passes through the critical state at point *B*. Hence, the depth of flow at point *B* will be 0.70 ft. The problem now becomes one of determining the flow profile from the critical depth of 0.70 ft at point *B* upstream to the point where the normal depth of 1.50 ft is reached and downstream to the point where the normal depth of 0.38 ft is reached. These computations are carried out in the tables below. Each profile will be analyzed using five equal depth increments.

For Segment *AB*

(1) d, ft	(2) v, ft/s $\frac{16.5}{5.0 \times (1)}$	(3) v_m, ft/s	(4) $v^2/2g$, ft $\frac{(2)^2}{2 \times 32.2}$	(5) R, ft $\frac{5.0 \times (1)}{5.0 + 2 \times (1)}$	(6) R_m, ft	(7) s $\left[\frac{0.013 \times (3)}{1.486 \times (6)^{2/3}}\right]^2$	(8) L, ft $\frac{[(4)+(1)]_2 - [(4)+(1)]_1}{0.00040 - (7)}$
0.70	4.714		0.3451	0.5469			
		4.276			0.5934	0.0028063	−18
0.86	3.837		0.2286	0.6399			
		3.536			0.6822	0.0015934	−79
1.02	3.235		0.1625	0.7244			
		3.016			0.7630	0.0009985	−199
1.18	2.797		0.1215	0.8016			
		2.630			0.8370	0.0006711	−489
1.34	2.463		0.0942	0.8724			
		2.332			0.9050	0.0004755	−1868
1.50	2.200		0.0752	0.9375			

For Segment *BC*

(1) d, ft	(2) v, ft/s $\frac{16.5}{5.0 \times (1)}$	(3) v_m, ft/s	(4) $v^2/2g$, ft $\frac{(2)^2}{2 \times 32.2}$	(5) R, ft $\frac{5.0 \times (1)}{5.0 + 2 \times (1)}$	(6) R_m, ft	(7) s $\left[\frac{0.013 \times (3)}{1.486 \times (6)^{2/3}}\right]^2$	(8) L, ft $\frac{[(4)+(1)]_2 - [(4)+(1)]_1}{0.025 - (7)}$
0.70	4.714		0.3451	0.5469			
		4.935			0.5283	0.004364	0.4
0.64	5.156		0.4128	0.5096			
		5.473			0.4869	0.005985	2.0
0.57	5.789		0.5204	0.4642			
		6.130			0.4439	0.008493	4.2
0.51	6.471		0.6502	0.4236			
		6.986			0.3989	0.012720	12.5
0.44	7.500		0.8734	0.3741			
		8.092			0.3520	0.020164	49.1
0.38	8.684		1.1710	0.3299			

Based on the values computed above, the water-surface profile is illustrated in Fig. 14-68*c*.

14.267 Prepare a computer program to solve nonuniform flow problems like those of Probs. 14.265 and 14.266. The program should handle data in both the English Gravitational Unit System and the International System of Units.

```
C     THIS PROGRAM DETERMINES THE FLOW PROFILE FOR A NON-UNIFORM FLOW
C     IN A RECTANGULAR OPEN CHANNEL.  IT CAN BE USED FOR PROBLEMS IN
C     BOTH THE ENGLISH SYSTEM OF UNITS AND THE INTERNATIONAL SYSTEM OF
C     UNITS.
C
C     THE PROGRAM CONSIDERS TWO ADJACENT CHANNEL SEGMENTS.  CONSIDER
C     ONE SEGMENT TO GO FROM POINT "A" TO POINT "B" AND THE OTHER
C     SEGMENT TO GO FROM POINT "B" TO POINT "C" IN THE DOWNSTREAM
C     DIRECTION.  THE PROGRAM COMPUTES THE FLOW PROFILE IN SEGMENT AB
C     IN THE UPSTREAM DIRECTION (I.E., FROM "B" TO "A") BASED ON SUB-
C     CRITICAL FLOW IN SEGMENT AB AND THE FLOW PROFILE IN SEGMENT BC IN
C     THE DOWNSTREAM DIRECTION (I.E., FROM "B" TO "C") BASED ON SUPER-
C     CRITICAL FLOW IN SEGMENT BC.  THE PROGRAM CAN BE USED EITHER FOR
C     TWO SEGMENTS AS DESCRIBED ABOVE OR FOR A SINGLE SEGMENT WITH
C     COMPUTATIONS TO PROCEED UPSTREAM (SEGMENT AB) OR DOWNSTREAM (SEG-
C     MENT BC), AS DESIRED.  IN THE CASE OF A SINGLE SEGMENT, ENTER DATA
C     FOR SEGMENT AB IF FLOW IS SUBCRITICAL AND FOR SEGMENT BC IF FLOW
C     IS SUPERCRITICAL.
C
C     THIS PROGRAM IS BASED ON CONSTANT CHANNEL WIDTH, CONSTANT FLOW
C     RATE, AND CONSTANT MANNING N-VALUE THROUGHOUT AND ON A SEPARATE
C     CONSTANT CHANNEL SLOPE IN EACH OF SEGMENTS AB AND BC.
C
C     INPUT DATA MUST BE SET UP AS FOLLOWS.
C
C     CARD 1    COLUMN 1        ENTER 0 (ZERO) OR BLANK IF THE ENGLISH
C                               SYSTEM OF UNITS IS TO BE USED.  ENTER 1
C                               (ONE) IF THE INTERNATIONAL SYSTEM OF
C                               UNITS IS TO BE USED.
C               COLUMN 2        ENTER 1 (ONE) IF ONLY A SINGLE UPSTREAM
C                               COMPUTATION IS DESIRED.  (IN THIS CASE,
C                               ENTER DEPTHS AT POINTS A AND B AND SLOPE
C                               IN SEGMENT AB.  LEAVE DEPTH AT POINT C
C                               AND SLOPE IN SEGMENT BC BLANK.)  ENTER 2
C                               (TWO) IF ONLY A SINGLE DOWNSTREAM COM-
C                               PUTATION IS DESIRED.  (IN THIS CASE, ENTER
C                               DEPTHS AT POINTS B AND C AND SLOPE IN
C                               SEGMENT BC.  LEAVE DEPTH AT POINT A AND
C                               SLOPE IN SEGMENT AB BLANK.)  ENTER 3
C                               (THREE) IF COMPUTATIONS FOR BOTH SEGMENTS
C                               ARE DESIRED. (IN THIS CASE, ENTER DEPTHS
C                               AT POINTS A, B, AND C AND SLOPES IN SEG-
C                               MENTS AB AND BC.)
C               COLUMNS 3-5     ENTER INTEGER NUMBER (RIGHT ADJUSTED)
C                               GIVING NUMBER OF LENGTH INCREMENTS TO BE
C                               USED IN COMPUTING THE FLOW PROFILE IN
C                               EACH SEGMENT.
C               COLUMNS 6-80    ENTER TITLE, DATE, AND OTHER INFORMATION,
C                               IF DESIRED.
C     CARD 2    COLUMNS 1-10    ENTER NUMBER INCLUDING DECIMAL GIVING
C                               WIDTH OF RECTANGULAR CHANNEL (IN FEET OR
C                               METERS).
C               COLUMNS 11-20   ENTER NUMBER INCLUDING DECIMAL GIVING
C                               FLOW RATE (IN CUBIC FEET PER SECOND OR
C                               CUBIC METERS PER SECOND).
C               COLUMNS 21-30   ENTER NUMBER INCLUDING DECIMAL GIVING
C                               DEPTH AT POINT A (IN FEET OR METERS).
C                               THIS VALUE MAY BE LEFT BLANK, IN WHICH
C                               CASE THE "NORMAL DEPTH" IN SEGMENT AB
C                               WILL AUTOMATICALLY BE USED.
C               COLUMNS 31-40   ENTER NUMBER INCLUDING DECIMAL GIVING
C                               DEPTH AT POINT B (IN FEET OR METERS).
C                               THIS VALUE MAY BE LEFT BLANK, IN WHICH
C                               CASE THE "CRITICAL DEPTH" WILL AUTO-
C                               MATICALLY BE USED.
C               COLUMNS 41-50   ENTER NUMBER INCLUDING DECIMAL GIVING
C                               DEPTH AT POINT C (IN FEET OR METERS).
C                               THIS VALUE MAY BE LEFT BLANK, IN WHICH
C                               CASE THE "NORMAL DEPTH" IN SEGMENT BC
C                               WILL AUTOMATICALLY BE USED.
C               COLUMNS 51-60   ENTER NUMBER INCLUDING DECIMAL GIVING
C                               CHANNEL SLOPE IN SEGMENT AB.
C               COLUMNS 61-70   ENTER NUMBER INCLUDING DECIMAL GIVING
```

```
C                            CHANNEL SLOPE IN SEGMENT BC.
C             COLUMNS 71-80  ENTER NUMBER INCLUDING DECIMAL GIVING
C                            MANNING N-VALUE.
C
C     MULTIPLE DATA SETS FOR SOLVING ANY NUMBER OF PROBLEMS MAY BE
C     INCLUDED FOR PROCESSING.
C
      COMMON DMID,DUP,NSEG,Q,W,N,COEFF,G,SUP
      DIMENSION TITLE(13)
      REAL N
      INTEGER UNITS,CODE
1     READ(5,100,END=2)UNITS,CODE,NSEG,TITLE
  100 FORMAT(2I1,I3,12A6,A3)
      WRITE(6,106)TITLE
  106 FORMAT('1',12A6,A3,////)
      READ(5,101)W,Q,DUP,DMID,DDOWN,SUP,SDOWN,N
  101 FORMAT(8F10.0)
      COEFF=1.486
      IF(UNITS.EQ.1)COEFF=1.0
      G=32.2
      IF(UNITS.EQ.1)G=9.807
      IF(DMID.LT.0.0001)DMID=((Q/W)**2/G)**(1.0/3.0)
      IF(CODE.EQ.2)GO TO 110
      IF(DUP.LT.0.0001)DUP=DNORM(W,Q,SUP,N,COEFF)
      IF(CODE.EQ.1)GO TO 109
  110 IF(DDOWN.LT.0.0001)DDOWN=DNORM(W,Q,SDOWN,N,COEFF)
      IF(CODE.EQ.2)GO TO 105
  109 X1=' UPST'
      X2='REAM '
      IF(UNITS.EQ.0)WRITE(6,107)X1,X2,W,Q,DUP,DMID,SUP,N
  107 FORMAT(1X,'GIVEN DATA FOR ' ,2A5,' FLOW PROFILE FOR A RECTANGULAR
     *OPEN CHANNEL',//5X,'WIDTH OF CHANNEL =',F7.1,' FT',6X,'FLOW RATE O
     *F WATER =',F7.1,' CU FT/S',//5X,'DEPTH OF WATER AT UPSTREAM END OF
     * SEGMENT =',F7.2,' FT',//5X,'DEPTH OF WATER AT DOWNSTREAM END OF S
     *EGMENT =',F7.2,' FT',//5X,'SLOPE =',F10.7,20X,'MANNING N-VALUE =',
     *F6.3,///1X,'THE FLOW PROFILE WITHIN THE SEGMENT IS GIVEN IN THE TA
     *BLE BELOW'///4X,'DEPTH (FT)',10X,'VELOCITY (FT/S)',10X,'LENGTH OF
     *SUBSEGMENT (FT)')
      IF(UNITS.EQ.1)WRITE(6,108)X1,X2,W,Q,DUP,DMID,SUP,N
  108 FORMAT(1X,'GIVEN DATA FOR ' ,2A5,' FLOW PROFILE FOR A RECTANGULAR
     *OPEN CHANNEL',//5X,'WIDTH OF CHANNEL =',F7.1,' M',6X,'FLOW RATE OF
     * WATER =',F7.1,' CU M/S',//5X,'DEPTH OF WATER AT UPSTREAM END OF S
     *EGMENT =',F7.2,' M',//5X,'DEPTH OF WATER AT DOWNSTREAM END OF SEGM
     *ENT =',F7.2,' M',//5X,'SLOPE =',F10.7,20X,'MANNING N-VALUE =',
     *F6.3,///1X,'THE FLOW PROFILE WITHIN THE SEGMENT IS GIVEN IN THE TA
     *BLE BELOW'///4X,'DEPTH (M)',10X,'VELOCITY (M/S)',10X,'LENGTH OF SU
     *BSEGMENT (M)')
      CALL LENGTH
      IF(CODE.EQ.1)GO TO 1
  105 X1='DOWNS'
      X2='TREAM'
      IF(UNITS.EQ.0)WRITE(6,107)X1,X2,W,Q,DMID,DDOWN,SDOWN,N
      IF(UNITS.EQ.1)WRITE(6,108)X1,X2,W,Q,DMID,DDOWN,SDOWN,N
      DUP=DDOWN
      SUP=SDOWN
      CALL LENGTH
      GO TO 1
2     STOP
      END
      FUNCTION DNORM(W,Q,S,N,COEFF)
      D=0.001
      TRY1=COEFF/N*(W*D/(W+2.0*D))**(2.0/3.0)*SQRT(S)-Q/W/D
  104 D=D+0.001
      TRY2=COEFF/N*(W*D/(W+2.0*D))**(2.0/3.0)*SQRT(S)-Q/W/D
      IF(TRY1*TRY2)102,102,103
  103 TRY1=TRY2
      GO TO 104
  102 DNORM=D-0.0005
      RETURN
      END
      SUBROUTINE LENGTH
      COMMON DMID,DUP,NSEG,Q,W,N,COEFF,G,SUP
      DINC=(DMID-DUP)/FLOAT(NSEG)
```

```
      D1=DMID
      TOTAL=0.0
      DO 102  J=1,NSEG
      D2=D1-DINC
      V1=Q/W/D1
      V2=Q/W/D2
      VMEAN=(V1+V2)/2.0
      V2G1=V1**2/2.0/G
      V2G2=V2**2/2.0/G
      HR1=W*D1/(W+2.0*D1)
      HR2=W*D2/(W+2.0*D2)
      HRMEAN=(HR1+HR2)/2.0
      SMEAN=(N*VMEAN/COEFF/HRMEAN**(2.0/3.0))**2
      SEGL=(V2G2+D2-(V2G1+D1))/(SUP-SMEAN)
      TOTAL=TOTAL+SEGL
      WRITE(6,103)D1,V1,SEGL
  103 FORMAT(1X,F10.2,11X,F12.3,/55X,F8.1)
  102 D1=D2
      WRITE(6,103)D2,V2
      WRITE(6,104)TOTAL
  104 FORMAT(41X,'TOTAL LENGTH =',F8.1,//)
      RETURN
      END
```

14.268 Solve Prob. 14.265 using the computer program developed in Prob. 14.267.

Input

```
11 10SAMPLE ANALYSIS ØF PRØFILE IN NØN-UNIFØRM FLØW
12.0       126.0                  4.55                   0.00086                0.013
```

Output

```
SAMPLE ANALYSIS OF PROFILE IN NON-UNIFORM FLOW

GIVEN DATA FOR  UPSTREAM  FLOW PROFILE FOR A RECTANGULAR OPEN CHANNEL

    WIDTH OF CHANNEL =   12.0 M      FLOW RATE OF WATER =  126.0 CU M/S

    DEPTH OF WATER AT UPSTREAM END OF SEGMENT =   2.95 M

    DEPTH OF WATER AT DOWNSTREAM END OF SEGMENT =   4.55 M

    SLOPE = 0.0008600                        MANNING N-VALUE = 0.013

THE FLOW PROFILE WITHIN THE SEGMENT IS GIVEN IN THE TABLE BELOW
```

DEPTH (M)	VELOCITY (M/S)	LENGTH OF SUBSEGMENT (M)
4.55	2.308	
		-235.2
4.39	2.392	
		-242.8
4.23	2.482	
		-252.6
4.07	2.579	
		-265.6
3.91	2.685	
		-283.6
3.75	2.799	
		-309.9
3.59	2.924	
		-351.8
3.43	3.060	
		-427.9
3.27	3.209	
		-607.7
3.11	3.374	
		-1530.9
2.95	3.556	

TOTAL LENGTH = -4508.0

14.269 Solve Prob. 14.266 using the computer program developed in Prob. 14.267.

▌ Input

```
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
03  5SAMPLE ANALYSIS ØF PRØFILE IN NØN-UNIFØRM FLØW
5.0        16.5                                          0.00040    0.025      0.013
```

Output

```
SAMPLE ANALYSIS OF PROFILE IN NON-UNIFORM FLOW


GIVEN DATA FOR  UPSTREAM  FLOW PROFILE FOR A RECTANGULAR OPEN CHANNEL

    WIDTH OF CHANNEL =    5.0 FT       FLOW RATE OF WATER =    16.5 CU FT/S

    DEPTH OF WATER AT UPSTREAM END OF SEGMENT =    1.50 FT

    DEPTH OF WATER AT DOWNSTREAM END OF SEGMENT =    0.70 FT

    SLOPE = 0.0004000                    MANNING N-VALUE = 0.013

THE FLOW PROFILE WITHIN THE SEGMENT IS GIVEN IN THE TABLE BELOW

   DEPTH (FT)          VELOCITY (FT/S)          LENGTH OF SUBSEGMENT (FT)
      0.70                  4.736
                                                       -17.6
      0.86                  3.845
                                                       -78.9
      1.02                  3.236
                                                      -201.2
      1.18                  2.793
                                                      -501.2
      1.34                  2.457
                                                     -1986.4
      1.50                  2.193

                                     TOTAL LENGTH =  -2785.2

GIVEN DATA FOR DOWNSTREAM FLOW PROFILE FOR A RECTANGULAR OPEN CHANNEL

    WIDTH OF CHANNEL =    5.0 FT       FLOW RATE OF WATER =    16.5 CU FT/S

    DEPTH OF WATER AT UPSTREAM END OF SEGMENT =    0.70 FT

    DEPTH OF WATER AT DOWNSTREAM END OF SEGMENT =    0.38 FT

    SLOPE = 0.0250000                    MANNING N-VALUE = 0.013

THE FLOW PROFILE WITHIN THE SEGMENT IS GIVEN IN THE TABLE BELOW

   DEPTH (FT)          VELOCITY (FT/S)          LENGTH OF SUBSEGMENT (FT)
      0.70                  4.736
                                                         0.5
      0.63                  5.208
                                                         1.9
      0.57                  5.783
                                                         4.5
      0.51                  6.501
                                                        11.0
      0.44                  7.423
                                                        46.1
      0.38                  8.650

                                     TOTAL LENGTH =     63.9
```

14.270 A rectangular channel 12.0 m wide is laid on a slope of 0.0028. The depth of flow at one section is 1.50 m, while the depth of flow at another section 500 ft downstream is 1.80 m. Determine the probable rate of flow, if $n = 0.026$.

▌ $Q = A_m v_m = (A_m)(1.0/n)R_m^{2/3}s^{1/2}$ $A_m = [(12.0)(1.50) + (12.0)(1.80)]/2 = 19.8\ \text{m}^2$

$$R_m = [(12.0)(1.50)/(1.50 + 12.0 + 1.50) + (12.0)(1.80)/(1.80 + 12.0 + 1.80)]/2 = 1.29\ \text{m}$$

$$Q = (19.8)(1.0/0.026)(1.29)^{2/3}s^{1/2} = 902.44s^{1/2} \qquad (1)$$

$$v_1^2/2g + d_1 + s_0L = v_2^2/2g + d_2 + sL \qquad v_1 = Q/A_1 = Q/[(12.0)(1.50)] = Q/18.0$$

$$v_2 = Q/A_2 = Q/[(12.0)(1.80)] = Q/21.6$$

$$(Q/18.0)^2/[(2)(9.807)] + 1.50 + (0.0028)(500) = (Q/21.6)^2/[(2)(9.807)] + 1.80 + 500s \qquad (2)$$

Substituting (*1*) into (*2*) gives $(902.44s^{1/2}/18.0)^2/[(2)(9.807)] + 1.50 + 1.40 = (902.44s^{1/2}/21.6)^2[(2)(9.807)] + 1.80 + 500s$, $460.8s = 1.10$, $s = 0.002387$; $Q = (902.44)(0.002387)^{1/2} = 44.1\ \text{m}^3/\text{s}$.

14.271 Solve Prob. 14.265 using five equal depth increments to determine the distance upstream to the point where the water surface is at the normal depth. Compare the answer with the one obtained in Prob. 14.265.

▌ Using data found in Prob. 14.265,

d	v	v_m	$v^2/2g$	R	R_m	s_g	L, m
4.55	2.308		0.2716	2.588			
		2.395			2.535	0.0002805	−479
4.23	2.482		0.3141	2.481			
		2.584			2.424	0.0003465	−519
3.91	2.685		0.3676	2.367			
		2.805			2.307	0.0004362	−593
3.59	2.925		0.4362	2.246			
		3.068			2.182	0.0005621	−774
3.27	3.211		0.5257	2.117			
		3.385			2.048	0.0007446	−1732
2.95	3.559		0.6458	1.978			
							−4097

The answer using five increments is $(4568 - 4097)/4568 = 0.103$, or 10.3 percent smaller than that using ten increments.

14.272 Water flows in a rectangular concrete channel similar to the one depicted in Fig. 14-68*a*. If the channel width is 3.0 ft and the discharge is 12.0 cfs, determine the water-surface profile throughout the channel shown.

▌ $d_c = [(Q/B)^2/g]^{1/3} = [(12.0/3.0)^2/32.2]^{1/3} = 0.79\ \text{ft}$ $Q = Av = (A)(1.486/n)(R^{2/3})(s^{1/2})$

$$12.0 = (3.0d_{AB})(1.486/0.013)[3.0d_{AB}/(3.0 + 2d_{AB})]^{2/3}(0.00052)^{1/2}$$

By trial and error, $d_{AB} = 1.76$ ft. $12.0 = (3.0d_{BC})(1.486/0.013)[3.0d_{BC}/(3.0 + 2d_{BC})]^{2/3}(0.030)^{1/2}$. By trial and error, $d_{BC} = 0.42$ ft. Hence, the water-surface profile changes from the normal depth in segment *AB* (1.76 ft) to the critical depth at *B* (0.79 ft) to the normal depth in segment *BC* (0.42 ft).

For Segment *AB* (*B* to *A*)

d	v	v_m	$v^2/2g$	R	R_m	s_e	L, ft
0.79	5.063		0.398	0.517			
		4.572			0.555	0.003508	−17
0.98	4.082		0.259	0.593			
		3.736			0.626	0.001995	−81
1.18	3.390		0.178	0.660			
		3.155			0.688	0.001254	−196
1.37	2.920		0.132	0.716			
		2.734			0.742	0.0008516	−510
1.57	2.548		0.101	0.767			
		2.410			0.788	0.0006107	−1863
1.76	2.273		0.080	0.810			

For Segment *BC* (*B* to *C*)

d	v	v_m	$v^2/2g$	R	R_m	s_e	L, ft
0.79	5.063		0.398	0.517			
		5.310			0.502	0.005409	0.4
0.72	5.556		0.479	0.486			
		5.903			0.468	0.007340	2.1
0.64	6.250		0.607	0.449			
		6.634			0.431	0.010346	4.5
0.57	7.018		0.765	0.413			
		7.591			0.391	0.015425	13.0
0.49	8.163		1.035	0.369			
		8.844			0.349	0.024362	53.7
0.42	9.524		1.408	0.328			

14.273 In Fig. 14-69, 400 cfs flows through the transition. The rectangular section is 8 ft wide and $y_1 = 8$ ft. The trapezoidal section is 6 ft wide at the bottom with side slopes 1:1, and $y_2 = 7.5$ ft. Determine the rise z in the bottom through the transition.

▌ $$v_1^2/2g + y_1 = v_2^2/2g + y_2 + z + E_1 \qquad v_1 = Q/A_1 = 400/[(8)(8)] = 6.250 \text{ ft/s}$$

$$v_2 = 400/[(6)(7.5) + (7.5)(7.5)] = 3.951 \text{ ft/s}$$

$$E_1 = (0.3)(v_1^2/2g - v_2^2/2g) = (0.3)\{6.250^2/[(2)(32.2)] - 3.951^2/[(2)(32.2)]\} = 0.109 \text{ ft}$$

$$6.250^2/[(2)(32.2)] + 8 = 3.951^2/[(2)(32.2)] + 7.5 + z + 0.109 \qquad z = 0.755 \text{ ft}$$

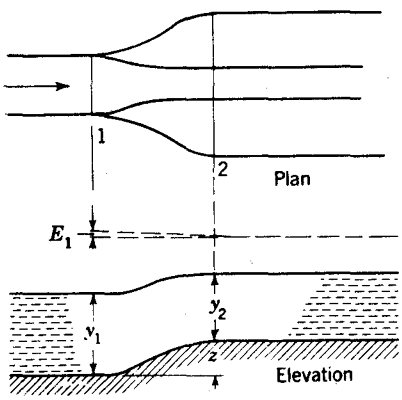


Fig. 14-69

14.274 In a critical-depth meter 2 m wide with $z = 0.3$ m, the depth y_1 is measured to be 0.75 m. Find the discharge.

▌ $q = (0.517)(g^{1/2})[y_1 - z + (0.55/g)(q^2/y_1^2)]^{3/2}$. Initially, neglect the last term in the equation above. $q = (0.517)(9.807)^{1/2}(0.75 - 0.3)^{3/2} = 0.489$ (m³/s)/m. Now, try $q = 0.500$ (m³/s)/m in the whole equation:

$q = (0.517)(9.807)^{1/2}[0.75 - 0.3 + (0.55/9.807)(0.500^2/0.75^2)]^{3/2} = 0.530\ (\text{m}^3/\text{s})/\text{m}$. Try $q = 0.536\ (\text{m}^3/\text{s})/\text{m}$: $q = (0.517)(9.807)^{1/2}[0.75 - 0.3 + (0.55/9.807)(0.536^2/0.75^2)]^{3/2} = 0.536\ (\text{m}^3/\text{s})/\text{m}$, $Q = (0.536)(2) = 1.07\ \text{m}^3/\text{s}$.

14.275 At section 1 of a canal, the cross section is trapezoidal with bottom width $b_1 = 10$ m, depth $y_1 = 7$ m, and side slopes of 2 horizontal to 1 vertical. At section 2, 200 m downstream, the bottom is 0.08 m higher than at section 1, $b_2 = 15$ m, and side slopes are 3 horizontal to 1 vertical ($Q = 200\ \text{m}^3/\text{s}$ and $n = 0.035$). Determine the depth of water at section 2.

$$\Delta L = [(v_1^2 - v_2^2)/2g + y_1 - y_2]/(s - s_0) \qquad s = (nQ/1.0AR^{2/3})^2$$

Try $y = 6.92$ m:

$$A_1 = (10)(7) + (7)[(2)(7)] = 168.0\ \text{m}^2 \qquad A_2 = (15)(6.92) + (6.92)[(3)(6.92)] = 247.5\ \text{m}^2$$

$$A_{avg} = (168.0 + 247.5)/2 = 207.8\ \text{m}^2 \qquad R = A/p_w \qquad (p_w)_1 = 10 + (2)(7)(\sqrt{2^2+1}) = 41.30\ \text{m}$$

$$(p_w)_2 = 15 + (2)(6.92)(\sqrt{3^2+1}) = 58.77\ \text{m} \qquad (p_w)_{avg} = (41.30 + 58.77)/2 = 50.04\ \text{m}$$

$$R = 207.8/50.04 = 4.153\ \text{m}$$

$$s = \{(0.035)(200)/[(1.0)(207.8)(4.153)^{2/3}]\}^2 = 0.0001700$$

$$v_1 = Q/A_1 = 200/168.0 = 1.190\ \text{m/s} \qquad v_2 = 200/247.5 = 0.8081\ \text{m/s}$$

$$\Delta L = \frac{(1.190^2 - 0.8081^2)/[(2)(9.807)] + 7 - 6.92}{0.0001700 - (-0.08/200)} = 209\ \text{m}$$

This value of ΔL (209 m) is not equal to the given value (200 m). However, reworking the problem with $y_2 = 6.93$ m (not shown) yields a value of ΔL of 191 m. Thus y_2 must be approximately 6.925 m.

14.276 A trapezoidal channel, $b = 3$ m, side slopes of 1:1, $n = 0.014$, $s_0 = 0.001$, carries $28\ \text{m}^3/\text{s}$. If the depth is 3 m at section 1, determine the water-surface profile for the next 700 m downstream.

To determine whether the depth increases or decreases, the slope of the energy grade line must be computed.

$$A = (3)(3) + (3)(3) = 18.00\ \text{m}^2 \qquad R = A/p_w = 18.00/[3 + (2)(3)(\sqrt{1^2+1^2})] = 1.567\ \text{m}$$

$$s = (nQ/1.0AR^{2/3})^2 = \{(0.014)(28)/[(1.0)(18.00)(1.567)^{2/3}]\}^2 = 0.0002606$$

$$(Q^2/gA_c^3)(T_c) = \{28^2/[(9.807)(18.00)^3]\}(9) = 0.1234$$

Since $(Q^2/gA_c^3)(T_c) < 1.0$, the depth is above critical. With the depth greater than critical and the energy grade line less steep than the bottom of the channel, the specific energy is increasing. When the specific energy increases above critical, the depth of flow increases, y is then positive.

$$L = \int_{y_1}^{y_2} \frac{1 - Q^2T/gA^3}{s_0 - (nQ/1.0AR^{2/3})^2}\,dy$$

$$L = \int_3^y \frac{1 - (28^2)(T)/[(9.807)(A)^3]}{0.001 - \{(0.014)(28)/[(1.0)(AR^{2/3})]\}^2}\,dy$$

$$L = \int_3^y \frac{1 - 79.94T/A^3}{0.001 - [0.1537/(A^2R^{4/3})]}\,dy$$

The following table evaluates the terms of the integrand:

y	A	P	R	T	numerator	$10^6 \times$ denominator	$F(y)$	L
3	18	11.48	1.57	9	0.8766	739	1185	0
3.2	19.84	12.05	1.65	9.4	0.9038	799	1131	231.6
3.4	21.76	12.62	1.72	9.8	0.9240	843	1096	454.3
3.6	23.76	13.18	1.80	10.2	0.9392	876	1072	671.1
3.8	25.84	13.75	1.88	10.6	0.9509	901	1056	883.9

The integral $\int F(y)\,dy$ can be evaluated by plotting the curve and taking the area under it between $y = 3$ and the following values of y. As $F(y)$ does not vary greatly in this example, the average of $F(y)$ can be used for each reach (the trapezoidal rule); and when it is multiplied by Δy, the length of reach is obtained. Between $y = 3$ and $y = 3.2$, $[(1185 + 1131)/2](0.2) = 231.6$. Between $y = 3.2$ and $y = 3.4$, $[(1131 + 1096)/2](0.2) = 222.7$ and so on. Five points on it are known, so the water surface can be plotted.

14.277 After contracting below a sluice gate, water flows onto a wide horizontal floor with a velocity of 15 m/s and a depth of 0.7 m. Find the equation for the water-surface profile ($n = 0.015$).

▌ $$x = -(\tfrac{3}{13})(1.0/nq)^2(y^{13/3} - y_1^{13/3}) + (3/4g)(1.0/n)^2(y^{4/3} - y_1^{4/3})$$

$$q = (0.7)(15) = 10.5\ (\text{m}^3/\text{s})/\text{m}$$

$$x = -(\tfrac{3}{13})\{1.0/[(0.015)(10.5)]\}^2(y^{13/3} - 0.7^{13/3}) + \{3/[(4)(9.807)]\}(1.0/0.015)^2(y^{4/3} - 0.7^{4/3})$$

$$= -9.303y^{13/3} + 339.9y^{4/3} - 209.3$$

$$y_c = (q^2/g)^{1/3} = (10.5^2/9.807)^{1/3} = 2.240\ \text{m}$$

The depth must increase downstream, since the specific energy decreases, and the depth must move toward the critical value for less specific energy. The equation does not hold near the critical depth because of vertical accelerations that have been neglected in the derivation of gradually varied flow. If the channel is long enough for critical depth to be attained before the end of the channel, the high-velocity flow downstream from the gate may be drowned or a jump may occur. The water-surface calculation for the subcritical flow must begin with critical depth at the downstream end of the channel.

14.278 Prepare a computer program in BASIC to calculate the steady gradually varied water-surface profile in any prismatic rectangular, symmetric trapezoidal, or triangular channel.

▌

```
10 REM B:PROFILES            WATER SURFACE PROFILES--STEADY STATE
20 ' WATER SURFACE PROFILE IN RECT, TRAPEZOIDAL, OR TRIANGULAR CHANNEL.
30 ' XL=LENGTH, B=BOT WIDTH, Z=SIDE SLOPE, RN=MANNING N,SO=BOT SLOPE,Q=FLOW.
40 ' YCONT=CONTROL DEPTH. IF YCONT=O IN DATA, YCONT IS SET EQUAL TO YC.
50 ' IN SUBCRITICAL FLOW, CONTROL IS DOWNSTREAM AND DISTANCES ARE MEASURED
60 ' IN THE UPSTREAM DIRECTION
70 ' IN SUPERCRITICAL FLOW, CONTROL IS U.S. AND DISTANCES ARE MEASURED D.S.
80 LPRINT CHR$(14);"STEADY-STATE WATER-SURFACE PROFILES"
90 LPRINT CHR$(14);"    DATE=";DATE$;" TIME=";TIME$
100 DEF FNAREA(YY)=YY*(B+Z*YY): DEF FNPER(YY)=B+2!*YY*SQR(1'+Z^2)
110 DEF FNYCRIT(YY)=1!-Q^2*(B+2!*Z*YY)/(G*FNAREA(YY)^3)
120 DEF FNYNORM(YY)=1!-Q^2*CON/(FNAREA(YY)^3.333/FNPER(YY)^1.333)
130 DEFINT I:     DEF FNDL(YY)=FNYCRIT(YY)/(FNYNORM(YY)*SO)
140 DEF FNFPM(YY)=GAM*(YY^2*(.5*B+Z*YY/3!)+Q^2/(G*FNAREA(YY)))
150 ISI$="SI":     DEF FNENERGY(YY)=YY+Q^2/(2!*G*FNAREA(YY)^2)
160 READ IUNIT$,XL,B,Z,RN,SO,Q,YCONT:  DATA "SI",200.,2.5,.8,.012,.025,25.,.0
170 IF IUNIT$=ISI$ THEN 190
180 GAM=62.4:G=32.2:CON=(RN/1.486)^2/SO:LPRINT"'USC UNITS":LPRINT: GOTO 200
190 GAM=9802!:    G=9.806001:    CON=RN^2/SO:    LPRINT"SI UNITS":LPRINT
200 LPRINT" CHANNEL LENGTH=";XL;" DISCHARGE=";Q;" B=";B;" Z=";Z;"RN=";RN;
    " SO=";SO
210 ' DETERMINATION OF CRITICAL AND NORMAL DEPTHS
220 NN=30:    DN=0!:  UP=30!: YC=15!: FOR I= 1 TO 15: IF FNYCRIT(YC)=0! THEN 250
230 IF FNYCRIT(YC)<0! THEN DN=YC ELSE UP=YC
240 YC=.5*(UP+DN):    NEXT I
250 IF YCONT=0! THEN YCONT=YC
260 IF SO<=0! THEN 320
270 UP=40!:    DN=0!:    YN=20!:    FOR I= 1 TO '15
280 X=FNYNORM(YN):    IF X<0! THEN DN=YN:    GOTO 300
290 IF X>0! THEN UP=YN ELSE GOTO 310
300 YN=.5*(UP+DN):  'NEXT I
310 LPRINT:    LPRINT" NORMAL DEPTH=";YN;" CRITICAL DEPTH=";YC:    GOTO 330
320 YN=3!*YC:    LPRINT:    LPRINT" CRITICAL DEPTH=";YC
330 IF YN<YC THEN 410
340 ' MILD,ADVERSE,OR HORIZONTAL CHANNEL YN>YC
350 IF YCONT<YC THEN 390
360 ' SUBCRITICAL FLOW, YCONT>=YC
370 SIGN=-1!:    DY=(YCONT-YN)*.998/NN:    LPRINT:
    LPRINT"CONTROL IS DOWNSTREAM,DEPTH=";YCONT:    GOTO 460
380 ' SUPERCRITICAL FLOW
390 SIGN=1!:    DY=(YC-YCONT)/NN:    LPRINT:
    LPRINT"CONTROL IS UPSTREAM, DEPTH=";YCONT:    GOTO 460
400 ' STEEP CHANNEL,YN<YC
410 IF YCONT<=YC THEN 450
420 ' SUBCRITICAL FLOW, YCONT>YC
430 SIGN=-1!:    DY=(YCONT-YC)/NN:    LPRINT:
    LPRINT" CONTROL IS DOWNSTREAM, DEPTH=";YCONT:    GOTO 460
440 ' SUPERCRITICAL FLOW, YCONT<=YC
450 SIGN=1!:    NN=2*NN:    DY=(YN-YCONT)*.998/NN:    LPRINT:
    LPRINT" CONTROL IS UPSTREAM, DEPTH=";YCONT
460 SL=0!:    Y=YCONT:    E=FNENERGY(Y):    FM=FNFPM(Y):    LPRINT
470 LPRINT"          DISTANCE          DEPTH          ENERGY          F+M":    GOSUB 550
480 ' WATER-SURFACE PROFILE CALCULATION
490 FOR I=1 TO NN STEP 2:    Y2=YCONT+SIGN*DY*(I+1)
500 DX=DY*(FNDL(Y)+FNDL(Y2)+4!*FNDL(YCONT+SIGN*I*DY))/3!
510 SL=SL+DX:    IF SL>XL THEN 540
520 Y=Y2:    E=FNENERGY(Y):    FM=FNFPM(Y):    IF (I=NN-1) AND (SL<0!) THEN SL=XL
530 GOSUB 550:    NEXT I:    GOTO 160
540 Y=Y2-SIGN*2!*DY*(SL-XL)/DX:    E=FNENERGY(Y):    FM=FNFPM(Y):    SL=XL:
    GOSUB 550:    GOTO 160
550 LPRINT SPC(5) USING"    ####.#";SL;:    LPRINT USING"     ###.###";Y;E;:
    LPRINT USING"    ########.";FM:    RETURN
560 DATA "SI",600.,2.5,.8,.012,.0002,25.,.907
570 DATA "SI",600.,2.5,.8,.012,.0002,25.,2.
```

Input data include the specification of the system of units (SI or USC) in the first columns of the data, followed by the channel dimensions, discharge, and water-surface control depth. If the control depth is set to zero in data, it is automatically assumed to be the critical depth in the program. For subcritical flow the control is downstream, and distances are measured in the upstream direction. For supercritical flow the control depth is upstream, and distances are measured in the downstream direction.

The program begins with several line functions to compute the various variables and functions in the problem. After the necessary data input, critical depth is computed, followed by the normal-depth calculation if normal depth exists. The bisection method is used in these calculations. The type of profile is then categorized, and finally the water-surface profile, specific energy, and $F + M$ are calculated and printed. Simpson's rule is used in the integration for the water-surface profile.

The program begins with several line functions to compute the various variables and functions in the problem. After the necessary data input, critical depth is computed, followed by the normal-depth calculation if normal.

14.279 A trapezoidal channel, $B = 2.5$ m, side slope = 0.8, has two bottom slopes. The upstream portion is 200 m long, $S_0 = 0.025$, and the downstream portion, 600 m long, $S_0 = 0.0002$, $n = 0.012$. A discharge of 25 m^3/s enters at critical depth from a reservoir at the upstream end, and at the downstream end of the system the water depth is 2 m. Determine the water-surface profiles throughout the system, including jump location, using the computer program of Prob. 14.278.

I Three separate sets of data are included in the program and are needed to obtain the results used to plot the solution as shown in Fig. 14-70. The first set for the steep upstream channel has a control depth equal to zero since it will be automatically assumed critical depth in the program. The second set is for the supercritical flow in the mild channel. It begins at a control depth equal to the end depth from the upstream channel and computes the water surface downstream to the critical depth. The third set of data uses the 2-m downstream depth as the control depth and computes in the upstream direction. Computer output from the last two data sets are given below. The jump is located by finding the position of equal $F + M$ from the output of the last two data sets.

```
SI UNITS

 CHANNEL LENGTH= 600   DISCHARGE= 25   B= 2.5   Z= .8 RN= .012   SO= .0002

 NORMAL DEPTH= 3.190308   CRITICAL DEPTH= 1.780243

CONTROL IS UPSTREAM, DEPTH= .9070001
```

DISTANCE	DEPTH	ENERGY	F+M
0.0	0.907	4.630	225573
25.8	0.965	4.160	211573
51.2	1.023	3.786	199573
76.0	1.082	3.487	189272
100.1	1.140	3.247	180431
123.3	1.198	3.054	172859
145.5	1.256	2.900	166400
166.5	1.315	2.777	160926
186.0	1.373	2.679	156334
204.0	1.431	2.603	152534
220.0	1.489	2.544	149455
233.9	1.547	2.500	147035
245.4	1.606	2.469	145221
254.1	1.664	2.448	143970
259.6	1.722	2.437	143242
261.5	1.780	2.433	143007

```
SI UNITS

 CHANNEL LENGTH= 600   DISCHARGE= 25   B= 2.5   Z= .8 RN= .012   SO= .0002

 NORMAL DEPTH= 3.190308   CRITICAL DEPTH= 1.780243

CONTROL IS DOWNSTREAM,DEPTH= 2
```

DISTANCE	DEPTH	ENERGY	F+M
0.0	2.000	2.474	146109
33.0	2.079	2.504	148634
80.5	2.158	2.541	151844
145.4	2.238	2.583	155711
231.3	2.317	2.630	160211
343.0	2.396	2.681	165326
486.6	2.475	2.734	171040
600.0	2.524	2.769	174857

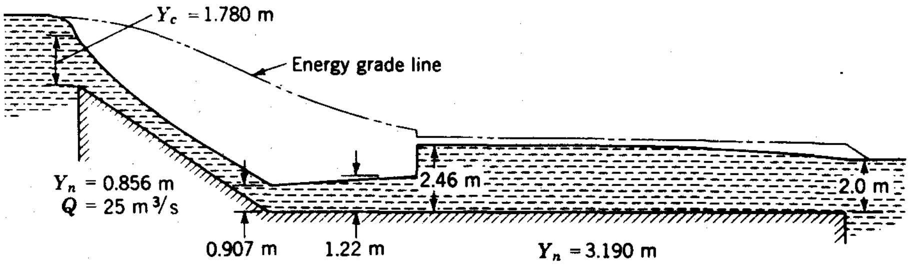


Fig. 14-70

14.280 A rectangular channel 3 m wide and 2 m deep, discharging 18 m^3/s, suddenly has the discharge reduced to 12 m^3/s at the downstream end. Compute the height and speed of the surge wave.

$$(v_1+c)(y_1)=(v_2+c)(y_2) \qquad (\gamma/2)(y_1^2-y_2^2)=(\gamma/g)(y_1)(v_1+c)(v_2+c-v_1-c)$$
$$v_1=Q/A_1=18/[(3)(2)]=3\text{ m/s} \qquad v_2=12/(3y_2) \qquad v_2y_2=4\text{ m}^2/\text{s}$$
$$(3+c)(2)=4+cy_2 \qquad 6=4+(c)(y_2-2)$$
$$(9.79/2)(2^2-y_2^2)=(9.79/9.807)(2)(3+c)(v_2+c-3-c)$$

Eliminating c and v_2 gives $y_2^2-4=(4/9.807)[2/(y_2-2)+3](3-4/y_2)$, $y_2=2.75$ m (by trial and error), $v_2=4/2.75=1.455$ m/s. The height of the surge wave is $2.75-2=0.75$ m, and the speed of the wave is $c=2/(y_2-2)=2/(2.75-2)=2.67$ m/s.

14.281 In Fig. 14-71 find the Froude number of the undisturbed flow such that the depth y_1 at the gate is just zero when the gate is suddenly closed. For $v_0=20$ ft/s, find the liquid-surface elevation.

It is required that $v_1=0$ when $y_1=0$ at $x=0$ for any time after $t=0$.

$$v=v_0-(2)(\sqrt{g})(\sqrt{y_0}-\sqrt{y}) \qquad 0=v_0-(2)(\sqrt{g})(\sqrt{y_0}-\sqrt{0})$$
$$v_0=(2)(\sqrt{gy_0}) \qquad N_F=v_0/\sqrt{gy_0}=2$$
$$x=[v_0-(2)(\sqrt{gy_0})+(3)(\sqrt{gy})](t) \qquad y_0=v_0^2/4g=20^2/[(4)(32.2)]=3.106\text{ ft}$$
$$x=\{20-(2)[\sqrt{(32.2)(3.106)}]+(3)(\sqrt{32.2y})\}(t)=17.02ty^{1/2}$$

The liquid surface is a parabola with vertex at the origin and surface concave upward.

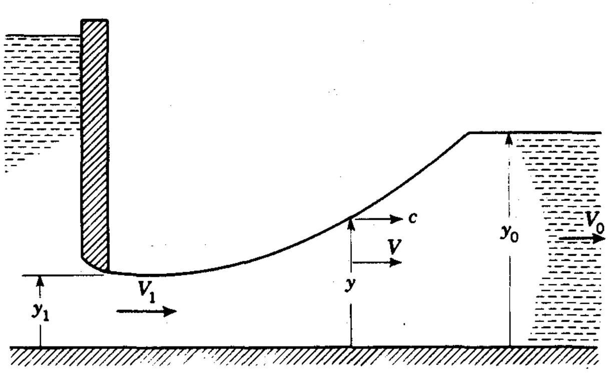


Fig. 14-71

14.282 In Fig. 14-71 the gate is partially closed at the instant $t=0$ so that the discharge is reduced by 50 percent ($v_0=6$ m/s, $y_0=3$ m). Find v_1, y_1, and the surface profile.

The new discharge is

$$q=(6)(3)/2=9=v_1y_1 \qquad v=v_0-(2)(\sqrt{g})(\sqrt{y_0}-\sqrt{y})$$
$$v_1=6-(2)(\sqrt{9.807})(\sqrt{3}-\sqrt{y_1})$$
$$v_1=4.25\text{ m/s} \quad\text{and}\quad y_1=2.11\text{ m} \quad\text{(by trial and error)}$$
$$x=[v_0-(2)(\sqrt{gy_0})+(3)(\sqrt{gy})](t)=\{6-(2)[\sqrt{(9.807)(3)}]+(3)[\sqrt{(9.807)(y)}]\}(t)$$
$$=[(9.39)(\sqrt{y})-4.85](t)$$

This holds for the range of values of y between 2.12 m and 3 m.

14.283 A discharge of 160 cfs occurs in a rectangular open channel 6 ft wide with $s_0 = 0.002$ and $n = 0.012$. If the channel ends in a free outfall, calculate the depth at the brink, y_n, and y_c. Determine the shape of the water-surface profile for a distance of 100 ft upstream from the brink.

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 160 = (6y_n)(1.486/0.012)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.002)^{1/2}$$

$$y_n = 3.5 \text{ ft} \qquad \text{(by trial and error)}$$

$$Q^2/g = A^3/B \qquad 160^2/32.2 = (6y_c)^3/6 \qquad y_c = 2.81 \text{ ft}$$

Since $y_n > y_c$, the flow is subcritical and the water-surface profile is M_2 (see Fig. A-19). The depth at the outfall is approximately $0.7y_c = (0.7)(2.81) = 2.0$ ft. Critical depth occurs at about $4y_c = (4)(2.81) = 11$ ft upstream from the brink. Computations for the water-surface profile are given below.

y, ft	A, ft²	$B+2y$, ft	R, ft	V, ft/s	$\frac{V^2}{2g}$, ft	$y+\frac{V^2}{2g}$, ft	$\Delta\left(y+\frac{V^2}{2g}\right)$, ft	V_{avg}, ft/s	R_{avg}, ft	S	$S-S_0$	x, ft	Σx,* ft
2.81	16.86	11.62	1.451	9.49	1.398	4.208							
							0.005	9.34	1.463	0.00341	0.00141	4	4
2.90	17.40	11.80	1.475	9.20	1.313	4.213							
							0.014	9.04	1.488	0.00312	0.00112	12	16
3.00	18.00	12.00	1.500	8.89	1.227	4.227							
							0.022	8.74	1.512	0.00284	0.00084	26	42
3.10	18.60	12.20	1.525	8.60	1.149	4.249							
							0.029	8.47	1.536	0.00262	0.00062	47	89
3.20	19.20	12.40	1.548	8.33	1.078	4.278							

* Summation x is measured from the point of critical depth 11 ft upstream from the brink.

The water-surface profile is sketched in Fig. 14-72.

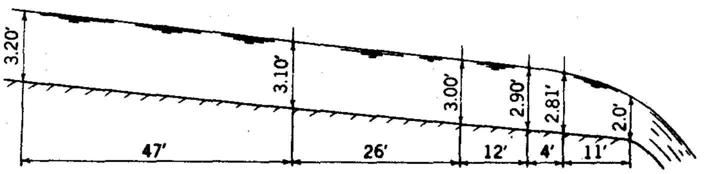


Fig. 14-72

14.284 Examine the flow conditions in a 10-ft-wide open rectangular channel of rubble masonry with $n = 0.017$ when the flow rate is 400 cfs. The channel slope is 0.020, and an ogee weir 5.0 ft high with $C_w = 3.8$ is located in the downstream end of the channel (see Fig. 14-73).

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 400 = (10y_n)(1.486/0.017)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.020)^{1/2}$$

$$y_n = 2.36 \text{ ft} \qquad \text{(by trial and error)} \qquad y_c = (q^2/g)^{1/3} \qquad y_c = [(\tfrac{400}{10})^2/32.2]^{1/3} = 3.68 \text{ ft}$$

Since $y_n < y_c$, the flow is supercritical. The head required at the weir to discharge the given flow is found from the equation $Q = (C_wL)(h + V_0^2/2g)^{3/2}$:

$$V_0 = Q/A = 400/[(5+h)(10)] = 400/(50+10h) \qquad 400 = (3.8)(10)\{h + [400/(50+10h)]^2/[(2)(32.2)]\}^{3/2}$$

$$h = 4.53 \text{ ft} \qquad \text{(by trial and error)}$$

Hence, the depth of water just upstream of the weir is $5 + 4.53 = 9.53$ ft, which is greater than y_c. The flow at this point is subcritical, and a hydraulic jump must occur upstream. The depth y_2 after the jump is found from $y_2 = -y_1/2 + (y_1^2/4 + 2V_1^2y_1/g)^{1/2}$, $V_1 = Q/A_1 = 400/[(2.36)(10)] = 16.95$ ft/s, $y_2 = -2.36/2 + [2.36^2/4 + (2)(16.95)^2(2.36)/32.2]^{1/2} = 5.42$ ft. The distance from the weir to the jump is determined by the equation $x = [(y_A + V_A^2/2g) - (y_B + V_B^2/2g)]/(s - s_0)$:

$$V_A = Q/A = 400/[(5.42)(10)] = 7.380 \text{ ft/s} \qquad V_B = 400/[(9.53)(10)] = 4.197 \text{ ft/s}$$

$$s = (nV/1.486R^{2/3})^2 \qquad V_m = (7.380 + 4.197)/2 = 5.788 \text{ ft/s}$$

$$(p_w)_A = 5.42 + 10 + 5.42 = 20.84 \text{ ft} \qquad (p_w)_B = 9.53 + 10 + 9.53 = 29.06 \text{ ft}$$

$$R_m = [(5.42)(10)/20.84 + (9.53)(10)/29.06]/2 = 2.940 \text{ ft}$$

$$s = \{(0.017)(5.788)/[(1.486)(2.940)^{2/3}]\}^2 = 0.001041$$

$$x = \{5.42 + 7.380^2/[(2)(32.2)] - 9.53 - 4.197^2/[(2)(32.2)]\}/(0.001041 - 0.020) = 187 \text{ ft}$$

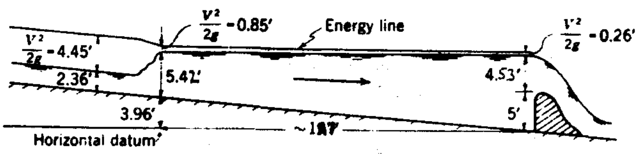


Fig. 14-73

14.285 A rectangular channel is 8 ft wide, has an 0.008 slope, discharge of 150 cfs, and $n = 0.014$. Find y_n and y_c. If the actual depth of flow is 5 ft, what type of profile exists?

I $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 150 = (8y_n)(1.486/0.014)[8y_n/(y_n + 8 + y_n)]^{2/3}(0.008)^{1/2}$

$y_n = 1.74$ ft (by trial and error) $\quad y_c = (q^2/g)^{1/3} = [(\frac{150}{8})^2/32.2]^{1/3} = 2.22$ ft

From Fig. A-19, this is an S_1 water-surface profile.

14.286 A rectangular channel is 3 m wide and ends in a free outfall. If the discharge is 10 m³/s, slope is 0.0025, and $n = 0.016$, find y_n, y_c, and the water-surface profile for a distance of 150 m upstream from the outfall.

I $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\quad 10 = (3y_n)(1.0/0.016)[3y_n/(y_n + 3 + y_n)]^{2/3}(0.0025)^{1/2}$

$y_n = 1.34$ m (by trial and error) $\quad y_c = (q^2/g)^{1/3} = [(\frac{10}{3})^2/9.807]^{1/3} = 1.04$ m

The depth at the outfall is approximately $0.7y_c = (0.7)(1.04) = 0.73$ m. Critical depth occurs at about $4y_c = (4)(1.04) = 4$ m upstream from the brink. Computations for the water-surface profile are given below $[s = (nv/1.0R^{2/3})^2,\ x = E/(s - s_0)]$.

y_n, m	v, m/s	$v^2/2g$, m	E, m	v_m, m/s	R_m, m	s	Σx, m	x, m
1.04	3.205	0.524	1.546					
				3.06	0.630	0.00443	6	6
1.14	2.924	0.436	1.576					
				2.80	0.662	0.00346	33	39
1.24	2.688	0.368	1.608					
				2.58	0.694	0.00276	185	224
1.34	2.488	0.316	1.656					

14.287 A rectangular drainage channel is 15 ft wide and is to carry 500 cfs. The channel is lined with rubble masonry ($n = 0.017$) and has a bottom slope of 0.0015. It discharges into a stream which may reach a stage 10 ft above the channel bottom during floods. Calculate y_n, y_c, and the distance from the channel outlet to the point where normal depth would occur under this condition. Use one step.

I $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 500 = (15y_n)(1.486/0.017)[15y_n/(y_n + 15 + y_n)]^{2/3}(0.0015)^{1/2}$

$y_n = 4.81$ ft (by trial and error) $\quad y_c = (q^2/g)^{1/3} = [(\frac{500}{15})^2/32.2]^{1/3} = 3.26$ ft

$\Delta x = [(y_1 + V_1^2/2g) - (y_2 + V_2^2/2g)]/(s - s_0)$ $\quad V_1 = Q/A = 500/[(4.81)(15)] = 6.930$ ft/s

$V_2 = 500/[(10)(15)] = 3.333$ ft/s

$s = (nV/1.486R^{2/3})^2$ $\quad V_m = (6.930 + 3.333)/2 = 5.132$ ft/s

$(p_w)_1 = 4.81 + 15 + 4.81 = 24.62$ ft $\quad (p_w)_2 = 10 + 15 + 10 = 35.00$ ft

$R_m = [(4.81)(15)/24.62 + (10)(15)/35.00]/2 = 3.608$ ft

$s = \{(0.017)(5.132)/[(1.486)(3.608)^{2/3}]\}^2 = 0.0006229$

$\Delta x = \{4.81 + 6.930^2/[(2)(32.2)] - 10 - 3.333^2/[(2)(32.2)]\}/(0.0006229 - 0.0015) = 5264$ ft

14.288 Solve Prob. 14.287 using three steps.

Using data from Prob. 14.287, $s = (nv/1.486R^{2/3})^2$, $x = \Delta E/(s - s_0)$.

y_n, ft	V, ft/s	$V^2/2g$, ft	E, ft	V_m, ft/s	R_m, ft	s	x, ft
10	3.33	0.17	10.17				
				3.75	4.09	0.000285	1560
8	4.17	0.27	8.27				
				4.86	3.60	0.000559	1900
6	5.55	0.48	6.48				
				6.28	3.12	0.000112	2420
4.8	7.00	0.76	5.56				
							5880

14.289 A trapezoidal channel with a bottom width of 10 ft and side slopes of 2 horizontal to 1 vertical has a horizontal curve with a radius of 100 ft without superelevation. If the discharge is 800 cfs and the water surface at the inside of the curve is 5 ft above the channel bottom, find the water-surface elevation at the outside of the curve. Assume the flow is subcritical.

See Fig. 14-74. $y_2 - y_1 = v^2B/gr$, $x - 5 = \{800/[(x+5)(10+x) - 25 - x^2]\}^2(10 + 10 + 2x)/[(32.2)(100)]$, $x = 5.53$ ft (by trial and error).

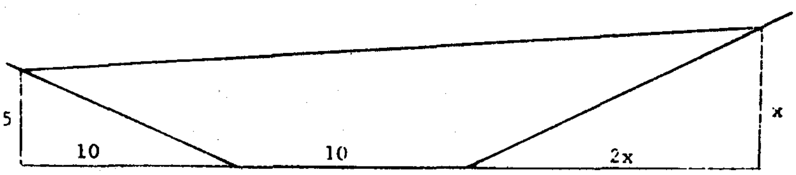


Fig. 14-74

14.290 A rectangular flume of planed timber ($n = 0.011$) is 1.5 m wide and carries 1.7 m³/s of water. The bed slope is 0.0005, and at section 1 the depth is 0.9 m. Find the distance to section 2, where the depth is 0.75 m. Is 2 upstream or downstream of 1?

$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0)$. Assume the 0.75 m depth is downstream.

$$v_1 = Q/A_1 = 1.7/[(0.9)(1.5)] = 1.259 \text{ m/s}$$
$$v_2 = Q/A_2 = 1.7/[(0.75)(1.5)] = 1.511 \text{ m/s} \qquad s = (nv/1.0R^{2/3})^2$$
$$v_m = (1.259 + 1.511)/2 = 1.385 \text{ m/s} \qquad (p_w)_1 = 0.9 + 1.5 + 0.9 = 3.30 \text{ m}$$
$$(p_w)_2 = 0.75 + 1.5 + 0.75 = 3.00 \text{ m}$$
$$R_m = [(0.9)(1.5)/3.30 + (0.75)(1.5)/3.00]/2 = 0.3920 \text{ m}$$
$$s = \{(0.011)(1.385)/[(1.0)(0.3920)^{2/3}]\}^2 = 0.000809$$
$$\Delta x = \{0.9 + 1.259^2/[(2)(9.807)] - 0.75 - 1.511^2/[(2)(9.807)]\}/(0.000809 - 0.0005) = 370 \text{ m}$$

Since Δx is positive, the 0.75 m depth is downstream, as assumed.

14.291 The flume of Prob. 14.290 still carries 1.7 m³/s, but now the depth varies from 1.2 m at one section to 0.9 m at a section 200 m downstream. Find the new bed slope.

$$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad v_1 = Q/A_1 = 1.7/[(1.2)(1.5)] = 0.9444 \text{ m/s}$$
$$v_2 = Q/A_2 = 1.7/[(0.9)(1.5)] = 1.259 \text{ m/s} \qquad s = (nv/1.0R^{2/3})^2$$
$$v_m = (0.9444 + 1.259)/2 = 1.102 \text{ m/s}$$
$$(p_w)_1 = 1.2 + 1.5 + 1.2 = 3.90 \text{ m} \qquad (p_w)_2 = 0.9 + 1.5 + 0.9 = 3.30 \text{ m}$$
$$R_m = [(1.2)(1.5)/3.90 + (0.9)(1.5)/3.30]/2 = 0.4353 \text{ m}$$
$$s = \{(0.012)(1.102)/[(1.0)(0.4353)^{2/3}]\}^2 = 0.0005301$$
$$200 = \{1.2 + 0.9444^2/[(2)(9.807)] - 0.9 - 1.259^2/[(2)(9.807)]\}/(0.0005301 - s_0) \qquad s_0 = -0.000793$$

See Fig. 14-75.

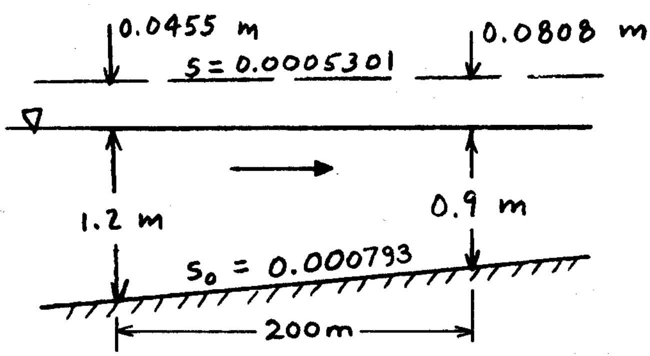


Fig. 14-75

14.292 The slope of the flume in Prob. 14.290 is now increased to 0.01. For the same flow as before, find the depth 35 m downstream from a section where the flow is 0.40 m deep.

▌ $y_c = (q^2/g)^{1/3} = [(1.7/1.5)^2/9.807]^{1/3} = 0.508$ m. Since $y < y_c$, the flow is supercritical.

$$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad v_1 = Q/A_1 = 1.7/[(0.40)(1.5)] = 2.833 \text{ m/s}$$

$$v_2 = Q/A_2 = 1.7/(1.5y_2) = 1.133/y_2 \qquad s = (nv/1.0R^{2/3})^2$$

$$v_m = (2.833 + 1.133/y_2)/2 = 1.417 + 0.5665/y_2$$

$$(p_w)_1 = 0.4 + 1.5 + 0.4 = 2.30 \text{ m} \qquad (p_w)_2 = y_2 + 1.5 + y_2 = 1.5 + 2y_2$$

$$R_m = [(0.4)(1.5)/2.30 + 1.5y_2/(1.5 + 2y_2)]/2 = 0.2609 + 0.75y_2/(1.5 + 2y_2)$$

$$s = \left\{\frac{(0.011)(1.417 + 0.5665/y_2)}{1.0[0.2609 + 0.75y_2/(1.5 + 2y_2)]^{2/3}}\right\}^2$$

$$35 = \{0.40 + 2.833^2/[(2)(9.807)] - y_2 - (1.133/y_2)^2/[(2)(9.807)]\}/(s - 0.01)$$

$$y_2 = 0.31 \text{ m} \qquad \text{(by trial and error)}$$

14.293 A rectangular flume 12 in wide and 30 ft long yielded the following test results: with still water, $z_1 - z_2 = 0.010$ ft; with a measured flow of 0.20 cfs, $y_1 = 0.400$ ft, $y_2 = 0.405$ ft. Evaluate the roughness coefficient n.

▌ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A_1 = (0.400)(\tfrac{12}{12}) = 0.400 \text{ ft}^2 \qquad A_2 = (0.405)(\tfrac{12}{12}) = 0.405 \text{ ft}^2$$

$$A_m = (0.400 + 0.405)/2 = 0.4025 \text{ ft}^2 \qquad (p_w)_1 = 0.40 + \tfrac{12}{12} + 0.40 = 1.800 \text{ ft}$$

$$(p_w)_2 = 0.405 + \tfrac{12}{12} + 0.405 = 1.810 \text{ ft}$$

$$R_m = (0.40/1.80 + 0.405/1.81)/2 = 0.2230 \text{ ft} \qquad x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0)$$

$$v_1 = Q/A_1 = 0.20/0.40 = 0.500 \text{ ft/s} \qquad v_2 = Q/A_2 = 0.2/0.405 = 0.4938 \text{ ft/s}$$

$$30 = \{0.40 + 0.500^2/[(2)(32.2)] - 0.405 - 0.4938^2/[(2)(32.2)]\}/(s - 0.01/30) \qquad s = 0.0001699$$

$$0.20 = (0.4025)(1.486/n)(0.2230)^{2/3}(0.0001699)^{1/2} \qquad n = 0.0143$$

14.294 A rectangular flume 10 ft wide is built of planed timber ($n = 0.012$) on a bed slope of 0.2 ft per 1000 ft ending in a free overfall. If the measured depth at the fall is 1.82 ft, find **(a)** the rate of flow and **(b)** the distance upstream from the fall to where the depth is 4 ft.

▌ **(a)** $$y_{brink} = 0.72y_c \qquad 1.82 = 0.72y_c \qquad y_c = 2.528 \text{ ft}$$

$$y_c = (q^2/g)^{1/3} \qquad 2.528 = (q^2/32.2)^{1/3} \qquad q = 22.81 \text{ cfs/ft} \qquad Q = (22.81)(10) = 228 \text{ ft}^3/\text{s}$$

(b) Between y_{brink} and y_c the flow is rapidly varying. So Manning's equation is not valid there and cannot be used there to determine Q. $s = (nV/1.486R^{2/3})^2$, $\Delta x = (y_1 + V_1^2/2g - y_2 - V_2^2/2g)/(s - s_0)$.

y, ft	V, fps	$V^2/2g$, ft	E, ft	A, ft²	P, ft	R, ft	R_m, ft	V_m, fps	s	Δx, ft
4.0	5.70	0.505	4.505	40	18.0	2.22				
							2.12	6.20	0.000915	568
3.4	6.71	0.698	4.095	34	16.8	2.02				
							1.949	7.15	0.001363	173
3.0	7.60	0.897	3.897	30	16.0	1.875				
							1.814	8.02	0.001888	53
2.7	8.44	1.108	3.808	27	15.4	1.753				
							1.717	8.73	0.00240	8
2.53	9.01	1.261	3.791	25.3	15.1	1.680				
										$4y_c = 10$
y_b										$\Sigma(\Delta x) = 812$

14.295 For a wide rectangular channel dredged in earth ($n = 0.030$), of slope 0.002273 and carrying 100 (ft³/s)/ft, find the water depth 1.5 mi upstream of a location (2) where the depth is 30 ft.

▮

$$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad v_1 = q/y_1 = 100/y_1$$

$$v_2 = q/y_2 = 100/30 = 3.333 \text{ ft/s}$$

$$v_m = (100/y_1 + 3.333)/2 = 50.0/y_1 + 1.667 \qquad s = (nv/1.486R^{2/3})^2$$

$$R_m = (y_1 + 30)/2 = y_1/2 + 15.00$$

$$s = \{(0.030)(50.0/y_1 + 1.667)/[(1.486)(y_1/2 + 15.00)^{2/3}]\}^2$$

$$(1.5)(5280) = \{y_1 + (100/y_1)^2/[(2)(32.2)] - 30 - 3.333^2/[(2)(32.2)]\}/(s - 0.002273)$$

$$y_1 = 11.9 \text{ ft} \qquad \text{(by trial and error)}$$

14.296 The slope of a stream of rectangular cross section is 0.00022, the width is 150 ft, and the value of the Chezy C is 80 ft$^{1/2}$/s. Find the depth for a uniform flow of 128.45 cfs per foot of width of the stream.

▮

$$v_0 = q/y_0 = 88.55/y_0 \qquad v_0 = C\sqrt{Rs} = (80.0)\{\sqrt{[150y_0/(y_0 + 150 + y_0)](0.00022)}\}$$

$$128.45/y_0 = 80\{\sqrt{[150y_0/(y_0 + 150 + y_0)](0.00022)}\} \qquad y_0 = 25.0 \text{ ft} \qquad \text{(by trial and error)}$$

14.297 If the stream of Prob. 14.296 is dammed, find the distance between a section where the increase in depth is 5 ft and one where the increase is 1 ft. Use reaches with 1-ft depth increments.

▮

$$v = C\sqrt{Rs} \qquad \Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0)$$

y, ft	A (150y), ft²	p, ft	R (A/p), ft	v (q/y), fps	$v^2/2g$, ft	E, ft	ΔE ($E_1 - E_2$), ft	v_m, fps	R_m, ft	s	Δx, ft
30	4500	210	21.43	4.28	0.284	30.284					
							−0.979	4.36	21.17	0.0001403	12 284
29	4350	208	20.91	4.43	0.305	29.305					
							−0.978	4.51	20.65	0.0001539	14 796
28	4200	206	20.39	4.59	0.327	28.327					
							−0.975	4.68	20.12	0.0001701	19 539
27	4050	204	19.85	4.76	0.352	27.352					
							−0.973	4.85	19.58	0.0001877	30 124
26	3900	202	19.31	4.94	0.379	26.379					$\Sigma(\Delta x) = 76\,743$

14.298 A portion of an outfall sewer is approximately a circular conduit 5 ft in diameter and with a slope of 1 ft in 1100 ft. It is of brick, for which $n = 0.013$. What would be its maximum capacity for uniform flow?

▮

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad Q_{\text{full}} = [(\pi)(5)^2/4](1.486/0.013)(\tfrac{5}{4})^{2/3}(\tfrac{1}{1100})^{1/2} = 78.53 \text{ ft}^3/\text{s}$$

From Fig. A-18, $(Q_{\max}/Q_{\text{full}}) = 1.06$. $Q_{\max} = (78.53)(1.06) = 83.2 \text{ ft}^3/\text{s}$.

14.299 If the outfall sewer in Prob. 14.298 discharges 120 cfs with a depth at the end of 2.90 ft, how far back from the end must it become a pressure conduit? Proceeding from the mouth upstream, find by tabular solution the length of sewer that is not flowing full. Use three reaches with equal depth increments.

▮ $s = (nV/1.486R^{2/3})^2 \qquad \Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0)$

y, ft	θ, deg	A, ft^2	P, ft	R (A/P), ft	V (Q/A), fps	$V^2/2g$, ft	E, ft	ΔE $(E_1 - E_2)$, ft	V_m, fps	R_m, ft	S	Δx, ft
2.9	99.2	11.81	8.66	1.364	10.16	1.603	4.503					
								0.073	9.05	1.429	0.00387	24.7
3.6	116.1	15.13	10.13	1.494	7.93	0.976	4.576					
								0.417	7.31	1.504	0.00236	287
4.3	136.1	17.96	11.87	1.513	6.68	0.693	4.993					
								0.587	6.40	1.382	0.00202	527
5.0	180	19.63	15.71	1.250	6.11	0.580	5.580					
												$\Sigma(\Delta x) = 839$

The sewer must become a pressure conduit 839 ft back from the end.

14.300 For the channel of Prob. 14.131, find the separation of a section where the depth is 2.0 ft and one where the depth is 3.0 ft.

▮ Assume the 2.0 ft depth is upstream. Then $y_1 = 2.0$ ft and $y_2 = 3.0$ ft. From Prob. 14.131, $Q = 100$ cfs, $s = 0.02$, and $n = 0.015$.

$$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad v_1 = Q/A_1 = 100/4.90 = 20.41\text{ ft/s}$$

$$v_2 = Q/A_2 = 100/10.48 = 9.542\text{ ft/s}$$

$$s = (nv/1.486R^{2/3})^2 \qquad v_m = (20.41 + 9.542)/2 = 14.98\text{ ft/s} \qquad R = A/p_w$$

$$R_1 = 4.90/6.12 = 0.801\text{ ft}$$

$$R_2 = 10.48/8.96 = 1.170\text{ ft} \qquad R_m = (0.801 + 1.170)/2 = 1.9855\text{ ft}$$

$$s = \{(0.015)(14.98)/[(1.486)(0.9855)^{2/3}]\}^2 = 0.02331$$

$$\Delta x = \{2.0 + 20.41^2/[(2)(32.2)] - 3.0 - 9.542^2/[(2)(32.2)]\}/(0.02331 - 0.02) = 1225\text{ ft}$$

Since Δx is positive, the assumption that the 2.0 ft depth was upstream is correct.

14.301 Sketch the water-surface profile in a long rectangular channel ($n = 0.014$), if the channel is 10 ft wide; the flow rate is 340 cfs; and there is an abrupt change in slope from 0.0016 to 0.0150.

▮ See Fig. 14-76. $y_c = (q^2/g)^{1/3} = [(340/10)^2/32.2]^{1/3} = 3.30$ ft, $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $340 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0016)^{1/2}$, $y_n = 4.50$ ft (by trial and error). In downstream segment, $340 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0150)^{1/2}$, $y_n = 2.04$ ft (by trial and error). Thus, the flow is subcritical before the break and supercritical after the break.

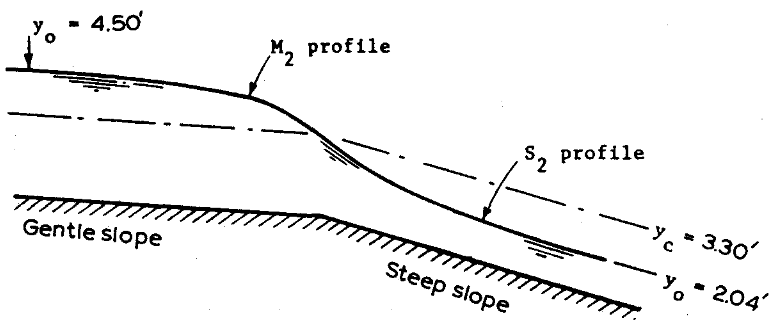


Fig. 14-76

14.302 Repeat Prob. 14.301 for the case where the flow rate is 130 cfs.

▮ $y_c = (q^2/g)^{1/3} = [(130/10)^2/32.2]^{1/3} = 1.74$ ft, $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $130 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0016)^{1/2}$, $y_n = 2.28$ ft (by trial and error). In downstream segment,

$130 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0150)^{1/2}$, $y_n = 1.08$ ft (by trial and error). Thus, the flow is subcritical before the break and supercritical after the break, so that Fig. 14-76 still applies.

14.303 Repeat Prob. 14.302 if the slope abruptly changes from 0.0016 to 0.0006. Estimate the distance upstream from the break to the point where normal depth occurs.

▌ $y_c = 1.74$ ft (from Prob. 14.302). In upstream segment, $y_n = 2.28$ ft (from Prob. 14.302). In downstream segment, $130 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0006)^{1/2}$, $y_n = 3.20$ ft (by trial and error). Thus, the flow is subcritical before and after the break. A water surface on a gentle slope cannot deviate from y_n in the upstream direction. Therefore, the depth at the break is 3.20 ft. See Fig. 14-77.

$$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad v_1 = Q/A_1 = 130/[(2.28)(10)] = 5.702 \text{ ft/s}$$

$$v_2 = Q/A_2 = 130/[(3.20)(10)] = 4.063 \text{ ft/s} \qquad s = (nv/1.486R^{2/3})^2$$

$$v_m = (5.702 + 4.063)/2 = 4.883 \text{ ft/s}$$

$$(p_w)_1 = 2.28 + 10 + 2.28 = 14.56 \text{ ft} \qquad (p_w)_2 = 3.20 + 10 + 3.20 = 16.40 \text{ ft/s}$$

$$R_m = [(2.28)(10)/14.56 + (3.20)(10)/16.40]/2 = 1.759 \text{ ft}$$

$$s = \{(0.014)(4.883)/[(1.486)(1.759)^{2/3}]\}^2 = 0.0009967$$

$$\Delta x = \{2.28 + 5.702^2/[(2)(32.2)] - 3.20 - 4.063^2/[(2)(32.2)]\}/(0.0009967 - 0.0016) = 1113 \text{ ft}$$

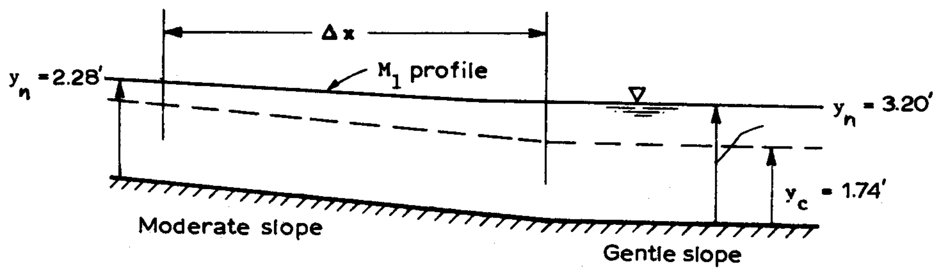


Fig. 14-77

14.304 In a 6-ft-wide rectangular channel ($s = 0.002$, $n = 0.014$), water flows at 240 cfs. A temporary dam increases the depth to 9.1 ft. Examine the water-surface profile upstream from the dam.

▌ $y_c = (q^2/g)^{1/3} = [(240/6)^2/32.2]^{1/3} = 3.68$ ft, $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $240 = (6y_n)(1.486/0.014)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.002)^{1/2}$, $y_n = 5.44$ ft (by trial and error). Since $y_n > y_c$, the channel slope is gentle. Also, since $9.1 \text{ ft} = y > y_n > y_c$, upstream of the dam is an M_1 profile (see Fig. A-19) with the depth gradually decreasing to the normal depth (5.44 ft). Critical depth (3.68 ft) occurs on the dam.

14.305 Solve Prob. 14.304 if the channel slope is 0.0004.

▌ $y_c = 3.68$ ft (from Prob. 14.304), $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $240 = (6y_n)(1.486/0.014)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.0004)^{1/2}$, $y_n = 10.7$ ft (by trial and error). Since $y_n > y_c$, the channel slope is gentle. But insertion of a dam cannot lower the water surface below y_n to 9.1 ft, so this situation is impossible.

14.306 Solve Prob. 14.304 if the channel slope is 0.0010.

▌ $y_c = 3.68$ ft (from Prob. 14.304), $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $240 = (6y_n)(1.486/0.014)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.0010)^{1/2}$, $y_n = 7.22$ ft (by trial and error). The conclusions follow Prob. 14.304.

14.307 Solve Prob. 14.304 if the channel slope is 0.006.

▌ $y_c = 3.68$ ft (from Prob. 14.304), $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $240 = (6y_n)(1.486/0.014)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.006)^{1/2}$, $y_n = 3.53$ ft (by trial and error). Since $y_n < y_c$, the channel slope is steep. Also, since $9.1 \text{ ft} = y > y_c > y_n$, upstream of the dam is an S_1 profile (see Fig. A-19), preceded by a hydraulic jump. Upstream of the jump is straight supercritical uniform flow with depth of 3.53 ft.

14.308 A rectangular channel changes in width from 3 ft to 5 ft, as shown in Fig. 14-78. Measurements indicate that $y_1 = 2.00$ ft and $Q = 60$ cfs. Determine the depth y_2, neglecting head loss.

▌ $$E_1 = E_2 + h_L \qquad E = y + v^2/2g \qquad v_1 = Q/A_1 = 60/[(3)(2.00)] = 10.00 \text{ ft/s}$$

$$E_1 = 2.0 + 10.00^2/[(2)(32.2)] = 3.553 \text{ ft} \qquad v_2 = Q/A_2 = 60/(5y_2) = 12.0/y_2$$

$$E_2 = y_2 + (12.0/y_2)^2/[(2)(32.2)] = y_2 + 2.236/y_2^2$$

$$h_L = 0 \qquad 3.553 = y_2 + 2.236/y_2^2 + 0 \qquad y_2 = 3.36 \text{ ft} \qquad \text{(by trial and error)}$$

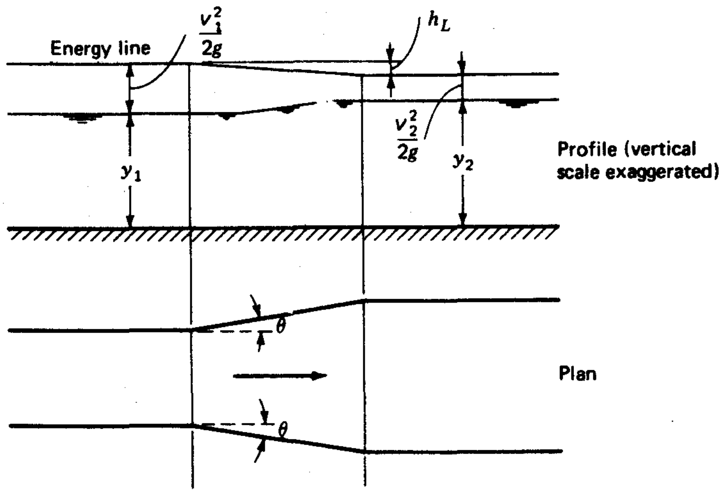


Fig. 14-78

14.309 Solve Prob. 14.308, assuming $h_L = (0.3)(v_1^2/2g - v_2^2/2g)$.

▌ Using data from Prob. 14.308, $h_L = 0.3\{10.00^2/[(2)(32.2)] - (12/y_2)^2/[(2)(32.2)]\} = 0.4658 - 0.6708/y_2^2$, $3.553 = y_2 + 2.236/y_2^2 + 0.4658 - 0.6708/y_2^2$, $y_2 = 2.90$ ft (by trial and error).

14.310 Consider a rectangular flume 4.5 m wide, built of unplaned planks ($n = 0.014$), leading from a reservoir in which the water surface is maintained constant at a height of 1.8 m above the bed of the flume at entrance (see Fig. 14-79). The flume is on a slope of 0.001. The depth 300 m downstream from the head end of the flume is 1.20 m. Assuming an entrance loss of $0.2V_1^2/2g$, find the flow rate for the given conditions.

▌ For a first approximate answer we shall consider the entire flume as one reach. The equations to be satisfied are

Energy at entrance: $$y_1 + (1.2V_1^2/2g) = 1.80 \qquad (1)$$

Energy equation for the entire reach:

$$y_1 + (V_1^2/2g) = 1.20 + (V_2^2/2g) + (S - 0.001)L \qquad (2)$$

where S is given by $$S = (nV_m/R_m^{2/3})^2 \qquad (3)$$

The procedure is to make successive trials of the upstream depth y_1. This determines corresponding values of V_1, q, V_2, V_m, R_m, and S. The trials are repeated until the value of Δx from Eq. (2) is close to 300 m. The solution is conveniently set in tabular form as follows:

Trial y_1, m	V_1, Eq. (1), m/s	$q = y_1V_1$, m^3/s/m	$V_2 = q/1.20$, m/s	V_m, m/s	R_{h_1}, m	R_{h_2}, m	R_{h_m}, m	S, Eq. (3)	Δx, Eq. (2), m
1.50	2.22	3.33	2.78	2.50	0.90	0.78	0.89	0.00143	358
1.48	2.29	3.39	2.82	2.56	0.89	0.78	0.835	0.00163	226

Thus $y_1 \approx 1.49$ m and the flow rate $Q = qB \approx 3.36 \times 4.5 \approx 15.1$ m^3/s. The accuracy of the result, of course, depends on one's ability to select the correct value for Manning's n. If n was assumed to be 0.015, for example, rather than 0.014, the result would have been quite different. Also, a more accurate result can be obtained by dividing the flume into reaches in which the depth change is about 10 percent of the depth.

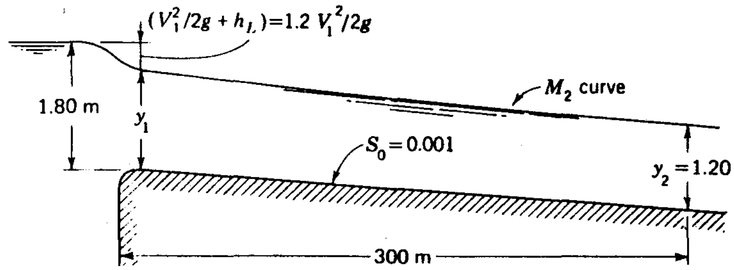


Fig. 14-79

14.311 A rectangular flume of planed timber ($n = 0.012$) 20 ft wide, 1000 ft long, with horizontal bed leads from a reservoir in which the still-water surface is 10 ft above the flume bed. Assume that the depth of the downstream end of the flume is fixed at 8 ft by some control section downstream. Allowing an entrance loss of 0.2 times the velocity head, find the flow rate in the flume using one reach.

▮ Energy at entrance:

$$y_1 + 1.2V_1^2/2g = 10 \tag{1}$$

Energy equation for the entire reach:

$$y_1 + V_1^2/2g = 8 + V_2^2/2g + (S - 0)(L) \tag{2}$$

where S is given by

$$S = (nV_m/1.486R_m^{2/3})^2 \tag{3}$$

This problem involves a trial-and-error solution exactly like that of Prob. 14.310. After successive trials (not shown), the solution is reached with $y_1 = 8.82$ ft and $Q = qB = (70.25)(20) = 1405$ ft³/s.

14.312 Find the flow rate in the flume of Prob. 14.311 if it ends in a free fall, all other conditions remaining the same. The critical depth may be supposed to occur at about $6y_c$ back from the fall. Thus the length of the reach is $1000 - 6y_c$, and $y_2 = y_c$.

▮ Take the flume in one reach with y_1 and V_1 at the upstream end and y_c and V_c at the downstream end at a distance $L = (1000 - 6y_c)$ ft. Then, energy at entrance:

$$y_1 + 1.2V_1^2/2g = 10 \tag{1}$$

Energy equation for the entire reach:

$$y_1 + V_1^2/2g = y_c + V_c^2/2g + (S - 0)(L) \tag{2}$$

where S is given by

$$S = (nV_m/1.486R_m^{2/3})^2 \tag{3}$$

First trial: Assume $y_1 = 7$ ft.

$$7 + 1.2V_1^2/[(2)(32.2)] = 10 \qquad V_1 = 12.69 \text{ ft/s} \qquad q = V_1y_1 = (12.69)(7) = 88.83 \text{ cfs/ft}$$

$$y_c = (q^2/g)^{1/3} = (88.83^2/32.2)^{1/3} = 6.26 \text{ ft}$$

$$V_c = q/y_c = 88.83/6.26 = 14.19 \text{ ft/s} \qquad V_m = (12.69 + 14.19)/2 = 13.44 \text{ ft/s}$$

$$R_m = [(20)(7)/(7 + 20 + 7) + (20)(6.26)/(6.26 + 20 + 6.26)]/2 = 3.984 \text{ ft}$$

$$S = \{(0.012)(13.44)/[(1.486)(3.984)^{2/3}]\}^2 = 0.001865$$

$$7 + 12.69^2/[(2)(32.2)] = 6.26 + 14.19^2/[(2)(32.2)] + 0.001865L \qquad L = 61.1 \text{ ft}$$

But $L = 1000 - (6)(6.26) = 962$ ft. Since the computed value of L (61.1 ft) is not equal to the actual length of approximately 962 ft, the assumed value of y_1 of 7 ft is incorrect. As a second trial, y_1 should be increased. After successive trials (not shown), a value of $y_1 = 8.46$ ft is found to be appropriate. Then, $8.46 + 1.2V_1^2/[(2)(32.2)] = 10$, $V_1 = 9.091$ ft/s, $q = (9.091)(8.46) = 76.91$ cfs/ft, $Q = (76.91)(20) = 1538$ ft³/s.

14.313 An 8-ft-wide rectangular channel has bottom slope $s_0 = 0.0015$ and roughness $n = 0.013$; it carries 250 ft³/s of water. Compute the separation between sections where depths are 4.00 ft and 4.20 ft.

$$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad s = (nv_{avg}/1.486R_{avg}^{2/3})^2$$

The calculations in the following table give the separation as 512 ft.

y, ft	A, ft^2	p $(8+2y)$, ft	R_h, ft	v, fps	$v^2/2g$, ft	$y+(v^2/2g)$	Numerator $\Delta[y+(v^2/2g)]$	v_{avg}, fps	R_{avg}, ft	s	Denominator $s-s_0$	Δx, ft
4.00	32.00	16.00	2.00	7.81	0.947	4.947						
							0.055	7.72	2.013	0.00179	0.00029	190
4.10	32.80	16.20	2.025	7.62	0.902	5.002						
							0.058	7.53	2.037	0.00168	0.00018	322
4.20	33.60	16.40	2.049	7.44	0.860	5.060						$\Sigma(\Delta x) = 512$

14.314 Assume the channel of Prob. 14.145 has a bottom slope of 0.0048. If $y_0 = 3$ ft at $x = 0$, how far along the channel $x = L$ does it take the depth to rise to $y_L = 4$ ft (see Fig. 14-80). Use increments of $y = 0.2$ ft and Manning's formula. Is the 4-ft-depth position upstream or downstream in Fig. 14-80?

$y_c = 4.27$ ft (from Prob. 14.145). Since $[y = 3\text{ ft}] < [y_c = 4.27\text{ ft}]$, the flow is supercritical; and the given channel slope (0.0048) is greater than s_c of 0.00435 (from Prob. 14.145). Therefore, we must be on an S profile. $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. Since the channel is "wide", use $R = y_n$: $50 = [(1)(y_n)](1.486/0.022)(y_n)^{2/3}(0.0048)^{1/2}$, $y_n = 4.14$ ft. Thus y_0 and y_L are less than y_n, which is less than y_c and we *must* be on an S_3 curve. For the numerical solution, we tabulate $y = 3.0$ to 4.0 in intervals of 0.2, computing six values of $V = q/y$, $E = y + (V^2/2g)$, and $S = n^2V^2/2.208y^{4/3}$, from which S_{av} and Δx follow. The slope $S_0 = 0.0048$ is constant.

y, ft	$V = 50/y$	$E = y + (V^2/2g)$	S	S_{av}	Δx	$x = \Sigma \Delta x$
3.0	16.67	7.313	0.01407			0.0
				0.01271	40.7	
3.2	15.62	6.991	0.01135			40.7
				0.01031	42.3	
3.4	14.71	6.758	0.00927			83.0
				0.00847	44.4	
3.6	13.89	6.595	0.00766			127.4
				0.00703	48.0	
3.8	13.16	6.488	0.00640			175.4
				0.00590	56.4	
4.0	12.50	6.426	0.00539			$L = 231.8$ ft

For this depth increment of 0.2 ft, gradually-varied-flow theory predicts that a length of about 232 ft is required for the depth to rise from 3 to 4 ft in this supercritical flow. Using 10 increments by reducing Δy to 0.1 ft would give an estimate of $L = 235.2$ ft (calculations not shown). It should be clear that the $y = 4$ ft position is downstream.

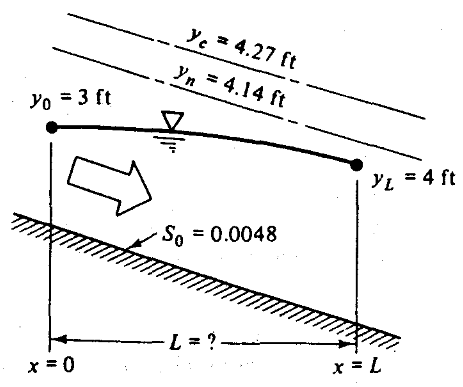


Fig. 14-80

14.315 If a rectangular channel 10 ft wide, with $n = 0.014$ and $s_0 = 0.0009$, carries $Q = 300$ cfs, is the slope steep or mild?

■ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 300 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0009)^{1/2}$$

$$y_n = 5.09 \text{ ft} \qquad \text{(by trial and error)} \qquad y_c = (q^2/g)^{1/3} = [(\tfrac{300}{10})^2/32.2]^{1/3} = 3.03 \text{ ft}$$

Since $y_c < y_n$, the slope is mild.

14.316 Repeat Prob. 14.315 if the slope is 0.006.

■ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 300 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.006)^{1/2}$$

$$y_n = 2.57 \text{ ft} \qquad \text{(by trial and error)} \qquad y_c = (q^2/g)^{1/3} = [(\tfrac{300}{10})^2/32.2]^{1/3} = 3.03 \text{ ft}$$

Since $y_c > y_n$, the slope is steep.

14.317 Water flows in a wide clean-earth ($n = 0.020$) channel at $s_0 = 0.006$ and $q = 10$ (m^3/s)/m. Give the ranges of depths corresponding to flow on a type 1, 2, or 3 curve.

■ $$Q/B = (A/B)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 10 = (y_n)(1.0/0.020)(y_n)^{2/3}(0.006)^{1/2}$$

$$y_n = 1.77 \text{ m} \qquad y_c = (q^2/g)^{1/3} = (10^2/9.807)^{1/3} = 2.17 \text{ m}$$

Since $y_c > y_n$, the slope is steep. Flow will be S_1 for $y > 2.17$ m (see Fig. A-19); S_2 for $1.77 \text{ m} < y < 2.17$ m; and S_3 for $y < 1.77$ m.

14.318 Repeat Prob. 14.317 if the channel slope is 0.0014.

■ $$Q/B = (A/B)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 10 = (y_n)(1.0/0.020)(y_n)^{2/3}(0.0014)^{1/2}$$

$$y_n = 2.73 \text{ m} \qquad y_c = (q^2/g)^{1/3} = (10^2/9.807)^{1/3} = 2.17 \text{ m}$$

Since $y_c < y_n$, the slope is mild. Flow will be M_1 for $y > 2.73$ m (see Fig. A-19); M_2 for $2.17 \text{ m} < y < 2.73$ m; and M_3 for $y < 2.17$ m.

14.319 Water flows at 130 cfs in an isosceles right-triangular sluice with $n = 0.015$ and $s_0 = 0.010$. For what ranges of depths will the flow be a type 1, 2, or 3 curve?

■ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = y_n y_n = y_n^2 \qquad p_w = 2y_n/(\cos 45°)$$

$$R = A/p_w = y_n^2/[2y_n/(\cos 45°)] = 0.3536y_n$$

$$130 = (y_n^2)(1.486/0.015)(0.3536y_n)^{2/3}(0.010)^{1/2} \qquad y_n = 3.41 \text{ ft}$$

$$A_c = (bQ^2/g)^{1/3} \qquad y_c^2 = [(2y_c)(130)^2/32.2]^{1/3} \qquad y_c = 4.02 \text{ ft}$$

Since $y_c > y_n$, the slope is steep. Flow will be S_1 for $y > 4.02$ ft (see Fig. A-19); S_2 for $3.41 \text{ ft} < y < 4.02$ ft; and S_3 for $y < 3.41$ ft.

14.320 A riveted-steel ($n = 0.014$) duct is 2 m in diameter and laid on a slope of 0.000758. The duct is half-full of water flowing at 2.5 m^3/s. Specify the type of the flow curve.

■ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $Q_{\text{full}} = [(\pi)(2)^2/4](1.0/0.014)(\tfrac{2}{4})^{2/3}(0.000758)^{1/2} = 3.89$ m^3/s, $Q/Q_{\text{full}} = 2.5/3.89 = 0.64$. From Fig. A-18, $D/D_{\text{full}} = 58$ percent. $y_n = (0.58)(2) = 1.16$ m, $A_c = (bQ^2/g)^{1/3}$. Try $y_c = 0.70$ m: $y_c/y_{\text{full}} = 0.70/2 = 0.35$. From Fig. A-18, $A_c/A_{\text{full}} = 30\%$.

$$A_c = 0.30[(\pi)(2)^2/4] = 0.9425 \text{ m}^2 \qquad b = 2[\sqrt{1^2 - (1 - y_c)^2}] \qquad \text{(see Fig. 14-81)}$$

$$b = 2[\sqrt{1^2 - (1 - 0.70)^2}] = 1.908 \text{ m} \qquad 0.9425 = (1.908Q^2/9.807)^{1/3} \qquad Q = 2.07 \text{ m}^3/\text{s}$$

Since the computed value of Q (2.07 m^3/s) is not equal to the given value (2.5 m^3/s), try $y_c = 0.75$ m: $y_c/y_{\text{full}} = 0.75/2 = 0.375$. From Fig. A-18, $A_c/A_{\text{full}} = 34\%$.

$$A_c = 0.34[(\pi)(2)^2/4] = 1.068 \text{ m}^2 \qquad b = 2[\sqrt{1^2 - (1 - 0.75)^2}] = 1.936 \text{ m}$$

$$1.068 = (1.936Q^2/9.807)^{1/3} \qquad Q = 2.48 \text{ m}^3/\text{s}$$

This value of Q is close enough to the given value of 2.5 m^3/s so that y_c can be taken as 0.75 m. Since $y_c = 0.75\ m < 1.16\ m = y_n$, the slope is mild, and an M_2 curve (Fig. A-19) is appropriate.

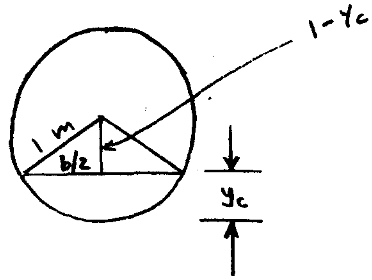


Fig. 14-81

14.321 Consider the gradual change from the profile beginning at point a in Fig. 14-82a, on a mild slope to a mild but steeper slope downstream. Sketch and label the curve $y(x)$ expected.

▌ See Fig. 14-82b.

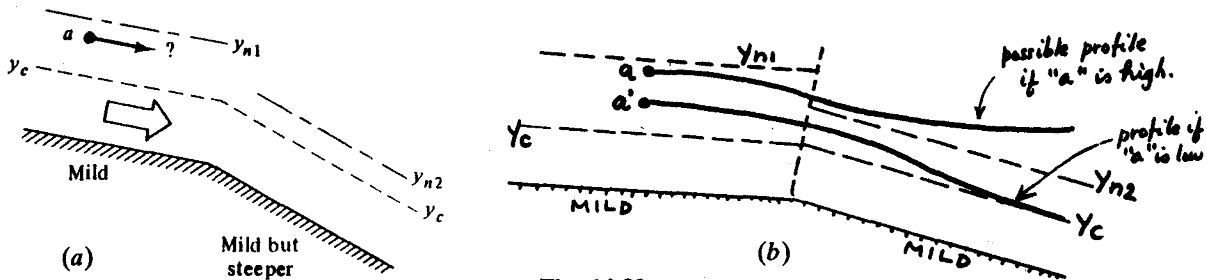


Fig. 14-82

14.322 Consider the wide-channel flow in Fig. 14-83a, which changes from a mild to a steep slope. Beginning at point a, sketch the water-surface profile $y(x)$ which is expected for gradually varied flow.

▌ See Fig. 14-83b.

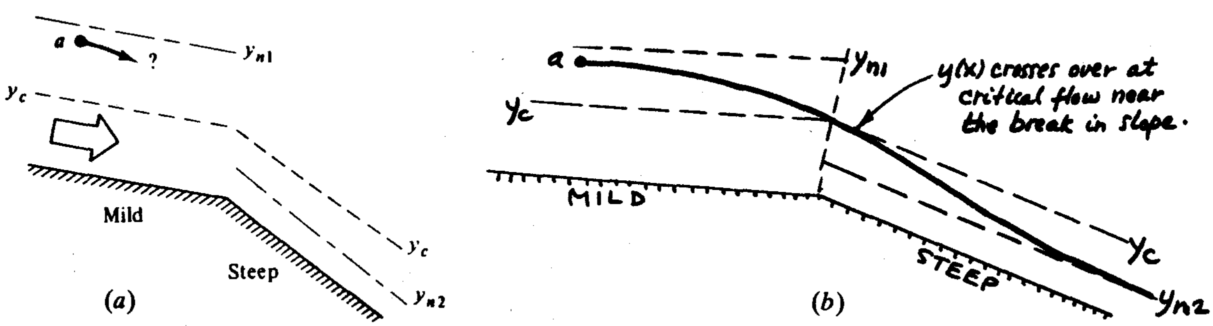


Fig. 14-83

14.323 In Fig. 14-84a the channel slope changes from steep to less steep. Beginning at point a, sketch and label the expected surface curve $y(x)$ from gradually-varied-flow theory.

▌ See Fig. 14-84b.

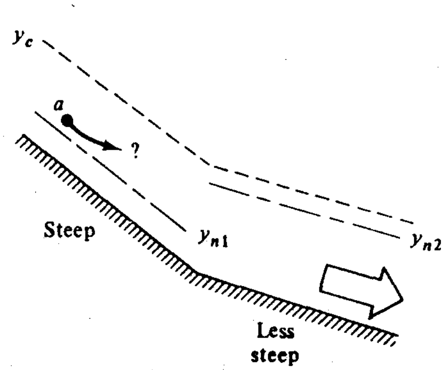


Fig. 14-84a

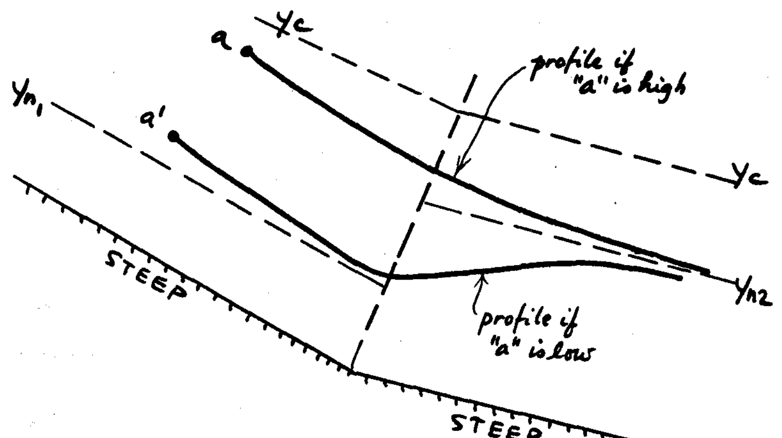


Fig. 14-84b *(continued)*

14.324 Repeat Prob. 14.314 to find the length L using only two depth increments of 0.5 ft each. Is the accuracy sufficient?

▮ Using data from Prob. 14.314,

y	V	E	S	S_{av}	Δx
3.0	16.67	7.313	0.01407		
				0.01125	100
3.5	14.29	6.669	0.00842		
				0.00691	115
4.0	12.50	6.426	0.00539		$L = 215$ ft

Using only two depth increments gives $(231.8 - 215)/231.8 = 0.07$, or 7 percent smaller length than that using five increments.

14.325 A wide-channel flow of $q = 4\ (\text{m}^3/\text{s})/\text{m}$, with $n = 0.016$, climbs an adverse slope $s_0 = -0.0009$. Using distance increments of 125 mm, approximate the distance required for the depth to change from 2.5 m to 2.0 m.

▮ $y_c = (q^2/g)^{1/3} = (4^2/9.807)^{1/3} = 1.18$ m. Hence we have an A_2 profile (see Fig. A-19).

y	$v = 4/y$	E	$S = n^2v^2/y^{4/3}$	s_{avg}	$\Delta x = \Delta E/(s_0 - s_{avg})$
2.5	1.600	2.6305	0.0001931		
				0.0002111	99.8
2.375	1.684	2.5196	0.0002291		
				0.0002518	94.2
2.25	1.778	2.4111	0.0002745		
				0.0003032	87.7
2.125	1.882	2.3056	0.0003319		
				0.0003692	80.1
2.0	2.000	2.2039	0.0004064		$\Sigma(\Delta x) = L \approx 362$ m

14.326 Figure 14-85 illustrates "free overfall" or "dropdown," where a channel flow accelerates down a slope and falls freely over a sharp edge. As shown, the flow becomes critical just before the overfall. Between y_c and the edge the flow fluctuates rapidly and does not obey gradually-varied-flow theory. If the flow rate is $q = 1.1\ (\text{m}^3/\text{s})/\text{m}$ and the surface is unfinished concrete ($n = 0.014$), estimate the water depth 300 m upstream.

▮ $$q = (1.0/n)(y)(y^{2/3})(s^{1/2}) \qquad 1.1 = (1.0/0.014)(y_n)(y_n)^{2/3}(0.001047)^{1/2} \qquad y_n = 0.64 \text{ m}$$

$$y_c = (q^2/g)^{1/3} = (1.1^2/9.807)^{1/3} = 0.50 \text{ m}$$

Hence we have an M_2 profile (see Fig. A-19). Use $\Delta y = 0.02$ m.

y	$v = 1.1/y$	E	s	s_{av}	Δx
0.50	2.200	0.74669	0.0023904		
				0.0022439	1.2
0.52	2.115	0.74808	0.0020975		
				0.0019735	3.7
0.54	2.037	0.75149	0.0018495		
				0.0017439	7.4
0.56	1.964	0.75666	0.0016384		
				0.0015480	13.3
0.58	1.897	0.76333	0.0014575		
				0.0013796	24.0
0.60	1.833	0.77131	0.0013018		
				0.0012344	48.0
0.62	1.774	0.78044	0.0011670		
				0.0011367	55.2
0.63	1.746	0.78538	0.0011064		
				0.0010920	57.3
0.635	1.732	0.78795	0.0010776		
				0.0010692	70.9
0.638	1.724	0.78951	0.0010608		
				0.0010594	21.6
0.6385	1.723	0.78977	0.0010580		
					$\Sigma\,(\Delta x) \approx 300$ m

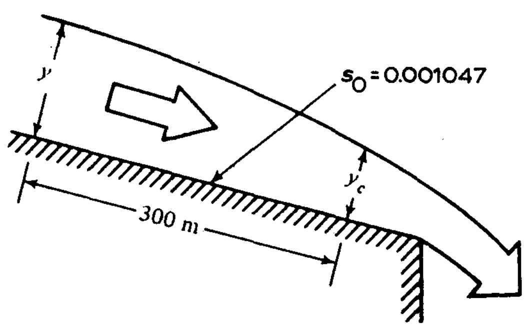


Fig. 14-85

14.327 The clean-earth ($n = 0.020$) channel in Fig. 14-86 is 6 m wide and laid on a slope of 0.005236. Water flows at 30 m³/s in the channel and enters a reservoir so that the channel depth is 3 m just before the entry. Assuming gradually varied flow, calculate the distance L.

▌ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\qquad 30 = (6y_n)(1.0/0.022)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.005236)^{1/2}$

$y_n = 1.52$ m (by trial and error) $\qquad y_c = (q^2/g)^{1/3} = [(30/6)^2/9.807]^{1/3} = 1.37$ m

Hence we have an M_1 profile (see Fig. A-19). Use $\Delta y = 0.25$ m.

y	$v = 30/6y$	E	R_h	$s = n^2v^2/R_h^{4/3}$	s_{av}	Δx
3.0	1.667	3.1416	1.500	0.0006474		
					0.0007321	50
2.75	1.818	2.9185	1.435	0.0008168		
					0.0009373	50
2.50	2.000	2.7039	1.364	0.0010577		
					0.001235	51
2.25	2.222	2.5017	1.286	0.0014122		
					0.0016864	52
2.0	2.500	2.3186	1.200	0.0019605		
						$L = \Sigma\,(\Delta x) \approx 203$ m

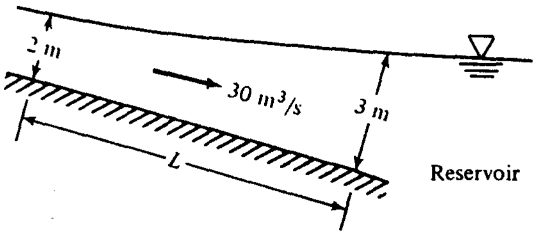


Fig. 14-86

14.328 The channel of Fig. 14-87 is sometimes called a *venturi flume*, because measurements of y_1 and y_2 can be used to meter the flow rate. Suppose that $b_1 = 4.5$ m, $b_2 = 3$ m, $y_1 = 2$ m, and $y_2 = 1.4$ m. Assuming no losses, compute the flow rate Q.

$$y_1 + v_1^2/2g = y_2 + v_2^2/2g \qquad Q = y_1 b_1 v_1 = y_2 b_2 v_2$$

Solve these equations simultaneously: $v_2 = \{(2g)(y_1 - y_2)/[1 - (y_2 b_2/y_1 b_1)^2]\}^{1/2}$, $Q = y_2 b_2 v_2 = \{(2g)(y_1 - y_2)/[(1/b_2^2 y_2^2) - (1/b_1^2 y_1^2)]\}^{1/2} = \{(2)(9.807)(2 - 1.4)/[1/(3^2)(1.4)^2 - 1/(4.5)^2(2)^2]\}^{1/2} = 16.3\ \text{m}^3/\text{s}$.

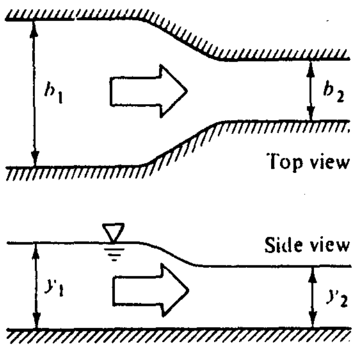


Fig. 14-87

14.329 In Prob. 14.328, holding y_1, b_1, and b_2 fixed, find y_2 for critical exit flow v_2.

$y_c = [(Q/b)^2/g]^{1/3}$. y_2 must equal y_c. $Q = \{(2g)(y_1 - y_2)/[(1/b_2^2 y_2^2) - (1/b_1^2 y_1^2)]\}^{1/2}$ (from Prob. 14.328). Try $y_2 = 1.5$ m: $Q = \{(2)(9.807)(2 - 1.5)/[1/(3^2)(1.5)^2 - 1/(4.5)^2(2)^2]\}^{1/2} = 16.27\ \text{m}^3/\text{s}$, $y_c = [(16.27/3)^2/9.807]^{1/3} = 1.44$ m. Since $y_c = 1.44$ m is not equal to the assumed $y_2 = 1.5$ m, try $y_2 = 1.45$ m: $Q = \{(2)(9.807)(2 - 1.45)/[1/(3^2)(1.45)^2 - 1/(4.5)^2(2)^2]\}^{1/2} = 16.32\ \text{m}^3/\text{s}$, $y_c = [(16.32/3)^2/9.807]^{1/3} = 1.45$ m (O.K.). Hence critical flow of $16.32\ \text{m}^3/\text{s}$ will occur at $y_2 = 1.45$ m.

14.330 In Prob. 14.328, holding y_2, b_1, and b_2 fixed, find y_1 for a flow rate of $12.2\ \text{m}^3/\text{s}$.

$$Q = \{(2g)(y_1 - y_2)/[(1/b_2^2 y_2^2) - (1/b_1^2 y_1^2)]\}^{1/2} \qquad \text{(from Prob. 14.328)}$$

$$12.2 = \{(2)(9.807)(y_1 - 1.4)/[(1/(3^2)(1.4)^2 - 1/(4.5)^2(y_1)^2]\}^{1/2} \qquad y_1 = 1.70\ \text{m} \qquad \text{(by trial and error)}$$

14.331 In the venturi flume of Fig. 14-87, let $b_1 = 5$ ft, $b_2 = 4$ ft, $y_1 = 3$ ft, and $Q = 75.9$ cfs. Compute y_2, assuming no losses.

$$Q = \{(2g)(y_1 - y_2)/[(1/b_2^2 y_2^2) - (1/b_1^2 y_1^2)]\}^{1/2} \qquad \text{(from Prob. 14.328)}$$

$$75.9 = \{(2)(32.2)(2 - y_2)/[(1/(3^2)(y_2)^2 - 1/(4)^2(2)^2]\}^{1/2} \qquad y_2 = 2.00\ \text{ft} \qquad \text{(by trial and error)}$$

14.332 A flow of $35\ \text{m}^3/\text{s}$ flows along a trapezoidal concrete channel where (see Fig. 14-88) the base a is 4 m and β is 45°. If at section 1, the depth of the flow is 3 m, what is the water-surface profile up to a distance 600 m downstream. The channel is finished concrete and has a constant slope S_0 of 0.001.

$$\Delta L = ([1 - (Q^2 b/gA^3)]/\{S_0 - (n/\kappa)^2[Q^2/(R_H^{4/3} A^2)]\})\ \Delta y$$

We start with $y_1 = 3$ m. At this section we know that

$$A_1 = (3)(4) + \tfrac{1}{2}(3)(3)(2) = 21\ \text{m}^2$$

$$(R_H)_1 = A_1/(p_w)_1 = 21/[4 + (2)(3/0.707)] = 1.6818\ \text{m} \qquad (1)$$

$$b_1 = 4 + (2)(3) = 10\ \text{m}$$

We take $n = 0.012$ and $\kappa = 1.00$ and let $y_2 = 3.1$ m. Now we compute A_2, $(R_H)_2$, and b_2.*

$$A_2 = (3.1)(4) + \tfrac{1}{2}(3.1)(3.1)(2) = 22.01\ \text{m}^2$$
$$(R_H)_2 = 22.01/[4 + 2(3.1/0.707)] = 1.7236\ \text{m} \qquad (2)$$
$$b_2 = 4 + (2)(3.1) = 10.20\ \text{m}$$

In the first interval, the average values of A, R_H, and b are

$$(A_{1\text{-}2})_{av} = 21.505\ \text{m}^2$$
$$[(R_H)_{1\text{-}2}]_{av} = 1.7027\ \text{m} \qquad (3)$$
$$(b_{1\text{-}2})_{av} = 10.10\ \text{m}$$

$$(\Delta L)_{1\text{-}2} = \left\{\frac{1 - (35^2)(10.10)/[(9.81)(21.505^3)]}{0.01 - (0.012/1)^2(35^2)/[(1.7027^{4/3})(21.505^2)]}\right\}(0.1) = 107.4\ \text{m}$$

We thus have two points on the free-surface profile. Next we compute A_3, R_{H_3}, and b_3 for $y = 3.2$ m. Using Eqs. (2) we now find the average values of these quantities in the interval 2–3. For instance, $(A_{2\text{-}3})_{av} = \tfrac{1}{2}\{(22.01) + [(3.2)(4) + (3.2^2)]\} = 22.525\ \text{m}^2$. We then proceed as indicated for the first interval.

The following table gives the results using six sections so that $L_{total} \approx 600$ m.

section	y, m	Δy, m	ΔL, m	L_{total}, m
1	3.0	0.1	0	0
2	3.1	0.1	107.4	107.4
3	3.2	0.1	106.9	214.3
4	3.3	0.1	105.6	319.9
5	3.4	0.1	104.7	424.6
6	3.5	0.1	104.1	528.7
7	3.6	0.1	103.8	632.5

We can now plot y versus L, that is, the second and last columns, starting with $y = 3$ for $L = 0$ m and going on to $y = 3.6$ m for $L = 632.5$ m. A smooth curve through these parts gives the approximate desired profile.

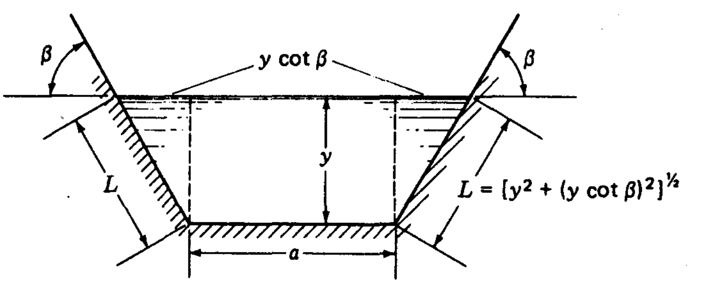


Fig. 14-88

14.333 Water enters a rectangular channel which is 3 ft wide at an average velocity of 0.8 ft/s and a depth of 3 in. The channel is laid on a slope of 0.003499. If Manning's n is 0.012, how far must the water travel before its depth becomes 4 in? Use four intermediary depths ($\Delta y = 0.2$ in) in the calculation.

$$\Delta x = \left([1 - (Q^2 b/gA^3)]/\{s_0 - (n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\}\right)\Delta y$$
$$s_0 = 0.003499 \qquad R_h = by/(b + 2y) = y$$

Point 1:

$$v_1 = 0.8\ \text{ft/s} \qquad (f_R)_1 = v\sqrt{b}/\sqrt{gA} = (0.8)(\sqrt{3})/\sqrt{(g)(3)(\tfrac{1}{4})} = 0.282 \qquad \Delta y = 0.2/12\ \text{ft}$$

Point 2:

$$v_2 = (y_0/y_1)v_1 = (3/3.2)(0.8) = 0.750\ \text{ft/s} \qquad (f_R)_2 = (0.750)(\sqrt{3})/\sqrt{(g)(3)(3.2/12)} = 0.256$$

* We are assuming that the depth y relative to the channel is increasing. If we get a positive result for ΔL, we know that our assumption is correct. If ΔL is negative, then the depth must be decreasing along the channel flow. Use y_2 less than 3 m in that case.

1–2:

$$(v_{1-2})_{av} = (0.8 + 0.750)/2 = 0.775 \text{ ft/s} \qquad (f_{R,1-2})_{av} = (0.282 + 0.256)/2 = 0.269$$

$$(\Delta x)_{1-2} = \frac{1 - 0.269^2}{0.00349 - (0.012/1.486)^2[0.775^2/(3.1/12)^{4/3}]}\left(\frac{0.2}{12}\right) = 4.75 \text{ ft}$$

Point 3:

$$v_3 = (3/3.4)(0.8) = 0.706 \text{ ft/s} \qquad (f_R)_3 = 0.706/[g(3.4/12)]^{1/2} = 0.234$$

2–3:

$$(v_{2-3})_{av} = (0.750 + 0.706)/2 = 0.728 \text{ ft/s} \qquad (f_{R,2-3})_{av} = (0.256 + 0.234)/2 = 0.245$$

$$(\Delta x)_{2-3} = \frac{1 - 0.245^2}{0.00349 - (0.012/1.486)^2[0.728^2/(3.3/12)^{4/3}]}\left(\frac{0.2}{12}\right) = 4.75 \text{ ft}$$

Point 4:

$$v_4 = (3/3.6)(0.8) = 0.667 \text{ ft/s} \qquad (f_R)_4 = 0.667/\sqrt{(g)(3.6/12)} = 0.215$$

3–4:

$$(v_{3-4})_{av} = (0.706 + 0.667)/2 = 0.686 \text{ ft/s} \qquad (f_{R,3-4})_{av} = (0.234 + 0.215)/2 = 0.224$$

$$(\Delta x)_{3-4} = \frac{1 - 0.224^2}{0.00349 - (0.012/1.486)^2[0.686^2/(3.5/12)^{4/3}]}\left(\frac{0.2}{12}\right) = 4.75 \text{ ft}$$

Point 5:

$$v_5 = (3/3.8)(0.8) = 0.632 \text{ ft/s} \qquad (f_R)_5 = 0.632/\sqrt{(g)(3.8/12)} = 0.198$$

4–5:

$$(v_{4-5})_{av} = (0.667 + 0.632)/2 = 0.650 \text{ ft/s} \qquad (f_{R,4-5})_{av} = (0.215 + 0.198)/2 = 0.206$$

$$(\Delta x)_{4-5} = \frac{1 - 0.206^2}{0.00349 - (0.012/1.486)^2[0.650^2/(3.7/12)^{4/3}]}\left(\frac{0.2}{12}\right) = 4.75 \text{ ft}$$

Point 6:

$$v_6 = (\tfrac{3}{4})(0.8) = 0.600 \text{ ft/s} \qquad (f_R)_6 = 0.600/[g(4.0/12)]^{1/2} = 0.183$$

5–6:

$$(v_{5-6})_{av} = (0.632 + 0.600)/2 = 0.616 \text{ ft/s} \qquad (f_{R,5-6})_{av} = (0.198 + 0.183)/2 = 0.190$$

$$(\Delta x)_{5-6} = \frac{1 - 0.190^2}{0.00349 - (0.012/1.486)^2[0.616^2/(3.9/12)^{4/3}]}\left(\frac{0.2}{12}\right) = 4.75 \text{ ft}$$

$$x = \sum (\Delta x) = 23.75 \text{ ft}$$

14.334 In Prob. 14.332, compute the distance L in one calculation where the free surface has a depth of 3.6 m. Do not average.

▮

$$\Delta L = ([1 - (Q^2b/gA^3)]/\{S_0 - (n/\kappa)^2[Q^2/(R_H^{4/3}A^2)]\})\,\Delta y$$

$$A = (3)(4) + (3)(3) = 21.00 \text{ m}^2 \qquad p_w = 4 + (2)(\sqrt{3^2 + 3^2}) = 12.49 \text{ m}$$

$$R_H = 21.00/12.49 = 1.681 \text{ m} \qquad b = 3 + 4 + 3 = 10 \text{ m}$$

$$\Delta L = \left\{\frac{1 - (35)^2(10)/(9.807)(21.00)^3}{0.001 - (0.012/1)^2[35^2/(1.681)^{4/3}(21.00)^2]}\right\}(3.6 - 3) = 649 \text{ m}$$

14.335 Do Prob. 14.334 using a linear average in the calculations. What is the percent error in not averaging?

▮

$$\Delta L = ([1 - (Q^2b/gA^3)]/\{S_0 - (n/\kappa)^2[Q^2/(R_H^{4/3}A^2)]\})\,\Delta y$$

At the section where $y = 3.6$ m,

$$A = (3.6)(4) + (3.6)(3.6) = 27.36 \text{ m}^2 \qquad p_w = 4 + (2)(\sqrt{3.6^2 + 3.6^2}) = 14.18 \text{ m}$$

$$R_H = 27.36/14.18 = 1.929 \text{ m} \qquad b = 3.6 + 4 + 3.6 = 11.20 \text{ m}$$

Using corresponding values where $y = 3$ m from Prob. 14.334,

$$A_{av} = (21.00 + 27.36)/2 = 24.18\ \text{m}^2 \qquad (R_H)_{av} = (1.681 + 1.929)/2 = 1.805\ \text{m}$$

$$b_{av} = (10 + 11.20)/2 = 10.60\ \text{m}$$

$$\Delta L = \left\{\frac{1 - (35)^2(10.60)/(9.807)(24.18)^3}{0.001 - (0.012/1)^2[35^2/(1.805)^{4/3}(24.18)^2]}\right\}(3.6 - 3) = 630\ \text{m}$$

Error = (649 − 630)/630 = 0.03, or 3 percent.

14.336 A wide sluice, made of finished concrete and having slope $s_0 = 0.0004$, is fed from a large reservoir through a sharp-edged gate (Fig. 14-89). The coefficient of contraction C_c is 0.82 and the coefficient of friction $C_f = 0.86$. How far from the vena contracta does the water increase depth by 30 mm? Make a one-stage calculation with linear averages.

I

$$v_1^2/2g + y_1 = v_2^2/2g + y_2 \qquad y_2 = (0.30)(0.82) = 0.2460\ \text{m}$$

$$0 + 12 = v_2^2/[(2)(9.807)] + 0.2460 \qquad v_2 = 15.18\ \text{m/s}$$

$$q_{\text{theoretical}} = y_2 v_2 = (0.2460)(15.18) = 3.734\ (\text{m}^3/\text{s})/\text{m} \qquad q_{\text{actual}} = (0.86)(3.734) = 3.21\ (\text{m}^3/\text{s})/\text{m}$$

$$\Delta x = ([1 - (Q^2b/gA^3)]/\{s_0 - (n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\})\ \Delta y$$

$$\Delta y_{\text{downstream}} = 0.246 + 0.03 = 0.276\ \text{m}$$

$$\Delta x = \left\{\frac{1 - (3.21)^2/(9.807)(0.276)^3}{0.0004 - (0.012/1)^2[3.21^2/(0.276)^{4/3}(0.276)^2]}\right\}(0.030) = 13.60\ \text{m}$$

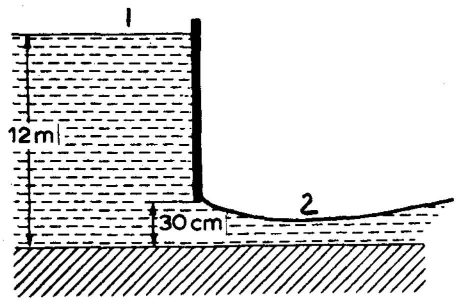


Fig. 14-89

14.337 Water is moving at 5 m/s in a very wide horizontal channel at a depth of 1 m. If $n = 0.024$, find analytically the distance x downstream to the section of depth 1.1 m.

I

$$x = (\kappa/n)^2\left[\frac{3}{4g}(y^{4/3} - y_1^{4/3}) - \frac{3}{13q^2}(y^{13/3} - y_1^{13/3})\right]$$

$$= \left(\frac{1}{0.024}\right)^2\left\{\left[\frac{3}{(4)(9.807)}\right](1.1^{4/3} - 1^{4/3}) - \left[\frac{3}{(13)(5)^2}\right](1.1^{13/3} - 1^{13/3})\right\} = 9.80\ \text{m}$$

14.338 In a wide rectangular earth channel obstructed with stones ($n = 0.032$), the depth of flow increases from 1 m to 1.1 m over a 10 m distance. Compute the volumetric flow per unit width.

I

$$x = \left(\frac{\kappa}{n}\right)^2\left[\frac{3}{4g}(y^{4/3} - y_1^{4/3}) - \frac{3}{13q^2}(y^{13/3} - y_1^{13/3})\right]$$

$$10 = \left(\frac{1}{0.032}\right)^2\left\{\left[\frac{3}{(4)(9.807)}\right](1.1^{4/3} - 1^{4/3}) - \left(\frac{3}{13q^2}\right)(1.1^{13/3} - 1^{13/3})\right\}$$

$$q = 30.96\ (\text{m}^3/\text{s})/\text{m}$$

14.339 In Prob. 14.196, for a flow of 4900 cfs, calculate the distance downstream over which the depth increases from 13 ft to 14 ft. Make a one-step calculation without averaging.

▮
$$\Delta x = \left([1-(Q^2b/gA^3)]/\{s_0-(n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\}\right)\Delta y$$

From Prob. 14.196, $A = 217.1\ \text{ft}^2$, $R_h = 5.802\ \text{ft}$, $n = 0.012$, $s_0 = 0.0016$, $b = 20\ \text{ft}$, and $\Delta y = 1\ \text{ft}$.

$$\Delta x = \left\{\frac{1-(4900)^2(20)/(32.2)(217.1)^3}{0.0016-(0.012/1.486)^2[4900^2/(5.802)^{4/3}(217.1)^2]}\right\}(1) = 288\ \text{ft}$$

14.340 Repeat Prob. 14.339, using a linear average over the interval.

▮
$$\Delta x = \left([1-(Q^2b/gA^3)]/\{s_0-(n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\}\right)\Delta y$$

At $y = 14$ ft, $A = (\frac{1}{2})(\pi)(10)^2 + (4)(10+10) = 237.1\ \text{ft}^2$ and $R_h = 237.1/[(\pi)(10)+4+4] = 6.015$ ft. Therefore,

$$A_{av} = (217.1+237.1)/2 = 227.1\ \text{ft}^2 \qquad (R_h)_{av} = (5.802+6.015)/2 = 5.908\ \text{ft}$$

$$\Delta x = \left\{\frac{1-(4900)^2(20)/(32.2)(227.1)^3}{0.0016-(0.012/1.486)^2[4900^2/(5.908)^{4/3}(227.1)^2]}\right\}(1) = 220\ \text{ft}$$

14.341 Water flows at a rate of 500 ft^3/s through a rectangular section 10.0 ft wide from a "steep" slope to a "mild" slope creating a hydraulic jump, in the manner illustrated in Fig. 14-90. The upstream depth of flow (d_1) is 3.1 ft. Find the (*a*) downstream depth, (*b*) energy (head) loss in the hydraulic jump, and (*c*) upstream and downstream velocities.

▮ (*a*) $d_2 = (d_1/2)(\sqrt{1+8q^2/gd_1^3}-1) = (3.1/2)\{\sqrt{1+(8)(500/10.0)^2/[(32.2)(3.1)^3]}-1\} = 5.7\ \text{ft}$

(*b*) $E_j = (d_2-d_1)^3/4d_1d_2 = (5.7-3.1)^3/[(4)(3.1)(5.7)] = 0.25$ ft of water

(*c*) $v_1 = Q/A_1 = 500/[(3.1)(10.0)] = 16.1\ \text{ft/s} \qquad v_2 = 500/[(5.7)(10.0)] = 8.8\ \text{ft/s}$

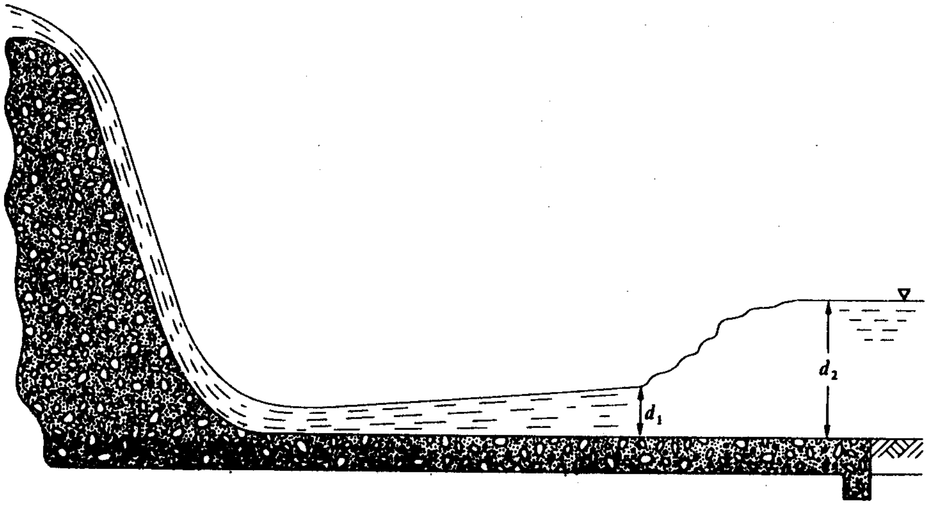


Fig. 14-90

14.342 Rework Prob. 14.341 for a flow rate of 20.0 m^3/s, a channel width of 4.0 m, and an upstream depth of flow of 1.20 m.

▮ (*a*) $d_2 = (d_1/2)(\sqrt{1+8q^2/gd_1^3}-1) = (1.20/2)\{\sqrt{1+(8)(20.0/4.0)^2/[(9.807)(1.20)^3]}-1\} = 1.55\ \text{m}$

(*b*) $E_j = (d_2-d_1)^3/4d_1d_2 = (1.55-1.20)^3/[(4)(1.20)(1.55)] = 0.006$ m of water

(*c*) $v_1 = Q/A_1 = 20.0/[(1.20)(4.0)] = 4.17\ \text{m/s} \qquad v_2 = 20.0/[(1.55)(4.0)] = 3.23\ \text{m/s}$

14.343 Water flows over a concrete spillway into a rectangular channel 9.0 m wide through a hydraulic jump, in the manner illustrated in Fig. 14-90. The depths before and after the jump are 1.55 m and 3.08 m, respectively. Find the rate of flow in the channel.

$$q = \{[(d_1 + d_2)/2](gd_1d_2)\}^{1/2} = \{[(1.55 + 3.08)/2][(9.807)(1.55)(3.08)]\}^{1/2} = 10.41\ (\text{m}^3/\text{s})/\text{m}$$
$$Q = (10.41)(9.0) = 93.7\ \text{m}^3/\text{s}$$

14.344 Rework Prob. 14.343 if the channel width is 20 ft and the depths of flow before and after the jump are 4.5 ft and 8.2 ft, respectively.

$$q = \{[(d_1 + d_2)/2](gd_1d_2)\}^{1/2} = \{[(4.5 + 8.2)/2][(32.2)(4.5)(8.2)]\}^{1/2} = 86.86\ (\text{ft}^3/\text{s})/\text{ft}$$
$$Q = (86.86)(20) = 1737\ \text{ft}^3/\text{s}$$

14.345 A hydraulic jump occurs downstream from a 15-m-wide sluice gate. The depth is 1.5 m, and the velocity is 20 m/s. Determine **(a)** the Froude number and the Froude number corresponding to the conjugate depth, **(b)** the depth and velocity after the jump, and **(c)** the power dissipated by the jump.

(a) $N_F = v/\sqrt{gy} \qquad (N_F)_1 = 20/\sqrt{(9.807)(1.5)} = 5.21$

$$(N_F)_2 = (2)(\sqrt{2})(N_F)_1/[\sqrt{1 + (8)(N_F)_1^2} - 1]^{3/2} = (2)(\sqrt{2})(5.21)/[\sqrt{1 + (8)(5.21)^2} - 1]^{3/2} = 0.288$$

(b) $(N_F)_2 = v_2/\sqrt{gy} \qquad v_2y_2 = v_1y_1 = (1.5)(20) = 30.00\ \text{m}^2/\text{s}$

$$v_2^2 = (N_F)_2^2gy_2 = (N_F)_2^2(g)(30.00/v_2) = (0.288)^2(9.807)(30.00/v_2)$$
$$v_2 = 2.90\ \text{m/s} \qquad y_2 = 30.00/2.90 = 10.34\ \text{m}$$

(c) $E_j = (y_2 - y_1)^3/4y_1y_2 = (10.34 - 1.5)^3/[(4)(1.5)(10.34)] = 11.13$ m of water

$$P = \gamma QE_j \qquad Q = Av = [(15)(1.5)](20) = 450\ \text{m}^3/\text{s} \qquad P = (9.79)(450)(11.13) = 49.0 \times 10^3\ \text{kW}$$

14.346 Water flows at a rate of 16 m³/s at half critical depth in a trapezoidal channel with $b = 4$ m and $m = 0.4$ (see Fig. 14-91), before a hydraulic jump occurs. Prepare and execute a computer program to find the height after the jump and the energy loss in kilowatts.

```
10 REM B:EX124                          JUMP IN A TRAPEZOIDAL CHANNEL
20 DEFINT I:     DEF FNC(DY)=Q^2*(B+2!*M*DY)-G*(DY*(B+M*DY))^3   '
30 DEF FNFM(DY)=.5*B*DY^2+M*DY^3/3!+Q^2/(G*DY*(B+M*DY))          '
40 READ G,Q,B,M,GAM:     DATA 9.806,16.,4.,.4,9806.
50 YMAX=16!:     YMIN=0!:     LPRINT:     LPRINT"G,Q,B,M,GAM=";G;Q;B;M;GAM
60 FOR I=1 TO 15:     YC=.5*(YMAX+YMIN)
70 IF FNC(YC)>0! THEN YMIN=YC ELSE YMAX=YC
80 PRINT YMAX;YMIN;YC
90 NEXT I:     LPRINT"Y1,YC=";.5*YC;YC
100 Y1=.5*YC:     YMIN=YC:     YMAX=3!*YC:     FM=FNFM(Y1)
110 FOR I=1 TO 15:     Y2=.5*(YMAX+YMIN)
120 IF FNFM(Y2)-FM>0! THEN YMAX=Y2 ELSE YMIN=Y2
130 PRINT"YMAX,YMIN,Y2=";YMAX;YMIN;Y2:     NEXT I
140 A1=Y1*(B+M*Y1):     A2=Y2*(B+M*Y2):     V1=Q/A1:     V2=Q/A2
150 LOSS=(V1^2-V2^2)/(2!*G)+Y1-Y2:     POWER=GAM*Q*LOSS/1000!
160 LPRINT"Y1,Y2,V1,V2,LOSS,POWER=";
170 LPRINT USING"###.### ";Y1;Y2;V1;V2;LOSS;POWER

G,Q,B,M,GAM= 9.806001  16  4  .4  9806
Y1,YC= .5661621  1.132324
Y1,Y2,V1,V2,LOSS,POWER=  0.566   1.973   6.687   1.693   0.726 113.970
```

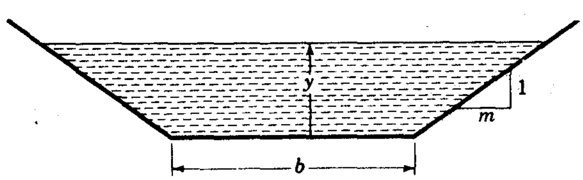


Fig. 14-91

14.347 A rectangular channel 10 ft wide carries 100 cfs in uniform flow at a depth of 1.67 ft. Suppose that an obstruction such as a submerged weir is placed across the channel, rising to a height of 6 in above the bottom. **(a)** Will this obstruction cause a hydraulic jump to form upstream? Why? **(b)** Find the water depth over the obstruction, and classify the surface profile, if possible, to be found upstream of the weir.

(a) $y_c = (q^2/g)^{1/3} = [(\frac{100}{10})^2/32.2]^{1/3} = 1.459$ ft. Since $[y_0 = 1.67\ \text{ft}] > [y_c = 1.459\ \text{ft}]$, the slope is mild, flow is subcritical, and no jump can occur.

(b) $E = y + (1/2g)(q^2/y^2) \qquad E_0 = 1.67 + \{1/[(2)(32.2)]\}[(\frac{100}{10})^2/1.67^2] = 2.227$ ft

$$E_{\min} = (\tfrac{3}{2})(y_c) = (\tfrac{3}{2})(1.459) = 2.188\ \text{ft} \qquad \Delta z_{\text{crit}} = E_0 - E_{\min} = 2.227 - 2.188 = 0.039\ \text{ft}$$

Since $[\Delta z = 0.5\ \text{ft}] > [\Delta z_{\text{crit}} = 0.039\ \text{ft}]$, y_c occurs over the obstruction. Hence, the water depth over the obstruction = 1.459 ft. Let y_1 = water depth just upstream of the obstruction. Assuming no energy loss over the obstruction, $E_1 = E_{min} + \Delta z$, or $y_1 + \{1/[(2)(32.2)]\}[(\frac{100}{10})^2/y_1^2] = 2.188 + 0.5 = 2.688$, $y_1 = 2.42$ ft or 0.943 ft (by trial and error). There is no mild profile that enables normal flow to become supercritical downstream, so $y_1 = 0.943$ ft is not possible here. Thus $y_1 = 2.42$ ft. Upstream of y_1, the water surface tends asymptotically to $y_0 = 1.67$ ft.

14.348 Suppose that the slope and roughness of the channel in Prob. 14.347 are such that uniform flow of 120 cfs occurs at 1.00 ft. Consider an obstruction rising 3 in above the bottom of the channel. Will a hydraulic jump form upstream? As in Prob. 14.347, classify the surface profile found just upstream from the obstruction.

I Since $y_0 = 1.000 \text{ ft} < 1.459 \text{ ft} = y_c$, the slope is steep and for $y = y_0$, the flow is supercritical.

$$E = y + (1/2g)(q^2/y^2) \qquad E_0 = 1.000 + \{1/[(2)(32.2)]\}[(\tfrac{120}{10})^2/1.00^2] = 3.236 \text{ ft}$$

$$E_{min} = 2.188 \text{ ft} \quad \text{(from Prob. 14.347)} \qquad \Delta z_{crit} = E_0 - E_{min} = 3.236 - 2.188 = 1.048 \text{ ft}$$

Since $\Delta z = 0.250 \text{ ft} < 1.048 \text{ ft} = \Delta z_{crit}$, the obstruction is not sufficiently high to produce critical flow. Hence, upstream the flow is straight supercritical uniform flow, and a hydraulic jump cannot form.

14.349 In a rectangular channel 4 m wide with a flow of 7.53 m^3/s the depth is 0.3 m. If a hydraulic jump occurs, find the depth downstream of it. Calculate the loss of specific energy through the jump.

I

$$y_2 = (y_1/2)(\sqrt{1 + 8q^2/gy_1^3} - 1) = (0.3/2)\{\sqrt{1 + (8)(7.53/4)^2/[(9.807)(0.3)^3]} - 1\} = 1.410 \text{ m}$$

$$E_j = (y_2 - y_1)^3/4y_1y_2 = (1.410 - 0.3)^3/[(4)(0.3)(1.410)] = 0.808 \text{ m}$$

14.350 In a horizontal rectangular channel 8 ft wide the depths before and after a hydraulic jump are 0.82 ft and 3.16 ft. Find the rate of flow and the specific energy loss.

I

$$q = \{[(d_1 + d_2)/2](gd_1d_2)\}^{1/2} = \{[(0.82 + 3.16)/2][(32.2)(0.82)(3.16)]\}^{1/2} = 18.18 \text{ (ft}^3\text{/s)/m}$$

$$Q = (18.18)(8) = 145 \text{ ft}^3\text{/s} \qquad E_j = (d_2 - d_1)^3/4d_1d_2 = (3.16 - 0.82)^3/[(4)(0.82)(3.16)] = 1.24 \text{ ft}$$

14.351 A tidal *bore* (progressive hydraulic jump) rises 13 ft above the normal low-tide river depth of 8 ft. The bore travels upstream at 16 mph (relative to the bank). Find the velocity of the undisturbed river. Is this subcritical or supercritical flow?

I

$$q = \{[(d_1 + d_2)/2](gd_1d_2)\}^{1/2} = \{[(13 + 8 + 8)/2][(32.2)(13 + 8)(8)]\}^{1/2} = 280.1 \text{ cfs/ft}$$

$$v_1 = q/d_1 = 280.1/8 = 35.01 \text{ ft/s} \quad \text{(relative to the bore)} \qquad v_{jump} = (16)(\tfrac{5280}{3600}) = 23.47 \text{ ft/s}$$

The bore is moving upstream at a velocity of 23.47 ft/s while the river is moving into the bore at 35.01 ft/s. Therefore, the river is moving downstream at 35.01 − 23.47 = 11.54 ft/s; $v^2/2g = 11.54^2/[(2)(32.2)] = 2.07$ ft. Since $v^2/2g = 2.07 \text{ ft} < 4 \text{ ft} = y/2$, the flow is subcritical.

14.352 A hydraulic jump occurs in a triangular flume having side slopes 1:1. The flow rate is 17.72 cfs and the depth before the jump is 1.0 ft. Find the specific-energy loss in the jump.

I $Q^2/gA_1 + (h_c)_1A_1 = Q^2/gA_2 + (h_c)_2A_2$. For the triangular section, $h_c = y/3$, $A = y^2$, and $y_1 = 1.0$ ft.

$$17.72^2/[(32.2)(1.0)^2] + (1.0/3)(1.0)^2 = 17.72^2/[(32.2)(y_2)^2] + (y_2/3)(y_2)^2$$

$$y_2 = 3.00 \text{ ft} \quad \text{(by trial and error)}$$

$$v_1 = Q/A_1 = 17.72/1.0^2 = 17.72 \text{ ft/s} \qquad v_2 = 17.72/3.00^2 = 1.97 \text{ ft}$$

$$E_j = (y_1 + v_1^2/2g) - (y_2 + v_2^2/2g) = 1.0 + 17.72^2/[(2)(32.2)] - 3.00 - 1.97^2/[(2)(32.2)] = 2.815 \text{ ft}$$

14.353 A hydraulic jump occurs in a 4-m-wide rectangular channel carrying 5 m^3/s on a slope of 0.004. The depth after the jump is 1.2 m. Find (*a*) the depth before the jump, (*b*) the losses of energy and power in the jump.

I (*a*)

$$y_1 = (y_2/2)(\sqrt{1 + 8q^2/gy_2^3} - 1) = (1.2/2)\{\sqrt{1 + (8)(\tfrac{5}{4})^2/[(9.807)(1.2)^3]} - 1\} = 0.191 \text{ m}$$

(*b*)

$$E_j = (y_2 - y_1)^3/4y_1y_2 = (1.2 - 0.191)^3/[(4)(0.191)(1.2)] = 1.120 \text{ m}$$

$$P = Q\gamma E_j = (5)(9.79)(1.120) = 54.8 \text{ kW}$$

14.354 Analyze the water-surface profile in a long rectangular channel with concrete lining ($n = 0.013$). The channel is 10 ft wide, the flow rate is 400 cfs, and there is an abrupt change in channel slope from 0.0150 to 0.0016. Find also the horsepower loss in the jump.

▮ $$Q = (1.486/n)(A)(R^{2/3})(s^{1/2}) \qquad 400 = (1.486/0.013)(10y_{0_1})[10y_{0_1}/(10 + 2y_{0_1})]^{2/3}(0.015)^{1/2}$$

By trial, $y_{0_1} = 2.17$ ft (normal depth on upper slope). Using a similar procedure, the normal depth y_{0_2} on the lower slope is found to be 4.80 ft. $y_c = (q^2/g)^{1/3} = [(\frac{400}{10})^2/32.2]^{1/3} = 3.68$ ft. Thus flow is supercritical ($y_{0_1} < y_c$) before break in slope and subcritical ($y_{0_2} > y_c$) after break, so a hydraulic jump must occur. $y_2 = (y_1/2)[-1 + \sqrt{1 + (8q^2/gy_1^3)}]$, $y_2' = (2.17/2)(-1 + \{1 + [8(40)^2/32.2(2.17)^3]\}^{1/2}) = 5.75$ ft. Therefore the depth conjugate to the upper-slope normal depth of 2.17 ft is 5.75 ft. This jump cannot occur because the normal depth y_{0_2} on the lower slope is less than 5.75 ft. $y_1' = (4.80/2)(-1 + \{1 + [8(40)^2/32.2(4.8)^3]\}^{1/2}) = 2.76$ ft. The lower conjugate depth of 2.76 ft will occur downstream of the break in slope. The location of the jump (i.e., its distance below the break in slope) may be found by $\Delta x = (E_1 - E_2)/(S - S_0)$, $E_1 = 2.17 + [(400/21.7)^2/64.4] = 7.45$ ft, $E_2 = 2.76 + [(400/27.6)^2/64.4] = 6.02$ ft, $V_m = (\frac{1}{2})[(400/21.7) + (400/27.6)] = 16.46$ fps, $R_m = (\frac{1}{2})[(21.7/14.34) + (27.6/15.52)] = 1.645$ ft, $S = (nv/1.486R^{2/3})^2 = [(0.013)(16.45)/1.49(1.645)^{2/3}]^2 = 0.01060$, $\Delta x = (7.45 - 6.02)/(0.0106 - 0.0016) = 160$ ft. Thus depth on the upper slope is 2.17 ft; downstream of the break the depth increases gradually (M_3 curve) to 2.76 ft over a distance of approximately 160 ft; then a hydraulic jump occurs from a depth of 2.76 ft to 4.80 ft; downstream of the jump the depth remains constant (i.e., normal) at 4.80 ft. HP loss $= \gamma Q h_{L_j}/550$ where $h_{L_j} = \Delta E$.

Before jump: $E_1' = 2.76 + [(400/27.6)^2/64.4] = 6.02$ ft. After jump: $E_{0_2} = 4.80 + [(400/48.0)^2/64.4] = 5.88$ ft. Hence HP loss $= 62.4(400)(6.02 - 5.88)/550 = 6.35$.

14.355 A very wide rectangular channel with bed slope $S_0 = 0.0003$ and roughness $n = 0.020$ carries a steady flow of 50 cfs/ft of width. If a sluice gate is so adjusted as to produce a minimum depth of 1.5 ft in the channel, determine whether a hydraulic jump will occur downstream, and if so, find (using one reach) the distance from the gate to the jump.

▮ $$q = (y)(1.486/n)(y^{2/3})(s^{1/2}) \qquad 50 = (y_0)(1.486/0.020)(y_0)^{2/3}(0.0003)^{1/2} \qquad y_0 = 8.99 \text{ ft}$$
$$y_c = (q^2/g)^{1/3} = (50^2/32.2)^{1/3} = 4.27 \text{ ft}$$

Since $y_c < y_0$, the slope is mild. Since $y = 1.5$ ft caused by the gate $< y_c < y_0$, downstream of the sluice gate is an M_3 profile, which must be followed by a hydraulic jump to enable the flow to return to normal depth.

$$y_1 = (y_2/2)(\sqrt{1 + 8q^2/gy_2^3} - 1) = (8.99/2)\{\sqrt{1 + (8)(50)^2/[(32.2)(8.99)^3]} - 1\} = 1.63 \text{ ft}$$
$$\Delta x = (E_1 - E_2)/(S - S_0) \qquad E = y + v^2/2g$$

For $y = 1.5$ ft, $v = q/y = 50/1.5 = 33.33$ ft/s, $E = 1.5 + 33.33^2/[(2)(32.2)] = 18.75$ ft. For $y = 1.63$ ft, $v = q/y = 50/1.63 = 30.67$ ft/s, $E = 1.63 + 30.67^2/[(2)(32.2)] = 16.24$ ft.

$$v_m = (33.33 + 30.67)/2 = 32.00 \text{ ft/s} \qquad R_m = (1.5 + 1.63)/2 = 1.565 \text{ ft}$$
$$S = (nv/1.486R^{2/3})^2 = \{(0.020)(32.00)/[(1.486)(1.565)^{2/3}]\}^2 = 0.1021$$
$$\Delta x = (18.75 - 16.24)/(0.1021 - 0.0003) = 24.7 \text{ ft}$$

14.356 A rectangular channel 8 ft wide carries 280 cfs in uniform flow at a depth of 5 ft. Around a 40-ft-radius bend, how much higher should the outside wall be than the inside wall?

▮ $v = Q/A = 280/[(5)(8)] = 7.000$ ft/s, $N_F = v/\sqrt{gy} = 7.000/\sqrt{(32.2)(5)} = 0.552$. Since $N_F < 1.0$, the flow is subcritical. $\Delta y = v^2B/gr = (7.000)^2(10)/[(32.2)(40)] = 0.380$ ft.

14.357 Repeat Prob. 14.356 if normal depth is 2 ft.

▮ $v = Q/A = 280/[(2)(8)] = 17.5$ ft/s, $N_F = v/\sqrt{gy} = 17.5/\sqrt{(32.2)(2)} = 2.18$. Since $N_F > 1.0$, the flow is supercritical. $\Delta y = v^2B/gr = (17.5)^2(8)/[(32.2)(40)] = 1.90$ ft. Because of (supercritical) wave action, maximum water depth at inside wall $= y_0$, and maximum water depth at outside wall $= (y_0 + \Delta y)$. Therefore, the required difference in wall elevations $= 1.90$ ft.

14.358 Rework Prob. 14.356 for these data: $b = 4$ m, $Q = 8\ \text{m}^3/\text{s}$, $y = 2.0$ m, $r = 30$ m.

▮ $v = Q/A = 8/[(4)(2.0)] = 1.000$ m/s, $N_F = v/\sqrt{gy} = 1.000/\sqrt{(9.807)(2.0)} = 0.226$. Since $N_F < 1.0$, the flow is subcritical. $\Delta y = v^2B/gr = (1.000)^2(4)/[(9.807)(30)] = 0.0136$ m.

14.359 Find the maximum volumetric flow in a 3-ft by 3-ft concrete box culvert ($n = 0.013$) with a rounded entrance ($k_e = 0.05$, $C_d = 0.95$) if the culvert slope is 0.0065, the length 100 ft, and the headwater level 5 ft above the culvert invert? Assume free outlet conditions. Neglect headwater and tailwater velocity heads.

▌ Headwater/$d = \frac{5}{3} = 1.7$. Since $1.7 > 1.2$, conditions are those of Fig. 14-92*b* or 14-92*c*. Assume case *b*.

$$R = A/p_w = (3)(3)/(3+3+3+3) = 0.750 \text{ ft}$$
$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0 L$$
$$\Delta h = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g) \qquad y_1 - y_3 + s_0 L = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g)$$
$$5 - 3 + (0.0065)(100) = [0.05 + (29)(0.013)^2(100)/0.75^{4/3} + 1]\{v^2/[(2)(32.2)]\} \qquad v = 9.82 \text{ ft/s}$$
$$Q = Av = [(3)(3)](9.82) = 88.38 \text{ ft}^3/\text{s} = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

Now find the depth y_0 which occurs with normal uniform flow at this flow rate: $88.38 = (3y_0)(1.486/0.013)[3y_0/(y_0 + 3 + y_0)]^{2/3}(0.0065)^{1/2}$, $y_0 = 3.16$ ft (by trial and error). Since $y_0 > d$, the culvert flows full. Free discharge at the outlet is given; therefore, the preceding assumption and computations are valid, and $Q = 88.38 \text{ ft}^3/\text{s}$.

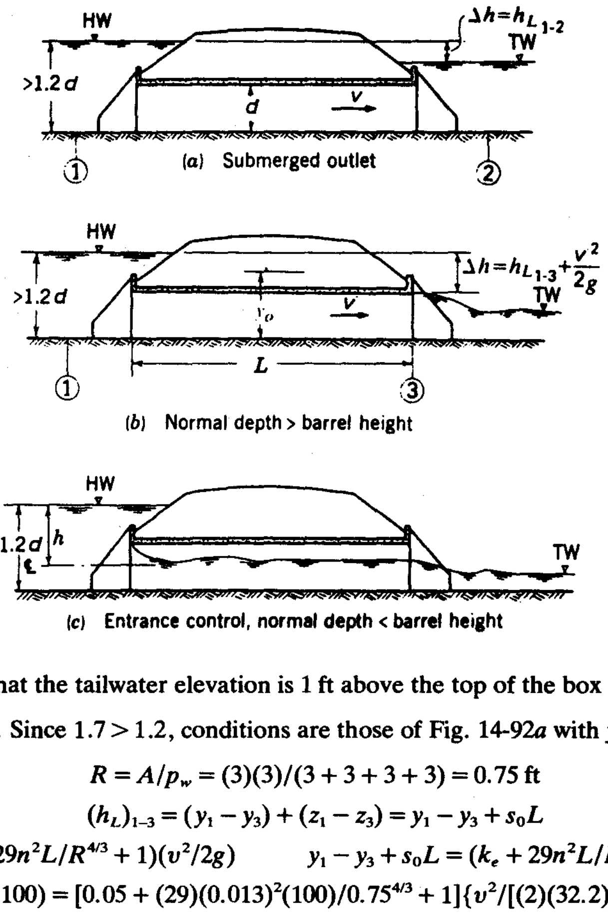


Fig. 14-92

14.360 Solve Prob. 14.359 given that the tailwater elevation is 1 ft above the top of the box at the outlet.

▌ Headwater/$d = \frac{5}{3} = 1.7$. Since $1.7 > 1.2$, conditions are those of Fig. 14-92*a* with $y_2 = 4$ ft.

$$R = A/p_w = (3)(3)/(3+3+3+3) = 0.75 \text{ ft}$$
$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0 L$$
$$\Delta h = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g) \qquad y_1 - y_3 + s_0 L = (k_e + 29n^2L/R^{4/3} + 1)(v^2/g)$$
$$5 - 4 + (0.0065)(100) = [0.05 + (29)(0.013)^2(100)/0.75^{4/3} + 1]\{v^2/[(2)(32.2)]\} \qquad v = 7.75 \text{ ft/s}$$
$$Q = Av = [(3)(3)](7.75) = 69.8 \text{ ft}^3/\text{s}$$

14.361 Solve Prob. 14.359 given that the tailwater elevation is 2 ft above the top of the box at the outlet.

▌ Headwater/$d = \frac{6}{4} = 1.5$. Since $1.5 > 1.2$, conditions are those of Fig. 14-92*a* with $y_2 = 6$ ft.

$$R = A/p_w = (4)(4)/(4+4+4+4) = 1.000 \text{ ft}$$
$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0 L$$
$$\Delta h = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g) \qquad y_1 - y_3 + s_0 L = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g)$$
$$5 - 5 + (0.0065)(100) = [0.05 + (29)(0.013)^2(100)/0.75^{4/3} + 1]\{v^2/[(2)(32.2)]\} \qquad v = 4.864 \text{ ft/s}$$
$$Q = Av = [(3)(3)](4.864) = 43.8 \text{ ft}^3/\text{s}$$

14.362 Repeat Prob. 14.359 if the culvert slope is 0.043.

▌ Headwater/$d = \frac{5}{3} = 1.7$. Since $1.7 > 1.2$, conditions are those of Fig. 14-92*b* or 14-92*c*. Assume case *b*.

$$R = A/p_w = (3)(3)/(3+3+3+3) = 0.75 \text{ ft}$$

$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0 L$$

$$\Delta h = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g) \qquad y_1 - y_3 + s_0L = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g)$$

$$5 - 3 + (0.043)(100) = [0.05 + (29)(0.013)^2(100)/0.75^{4/3} + 1]\{v^2/[(2)(32.2)]\} \qquad v = 15.14 \text{ ft/s}$$

$$Q = Av = [(3)(3)](15.14) = 136 \text{ ft}^3/\text{s} = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

Now find the depth y_0 which occurs with normal uniform flow at this flow rate: $136 = (3y_0)(1.486/0.013)[3y_0/(y_0+3+y_0)]^{2/3}(0.043)^{1/2}$, $y_0 = 2.1$ ft (by trial and error). Since $y_0 < d$, the situation is actually case *c*; therefore, $Q = C_dA\sqrt{2gh} = (0.95)[(3)(3)]\sqrt{(2)(32.2)(3)} = 119 \text{ ft}^3/\text{s}$.

14.363 Repeat Prob. 14.359 if the culvert slope is 0.08.

▌ Headwater/$d = \frac{5}{3} = 1.7$. Since $1.7 > 1.2$, conditions are those of Fig. 14-92*b* or 14-92*c*. Assume case *b*.

$$R = A/p_w = (3)(3)/(3+3+3+3) = 0.75 \text{ ft}$$

$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0 L$$

$$\Delta h = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g) \qquad y_1 - y_3 + s_0L = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g)$$

$$5 - 3 + (0.08)(100) = [0.05 + (29)(0.013)^2(100)/0.75^{4/3} + 1]\{v^2/[(2)(32.2)]\} \qquad v = 19.08 \text{ ft/s}$$

$$Q = Av = [(3)(3)](19.08) = 172 \text{ ft}^3/\text{s} = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

Now find the depth y_0 which occurs with normal uniform flow at this flow rate: $172 = (3y_0)(1.486/0.013)[3y_0/(y_0+3+y_0)]^{2/3}(0.08)^{1/2}$, $y_0 = 1.98$ ft (by trial and error). Since $y_0 < d$, our assumption was wrong and we have, as in Prob. 14.362, $Q = C_dA\sqrt{2gh} = (0.95)[(3)(3)]\sqrt{(2)(32.2)(3)} = 119 \text{ ft}^3/\text{s}$.

14.364 A culvert under a road must carry 4.3 m³/s. The culvert length will be 30 m and the slope will be 0.003. If the maximum permissible headwater level is 3.6 m above the culvert invert, what size corrugated-pipe culvert ($n = 0.025$) would you select? The outlet will discharge freely. Neglect velocity of approach. Assume square-edged entrance with $k_e = 0.5$ and $C_d = 0.65$.

▌ Assume $d < 3.0$ m, so that [headwater/d] > [3.6/3.0 = 1.2] and the conditions are those of Fig. 14-92*b* or 14-92*c*. Assume case *b*.

$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0L \qquad \Delta h = (k_e + 19.62n^2L/R^{4/3} + 1)(v^2/2g)$$

$$y_1 - y_3 + s_0L = (k_e + 19.62n^2L/R^{4/3} + 1)(v^2/2g) \qquad v = Q/A = 4.3/(\pi d^2/4) = 5.475/d^2$$

$$3.6 - d + (0.003)(30) = [0.5 + (19.62)(0.025)^2(30)/(d/4)^{4/3} + 1]\{(5.475/d^2)^2/[(2)(9.807)]\}$$

$$d = 1.20 \text{ m} \quad \text{(by trial and error)} \qquad Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$

Now find the diameter d_0 which just flows full with normal uniform flow: $4.3 = (\pi d_0^2/4)(1.0/0.025)(d_0/4)^{2/3}(0.003)^{1/2}$, $d_0 = 1.99$ m. Since $d_0 > d$, the culvert flows full, and free discharge at the outlet is given. Therefore, the above assumptions and analysis are valid, and $d = 1.20$ m.

14.365 Repeat Prob. 14.364 for a culvert length of 100 m.

▌ Assume $d < 3.0$ m, so that [headwater/d] > [3.6/3.0 = 1.2] and the conditions are those of Fig. 14-92*b* or 14-92*c*. Assume case *b*.

$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0L \qquad \Delta h = (k_e + 19.62n^2L/R^{4/3} + 1)(v^2/2g)$$

$$y_1 - y_3 + s_0L = (k_e + 19.62n^2L/R^{4/3} + 1)(v^2/2g) \qquad v = Q/A = 4.3/(\pi d^2/4) = 5.475/d^2$$

$$3.6 - d + (0.003)(100) = [0.5 + (19.62)(0.025)^2(100)/(d/4)^{4/3} + 1]\{(5.475/d^2)^2/[(2)(9.807)]\}$$

$$d = 1.41 \text{ m} \quad \text{(by trial and error)} \qquad Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$

Now find the diameter d_0 which just flows full with normal uniform flow: $4.3 = (\pi d_0^2/4)(1.0/0.025)(d_0/4)^{2/3}(0.003)^{1/2}$, $d_0 = 1.99$ m. Since $d_0 > d$, the culvert flows full, and free discharge at the outlet is given. Therefore, the above assumptions and analysis are valid, and $d = 1.41$ m. Use standard diameter 1.50 m.

14.366 Water flows in a wide channel at $q = 18\ (\mathrm{ft^3/s})/\mathrm{ft}$, $y_1 = 1$ ft, and then undergoes a hydraulic jump. Compute y_2, v_2, $(N_F)_2$, E_j, and the power dissipated per unit width.

$$N_F = v/\sqrt{gy} \qquad v_1 = q/y_1 = \tfrac{18}{1} = 18.00\ \mathrm{ft/s}$$
$$(N_F)_1 = 18.00/\sqrt{(32.2)(1)} = 3.17 \qquad E = y + v^2/2g$$
$$E_1 = 1 + 18.00^2/[(2)(32.2)] = 5.03\ \mathrm{ft}$$
$$y_2 = (y_1/2)(\sqrt{1 + 8q^2/gy_1^3} - 1) = (\tfrac{1}{2})\{\sqrt{1 + (8)(18)^2/[(32.2)(1)^3]} - 1\} = 4.01\ \mathrm{ft}$$
$$v_2 = q/y_2 = 18/4.01 = 4.49\ \mathrm{ft/s} \qquad (N_F)_2 = 4.49/\sqrt{(32.2)(4.01)} = 0.395$$
$$E_j = (y_2 - y_1)^3/4y_1y_2 = (4.01 - 1)^3/[(4)(4.01)(1)] = 1.70\ \mathrm{ft}$$
$$p_{\mathrm{dissipated}} = q\gamma E_j = (18)(62.4)(1.7) = 1909\ (\mathrm{ft\text{-}lb/s})/\mathrm{ft} = 3.47\ \mathrm{hp/ft}$$

14.367 A wide-channel flow at depth 60 cm passes through a hydraulic jump and emerges at a depth of 3.0 m. Compute the velocities on either side of the jump and the critical depth of flow.

$$2y_2/y_1 = -1 + [1 + 8(N_F)_1^2]^{1/2} \qquad (2)(3.0)/(0.60) = -1 + [1 + (8)(N_F)_1^2]^{1/2} \qquad (N_F)_1 = 3.87$$
$$N_F = v/\sqrt{gy} \qquad 3.87 = v_1/\sqrt{(9.807)(0.60)} \qquad v_1 = 9.39\ \mathrm{m/s}$$
$$v_1y_1 = v_2y_2 \qquad (9.39)(0.60) = (v_2)(3.0) \qquad v_2 = 1.88\ \mathrm{m/s}$$
$$q = yv = (3.0)(1.88) = 5.64\ (\mathrm{m^3/s})/\mathrm{m} \qquad y_c = (q^2/g)^{1/3} = (5.64^2/9.807)^{1/3} = 1.48\ \mathrm{m}$$

14.368 The flow downstream of a hydraulic jump in a rectangular channel is 8 m deep and has a velocity of 3.6 m/s. Obtain the velocity and depth upstream of the jump.

$$2y_1/y_2 = -1 + [1 + 8(N_F)_2^2]^{1/2} \qquad N_F = v/\sqrt{gy} \qquad (N_F)_2 = 3.6/\sqrt{(9.807)(8)} = 0.406$$
$$2y_1/8 = -1 + [1 + (8)(0.406)^2]^{1/2} \qquad y_1 = 2.09\ \mathrm{m} \qquad y_1v_1 = y_2v_2$$
$$2.09v_1 = (8)(3.6) \qquad v_1 = 13.78\ \mathrm{m/s}$$

14.369 Water in a horizontal channel accelerates smoothly over a bump and then undergoes a hydraulic jump, as in Fig. 14-93. If $y_1 = 1$ m and $y_3 = 30$ cm, estimate v_1, v_3, and y_4. Neglect friction.

$$E_1 = E_3 \qquad y_1 + v_1^2/2g = y_3 + v_3^2/2g \qquad 1 + v_1^2/[(2)(9.807)] = 0.30 + v_3^2/[(2)(9.807)]$$
$$y_1v_1 = y_3v_3 \qquad v_1 = y_3v_3/y_1 = (0.30)(v_3)/1 = 0.3000v_3$$
$$1 + (0.3000v_3)^2/[(2)(9.807)] = 0.30 + v_3^2/[(2)(9.807)]$$
$$v_3 = 3.88\ \mathrm{m/s} \qquad v_1 = (0.3000)(3.88) = 1.16\ \mathrm{m/s}$$
$$2y_4/y_3 = -1 + [1 + 8(N_F)_3^2]^{1/2} \qquad N_F = v/\sqrt{gy}$$
$$(N_F)_3 = 3.88/\sqrt{(9.807)(0.30)} = 2.26 \qquad 2y_4/(0.30) = -1 + [1 + (8)(2.26)^2]^{1/2} \qquad y_4 = 82\ \mathrm{cm}$$

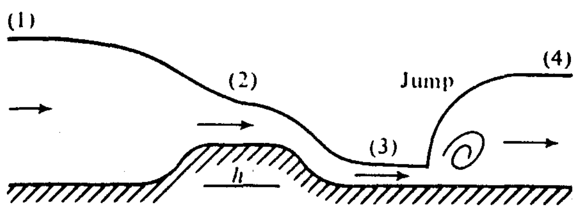


Fig. 14-93

14.370 Evaluate the bump height h in Prob. 14.369.

$$y_1 + v_1^2/2g = y_2 + v_2^2/2g + h$$
$$1 + 1.16^2/[(2)(9.807)] = y_2 + v_2^2/[(2)(9.807)] + h$$
$$y_1v_1 = y_2v_2 \qquad (1)(1.16) = y_2v_2 \qquad v_2 = 1.16/y_2$$

Since flow over the bump is critical,

$$v_2 = v_c = \sqrt{gy_2} \qquad 1.16/y_2 = \sqrt{9.807y_2} \qquad y_2 = 0.516\text{ m} \qquad v_2 = 1.16/0.516 = 2.248\text{ m/s}$$
$$1 + 1.16^2/[(2)(9.807)] = 0.516 + 2.248^2/[(2)(9.807)] + h \qquad h = 29.5\text{ m}$$

14.371 Repeat Prob. 14.369 if $y_1 = 3$ ft and $y_3 = 1$ ft.

$$E_1 = E_3 \qquad y_1 + v_1^2/2g = y_3 + v_3^2/2g \qquad 3 + v_1^2/[(2)(32.2)] = 1 + v_3^2/[(2)(32.2)]$$
$$y_1v_1 = y_3v_3 \qquad v_1 = y_3v_3/y_1 = (1)(v_3)/3 = 0.3333v_3$$
$$3 + (0.3333v_3)^2/[(2)(32.2)] = 1 + v_3^2/[(2)(32.2)]$$
$$v_3 = 12.04\text{ ft/s} \qquad v_1 = (0.3333)(12.04) = 4.01\text{ ft/s} \qquad 2y_4/y_3 = -1 + [1 + 8(N_F)_3^2]^{1/2}$$
$$N_F = v/\sqrt{gy} \qquad (N_F)_3 = 12.04/\sqrt{(32.2)(1)} = 2.12$$
$$2y_4/(1) = -1 + [1 + (8)(2.12)^2]^{1/2} \qquad y_4 = 2.54\text{ ft}$$

14.372 Evaluate the bump height h in Prob. 14.371.

$$y_1 + v_1^2/2g = y_2 + v_2^2/2g + h$$
$$3 + 4.01^2/[(2)(32.2)] = y_2 + v_2^2/[(2)(32.2)] + h$$
$$y_1v_1 = y_2v_2 \qquad (3)(4.01) = y_2v_2 \qquad v_2 = 12.03/y_2$$

Since flow over the bump is critical,

$$v_2 = v_c = \sqrt{gy_2} \qquad 12.03/y_2 = \sqrt{32.2y_2} \qquad y_2 = 1.65\text{ ft} \qquad v_2 = 12.03/1.65 = 7.29\text{ ft/s}$$
$$3 + 4.01^2/[(2)(32.2)] = 1.65 + 7.29^2/[(2)(32.2)] + h \qquad h = 0.775\text{ ft} = 9.3\text{ in.}$$

14.373 Find the power loss in a 100-ft-wide hydraulic jump from depth 3 ft to depth 12 ft.

$$E_j = (y_2 - y_1)^3/4y_1y_2 = (12-3)^3/[(4)(12)(3)] = 5.063\text{ ft} \qquad P = Q\gamma E_j \qquad N_F = v/\sqrt{gy}$$
$$2y_2/y_1 = -1 + [1 + 8(N_F)_1^2]^{1/2} \qquad (2)(12)/3 = -1 + [1 + (8)(N_F)_1^2]^{1/2} \qquad (N_F)_1 = 3.16$$
$$3.16 = v_1/\sqrt{(32.2)(3)} \qquad v_1 = 31.06\text{ ft/s} \qquad Q = A_1v_1 = [(100)(3)](31.06) = 9318\text{ ft}^3\text{/s}$$
$$P = (9318)(62.4)(5.063) = 2.944 \times 10^6\text{ ft-lb/s} = 5353\text{ hp}$$

14.374 At a certain section of a rectangular channel 10 ft wide, the depth of flow is 2 ft and the flow rate is 400 ft³/s. If a hydraulic jump occurs, will it be upstream or downstream of this section?

$$v_c = \sqrt{gy} = \sqrt{(32.2)(2)} = 8.02\text{ ft/s} \qquad v = Q/A = 400/[(10)(2)] = 20.00\text{ ft/s}$$

Since $v > v_c$, the flow is supercritical, and a hydraulic jump must occur downstream.

14.375 A still canal is 2 m deep and the water behind a bore (Prob. 14.351) is 4 m deep. Obtain the propagation speed of the bore.

$$N_F = v/\sqrt{gy} \qquad 2y_2/y_1 = -1 + [1 + 8(N_F)_1^2]^{1/2} \qquad (2)(4)/2 = -1 + [1 + (8)(N_F)_1^2]^{1/2}$$
$$(N_F)_1 = 1.73 \qquad 1.73 = v_1/\sqrt{(9.807)(2)} \qquad v_1 = c_{\text{bore}} = 7.66\text{ m/s}$$

14.376 In Prob. 14.375 suppose that the water upstream flows toward the bore at 4 m/s ground speed. What is the ground speed of the bore?

$$v_{\text{bore/ground}} = v_{\text{bore/water}} + v_{\text{water/ground}} = -7.66 + 4 = -3.66\text{ m/s.}$$

14.377 Consider the flow under the sluice gate of Fig. 14-94. If $y_1 = 9$ ft and all losses are neglected except the dissipation in the jump, calculate y_2 and y_3 and the percentage dissipation. The channel is horizontal and wide.

$$y_1 + V_1^2/2g = y_2 + V_2^2/2g \qquad 10 + 2^2/[(2)(32.2)] = y_2 + V_2^2/[(2)(32.2)]$$
$$V_1y_1 = V_2y_2 \qquad (2.2)(9) = V_2y_2 \qquad y_2 = 19.8/V_2$$

$$9 + 2.2^2/[(2)(32.2)] = 19.8/V_2 + V_2^2/[(2)(32.2)] \qquad V_2 = 23.0\text{ ft/s} \qquad \text{(by trial and error)}$$

$$y_2 = 19.8/23.0 = 0.861\text{ ft} \qquad 2y_3/y_2 = -1 + [1 + 8(N_F)_2^2]^{1/2} \qquad N_F = V/\sqrt{gy}$$

$$(N_F)_2 = 23.0/\sqrt{(32.2)(0.861)} = 4.37 \qquad 2y_3/0.861 = -1 + [1 + (8)(4.37)^2]^{1/2} \qquad y_3 = 4.91\text{ ft}$$

$$E_j = (y_3 - y_2)^3/4y_3y_2 = (4.91 - 0.861)^3/[(4)(4.91)(0.861)] = 3.93\text{ ft}$$

$$E_2 = y_2 + V_2^2/2g = 0.861 + 23^2/[(2)(32.2)] = 9.08\text{ ft}$$

$$\text{Percentage dissipation} = (3.93/9.08)(100\%) = 43.3\%$$

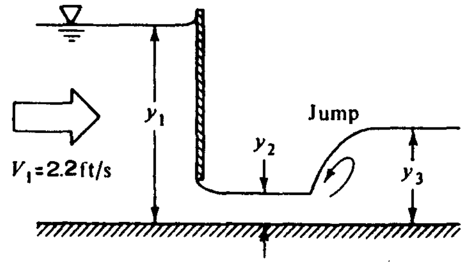


Fig. 14-94

14.378 If bottom friction is included in the sluice-gate flow of Fig. 14-94, the depths y_1, y_2, and y_3 will vary with x. What type of solution curves will we have in regions 1, 2, and 3?

▌ See Fig. 14-95.

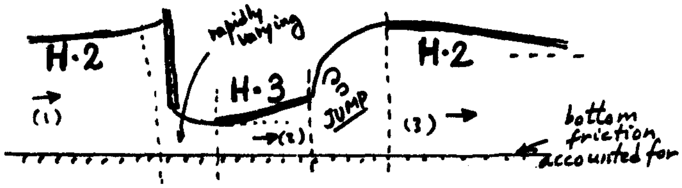


Fig. 14-95

14.379 A 10-cm-high bump in a wide horizontal water channel creates a hydraulic jump just upstream and the flow pattern in Fig. 14-96. Neglecting losses except in the jump, for the case $y_3 = 30$ cm, estimate V_4, y_4, V_1, and y_1.

▌ Since $y_4 < y_3$, assume flow over the hump is critical.

$$V = \sqrt{gy} \qquad V_3 = \sqrt{(9.807)(0.30)} = 1.715\text{ m}$$

$$E_2 = E_3 = E_4 = y_3 + V_3^2/2g + h = E_4 = 0.30 + 1.715^2/[(2)(9.807)] + 0.10 = E_4$$

$$E_2 = E_3 = E_4 = 0.5500\text{ m} \qquad E_2 = y_2 + V_2^2/2g = 0.5500\text{ m}$$

$$q = V_2y_2 = V_3y_3 = V_4y_4 = (1.715)(0.30) = 0.5145\text{ m}^2/\text{s} \qquad V_2 = 0.5145/y_2$$

$$y_2 + (0.5145/y_2)^2/[(2)(9.807)] = 0.5500$$

$$y_2 = 0.495\text{ m} \qquad \text{(by trial and error)} \qquad V_2 = 0.5145/0.495 = 1.039\text{ m/s}$$

$$N_F = V/\sqrt{gy} \qquad (N_F)_2 = 1.039/\sqrt{(9.807)(0.495)} = 0.472 \qquad \text{(subcritical)}$$

$$2y_1/y_2 = -1 + [1 + 8(N_F)_2^2]^{1/2} \qquad 2y_1/0.495 = -1 + [1 + (8)(0.472)^2]^{1/2} \qquad y_1 = 0.165\text{ m}$$

$$V_1 = q/y_1 = 0.5145/0.165 = 3.12\text{ m/s} \qquad E_3 = E_4 = 0.5500\text{ m} \qquad y_4 + V_4^2/2g = 0.5500\text{ m}$$

$$V_4y_4 = 0.5145\text{ m}^2/\text{s} \qquad V_4 = 0.5145/y_4 \qquad y_4 + (0.5145/y_4)^2/[(2)(9.807)] = 0.5500\text{ m}$$

$$y_4 = 0.195\text{ m} \qquad \text{(by trial and error)} \qquad V_4 = 0.5145/0.195 = 2.64\text{ m/s}$$

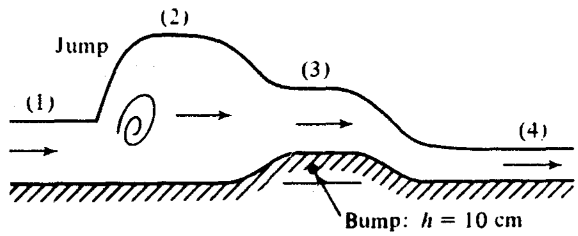


Fig. 14-96

14.380 Water flows in a wide channel at $q = 10\text{ (m}^3\text{/s)/m}$ and $y_1 = 1.25$ m. If the flow undergoes a hydraulic jump, compute (*a*) y_2, (*b*) V_2, (*c*) $(N_F)_2$, (*d*) E_j, (*e*) the percentage dissipation, (*f*) the power dissipated per unit width, and (*g*) the temperature rise due to dissipation if $c_p = 4200\,J/(\text{kg-K})$.

▌ (*a*) $V_1 = q/y_1 = 10/1.25 = 8.00 \text{ m/s}$ $\quad N_F = V/\sqrt{gy}$ $\quad (N_F)_1 = 8.00/\sqrt{(9.807)(1.25)} = 2.285$

$$2y_2/y_1 = -1 + [1 + 8(N_F)_1^2]^{1/2} \qquad 2y_2/1.25 = -1 + [1 + (8)(2.285)^2]^{1/2} \qquad y_2 = 3.46 \text{ m}$$

(*b*) $V_1 y_1 = V_2 y_2 \qquad (8.00)(1.25) = (V_2)(3.46) \qquad V_2 = 2.89 \text{ m/s}$

(*c*) $(N_F)_2 = 2.89/\sqrt{(9.807)(3.46)} = 0.496$

(*d*) $E_j = (y_2 - y_1)^3/4y_2y_1 = (3.46 - 1.25)^3/[(4)(3.46)(1.25)] = 0.624 \text{ m}$

(*e*) $E_1 = y_1 + v_1^2/2g = 1.25 + 8.00^2/[(2)(9.807)] = 4.51 \text{ m}$

Percentage dissipation $= E_j/E_1 = 0.624/4.51 = 0.138$ or 13.8 percent

(*f*) $P = q\gamma E_j \qquad P_{\text{dissipated}} = (10)(9.79)(0.624) = 61.1 \text{ kW/m}$

(*g*) $P = \dot{m}c_p\,\Delta T = \rho q c_p\,\Delta T \qquad (61.1)(1000) = (1000)(10)(4200)(\Delta T) \qquad \Delta T = 0.0015 \text{ K}$

14.381 Water flows over a spillway into a sluice 10 m wide. Before the jump the water has a depth of 1 m and a velocity of 18 m/s. Determine the Froude number before the jump and the depth of flow after the jump.

▌ $N_F = V/\sqrt{gy} \qquad (N_F)_1 = 18/\sqrt{(9.807)(1)} = 5.75$

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2 \qquad Q = A_1V_1 = [(10)(1)](18) = 180 \text{ ft}^3\text{/s}$$

$$y_2 = \{-1 + \sqrt{1^2 + [(8)(180)^2/(9.807)(10)^2](\tfrac{1}{1})}\}/2 = 7.64 \text{ m}$$

14.382 A rectangular channel has a width of 5 ft and a flow of 10 cfs; the depth in front of a hydraulic jump is 3 in. Determine the specific energy (in ft-lb/slug) behind the jump.

▌ $y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2$

$$= \{-\tfrac{3}{12} + \sqrt{(\tfrac{3}{12})^2 + [(8)(10)^2/(32.2)(5)^2][1/(\tfrac{3}{12})]}\}/2 = 0.880 \text{ ft}$$

$\text{KE} = v^2/2 \qquad v_2 = Q/A_2 = 10/[(0.880)(5)] = 2.27 \text{ ft/s} \qquad (\text{KE})_2 = 2.27^2/2 = 2.58 \text{ ft-lb/slug}$

14.383 Water in a rectangular, finished-concrete channel overflows a dam, as shown in Fig. 14-97, and goes into a stilling basin at which there is a hydraulic jump. If the channel is of width 6 m, has slope 0.003, and carries 18 m³/s, calculate the depth y at O.

▌ $\Delta x = ([1 - (Q^2b/gA^3)]/\{s_0 - (n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\})\,\Delta y$

$$A = (5)(6) = 30.00 \text{ m}^2 \qquad R_h = A/p_w = 30.00/(5 + 6 + 5) = 1.875 \text{ m}$$

$$60 = \left\{\frac{1 - (18^2)(6)/(9.807)(30.00)^3}{0.003 - (0.012/1.0)^2[18^2/(1.875)^{4/3}(30.00)^2]}\right\}(\Delta y)$$

$$\Delta y = 0.180 \text{ m} \qquad y = 5 + 0.180 = 5.180 \text{ m}$$

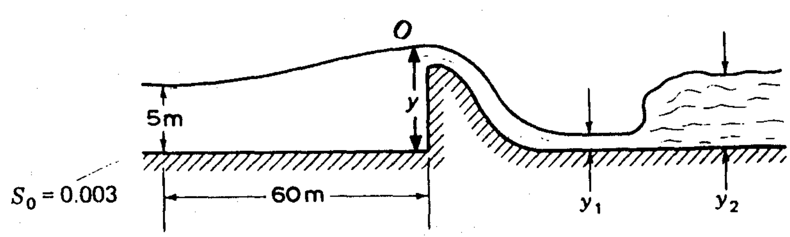


Fig. 14-97

14.384 Does linear averaging significantly improve the solution of Prob. 14.383?

▌ $\Delta x = ([1 - (Q^2b/gA^3)]/\{s_0 - (n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\})\,\Delta y$

At O, $A = (5.180)(6) = 31.08\ \text{m}^2$ and $R_h = A/p_w = 31.08/(5.180 + 6 + 5.180) = 1.900\ \text{m}$; therefore,

$$A_{av} = (30.00 + 31.08)/2 = 30.54\ \text{m}^2 \qquad (R_h)_{av} = (1.875 + 1.900)/2 = 1.888\ \text{m}$$

$$60 = \left\{\frac{1 - (18^2)(6)/(9.807)(30.54)^3}{0.003 - (0.012/1.0)^2[18^2/(1.888)^{4/3}(30.54)^2]}\right\}(\Delta y)$$

$$\Delta y = 0.180\ \text{m}$$

14.385 Determine the depth y_2 at the hydraulic jump of Prob. 14.383.

▌ Neglecting friction at the dam,

$$E_1 = E_0 \qquad E = y + q^2/2y^2g \qquad E_0 = 5.180 + (\tfrac{18}{6})^2/[(2)(5.180)^2(9.807)] = 5.197\ \text{m}$$

$$E_1 = y_1 + (\tfrac{18}{6})^2/[(2y_1^2)(9.807)] = 5.197\ \text{m} \qquad y_1 = 0.306\ \text{m} \qquad \text{(by trial and error)}$$

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2$$

$$= \{-0.306 + \sqrt{0.306^2 + [(8)(18)^2/(9.807)(6)^2](1/0.306)}\}/2 = 2.301\ \text{m}$$

14.386 Water flows in a 4-m-wide rectangular channel at Froude number $\sqrt{10}$; the depth of flow is 1 m. If the water undergoes a hydraulic jump, what is the Froude number downstream of the jump?

▌

$$N_F = v/\sqrt{gy} \qquad \sqrt{10} = v/\sqrt{(9.807)(1)} \qquad v = 9.903\ \text{m/s}$$

$$Q = Av = [(4)(1)](9.903) = 39.61\ \text{m}^3/\text{s}$$

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2$$

$$= \{-1 + \sqrt{1^2 + [(8)(39.61)^2/(9.807)(4)^2](1/1)}\}/2 = 4.000\ \text{m}$$

$$v_2 = Q/A_2 = 39.61/[(4.000)(4)] = 2.476\ \text{m/s} \qquad (N_F)_2 = 2.476/\sqrt{(9.807)(4.000)} = 0.395$$

14.387 Water in a 10-m-wide rectangular channel experiences a jump in depth from 2 m to 6 m. Find the Froude numbers on either side of the jump.

▌

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2 \qquad 6 = \{-2 + \sqrt{2^2 + [8Q^2/(9.807)(10)^2](\tfrac{1}{2})}\}/2$$

$$Q = 217\ \text{m}^3/\text{s}$$

$$N_F = v/\sqrt{gy} \qquad v_1 = Q/A_1 = 217/[(10)(2)] = 10.85\ \text{m/s} \qquad (N_F)_1 = 10.85/\sqrt{(9.807)(2)} = 2.45$$

$$v_2 = Q/A_2 = 217/[(10)(6)] = 3.62\ \text{m/s} \qquad (N_F)_2 = 3.62/\sqrt{(9.807)(6)} = 0.472$$

14.388 Water flows in a rectangular concrete channel and undergoes a hydraulic jump such that 60 percent of its mechanical energy is to be dissipated. If the volume flow rate is 100 m³/s and the width of the channel is 5 m, what must the Froude number be just before the jump? Set up the proper equations but do not actually solve.

▌

$$N_F = v/\sqrt{gy} \qquad E = y + v^2/2g \qquad v_1 = Q/A_1 = 100/(5y_1) = 20.00/y_1$$

$$E_1 = y_1 + (20.00/y_1)^2/[(2)(9.807)] = y_1 + 20.39/y_1^2 \qquad E_j = [y_2^3 - y_1^3 + y_1y_2(y_1 - y_2)]/4y_1y_2$$

$$0.60E_1 = (0.60)(y_1 + 20.39/y_1^2) = [y_2^3 - y_1^3 + y_1y_2(y_1 - y_2)]/4y_1y_2 \qquad (1)$$

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2 = \{-y_1 + \sqrt{y_1^2 + [(8)(100)^2/(9.807)(5)^2](1/y_1)}\}/2 \qquad (2)$$

Solve Eqs. (*1*) and (*2*) to find y_1 and y_2. Then, $v_1 = Q/A_1 = 100/5y_1$, $(F_R)_1 = (100/5y_1)/\sqrt{(9.807)(y_1)} = 6.386y^{-3/2}$.

14.389 Water is flowing from a spillway into a stilling basin, as shown in Fig. 14-98. The elevation y_A ahead of the spillway is 8 m. The width of the rectangular channel is 10 m. If the stilling basin dissipates half of the mechanical energy, what is the volume flow rate? Set up three simultaneous equations for y_1, y_2, and Q.

■
$$E = y + v^2/2g \qquad E_A = E_1$$
$$8 + Q^2/[(80)^2(2g)] = y_1 + Q^2/[(10y_1)^2(2g)] \tag{1}$$
$$0.50E_1 = E_j$$
$$0.50\{8 + Q^2/[(80)^2(2g)]\} = [y_2^3 - y_1^3 + y_1 y_2(y_1 - y_2)]/4y_1 y_2 \tag{2}$$
$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2 = [-y_1 + \sqrt{y_1^2 + [8Q^2/(9.807)(10)^2](1/y_1)}]/2 \tag{3}$$

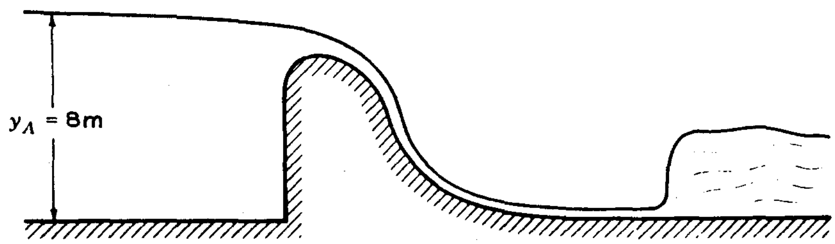


Fig. 14-98

14.390 Water having a steady known volumetric flow rate Q is moving in a rectangular channel at supercritical speed up an adverse slope. It undergoes a hydraulic jump as shown in Fig. 14-99. If we know y_B and y_A at positions L_1 and L_2 apart, how do we approximately locate the position of the hydraulic jump, i.e., how do we get L_1 and L_2? The channel has a known value of n. Explain the simplest method. The width is b.

■ Use the equation $\Delta L = \left([1 - (Q^2b/gA^3)]/\{S_0 - (n/\kappa)^2[Q^2/(R_H^{4/3}A^2)]\}\right)\Delta y$ and guess at a value L_1 and solve for $(\Delta y)_A$. Now compute $y_1 = y_A + (\Delta y)_A$ before the jump. Again go to the equation above and take L_2 and again solve for $(\Delta y)_B$ in the flow upstream of the jump: $y_2 = y_B - (\Delta y)_B$. Now insert y_1 and y_2 into the jump equation $y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2$ to see whether this equation is satisfied for the given volumetric flow Q. If not, go back and choose a different value of L_1 proceeding in this way until the jump equation is satisfied.

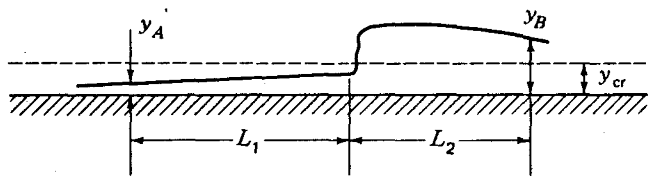


Fig. 14-99

14.391 The distance L_1 in Fig. 14-99 is 200 m and y_A is half the critical value. The channel is finished concrete with an adverse slope of -0.005. What is y_B at a distance $L_2 = 100$ m from the hydraulic jump? Solve in the simplest manner to get an approximate result. The flow rate is 200 m³/s and the width of the channel is 7 m.

■
$$y_c = (q^2/g)^{1/3} = [(\tfrac{200}{7})^2/9.807]^{1/3} = 4.366\text{ m} \qquad y_A = 4.366/2 = 2.183\text{ m}$$
$$\Delta L = \left([1 - (Q^2b/gA^3)]/\{S_0 - (n/\kappa)^2[Q^2/(R_H^{4/3}A^2)]\}\right)\Delta y$$
$$A_1 = (7)(2.183) = 15.28\text{ m}^2 \qquad R_H = A/p_w \qquad (R_H)_1 = 15.28/(2.183 + 7 + 2.183) = 1.344\text{ m}$$
$$200 = \left\{\frac{1 - (200)^2(7)/[(9.807)(15.28)^3]}{-0.005 - (0.012/1)^2[200^2/(1.344)^{4/3}(15.28)^2]}\right\}(\Delta y)_A \qquad (\Delta y)_A = 0.618\text{ m}$$
$$y_1 = 2.183 + 0.618 = 2.801\text{ m}$$
$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2 = \{-2.801 + \sqrt{2.801^2 + [(8)(200)^2/(9.807)(7)^2](1/2.801)}\}/2 = 6.435\text{ m}$$
$$A_2 = (7)(6.435) = 45.04\text{ m}^2 \qquad (R_H)_2 = 45.04/(6.435 + 7 + 6.435) = 2.267\text{ m}$$
$$100 = \left\{\frac{1 - (200)^2(7)/[(9.807)(45.04)^3]}{-0.005 - (0.012/1)^2[200^2/(2.267)^{4/3}(45.04)^2]}\right\}(\Delta y)_B \qquad (\Delta y)_B = -0.866\text{ m}$$
$$(\Delta y)_B = y_B - y_2 \qquad -0.866 = y_B - 6.435 \qquad y_B = 5.569\text{ m}$$

14.392 For a rectangular channel, develop an expression for the relation between the depths before and after a hydraulic jump. Refer to Fig. 14-100.

■ For the free body between sections 1 and 2, considering a unit width of channel and unit flow q, $P_1 = \gamma \bar{h}A = \gamma(\frac{1}{2}y_1)y_1 = \frac{1}{2}\gamma y_1^2$ and, similarly, $P_2 = \frac{1}{2}\gamma y_2^2$. From the principle of impulse and momentum, $\Delta P_x\, dt = \Delta$ linear momentum $= (W/g)(\Delta V_x)$, $\frac{1}{2}\gamma(y_2^2 - y_1^2)\,dt = (\gamma q\, dt/g)(V_1 - V_2)$.

Since $V_2 y_2 = V_1 y_1$ and $V_1 = q/y_1$, the above equation becomes

$$q^2/g = \tfrac{1}{2} y_1 y_2 (y_1 + y_2) \quad (1)$$

Since $q^2/g = y_c^3$,

$$y_c^3 = \tfrac{1}{2} y_1 y_2 (y_1 + y_2) \quad (2)$$

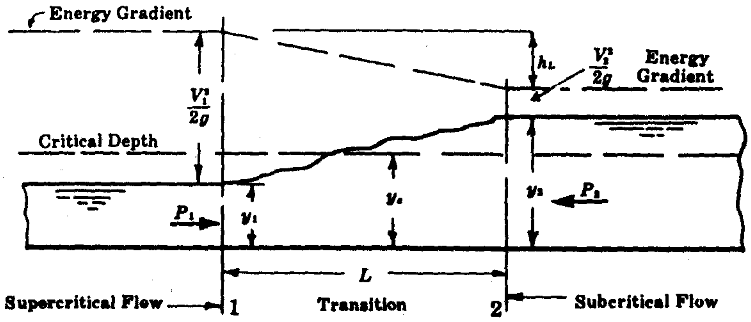


Fig. 14-100

14.393 A rectangular channel, 20 ft wide, carries 400 cfs and discharges onto a 20-ft-wide apron with no slope with a mean velocity of 20 fps. What is the height of the hydraulic jump? What energy is lost in the jump?

▌ $q^2/g = y_1 y_2 (y_1 + y_2)/2 \qquad y_1 = q/v_1 = (\frac{400}{20})/20 = 1.00\text{ ft} \qquad (\frac{400}{20})^2/32.2 = (1.00 y_2)(1.00 + y_2)/2$

$0.500 y_2^2 + 0.500 y_2 - 12.42 = 0 \qquad y_2 = 4.51\text{ ft} \qquad \text{Height of jump} = 4.51 - 1.00 = 3.51\text{ ft}$

$$E_j = (y_2 - y_1)^3/4y_1 y_2 = (4.51 - 1.00)^3/[(4)(4.51)(1.00)] = 2.40\text{ ft}$$

$$\text{Energy lost} = Q\gamma E_j = (400)(62.4)(2.40) = 59\,900\text{ ft-lb/s}$$

14.394 A rectangular channel, 16 ft wide, carries a flow of 192 cfs. The depth of water on the downstream side of a hydraulic jump is 4.20 ft. What is the upstream depth? What is the loss of head?

▌ $q^2/g = y_1 y_2 (y_1 + y_2)/2 \qquad (\frac{196}{16})^2/32.2 = (y_1)(4.20)(y_1 + 4.20)/2$

$$2.10 y_1^2 + 8.82 y_1 - 4.472 = 0 \qquad y_1 = 0.457\text{ ft}$$

$$E_j = (y_2 - y_1)^3/4y_1 y_2 = (4.20 - 0.457)^3/[(4)(0.457)(4.20)] = 6.83\text{ ft}$$

14.395 After flowing over the concrete spillway of a dam, 9000 cfs then passes over a level concrete apron ($n = 0.013$). The velocity of the water at the bottom of the spillway is 42.0 ft/s and the width of the apron is 180 ft. Conditions will produce a hydraulic jump, the depth in the channel below the apron being 10.0 ft. In order that the jump be contained on the apron, (*a*) how long should the apron be built? (*b*) How much energy is lost from the foot of the spillway to the downstream side of the jump?

▌ Refer to Fig. 14-101.

(*a*) $q^2/g = y_1 y_2 (y_1 + y_2)/2 \qquad (\frac{9000}{180})^2/32.2 = 10 y_2 (10 + y_2)/2$

$$5y_2^2 + 50 y_2 - 77.64 = 0 \qquad y_2 = 1.37\text{ ft}$$

$$y_1 = q/V_1 = (\tfrac{9000}{180})/42.0 = 1.19\text{ ft} \qquad L = [(V_2^2/2g + y_2) - (V_1^2/2g + y_1)]/(S_0 - S)$$

$$V_2 = q/y_2 = (\tfrac{9000}{180})/1.37 = 36.50\text{ ft/s} \qquad S = (nV/1.486R^{2/3})^2$$

$$V_{av} = (42.0 + 36.40)/2 = 39.20\text{ ft/s}$$

$$R = A/p_w \qquad R_1 = (180)(1.19)/(1.19 + 180 + 1.19) = 1.174\text{ ft}$$

$$R_2 = (180)(1.37)/(1.37 + 180 + 1.37) = 1.349\text{ ft}$$

$$R_{av} = (1.174 + 1.349)/2 = 1.262\text{ ft} \qquad S = \{(0.013)(39.20)/[(1.486)(1.262)^{2/3}]\}^2 = 0.08623$$

$$L = \{36.50^2/[(2)(32.2)] + 1.37 - 42.0^2/[(2)(32.2)] - 1.19\}/(0 - 0.08623) = 75.7\text{ ft}$$

The length of the jump L_3 from B to C is from $4.3y_3$ to $5.2y_3$. Assuming the conservative value of $5.0y_3$, $L_3 = (5.0)(10.0) = 50$ ft. Hence, $L_{ABC} = 76 + 50 = 126$ ft (approximately).

(*b*) $E = y + V^2/2g \qquad E_A = 1.19 + 42.0^2/[(2)(32.2)] = 28.58\text{ ft}$

$$V_3 = Q/A_3 = 9000/[(10.0)(180)] = 5.00\text{ ft/s}$$

$$E_C = 10.0 + 5.00^2/[(2)(32.2)] = 10.39\text{ ft} \qquad E_{lost} = 28.58 - 10.39 = 18.19\text{ ft}$$

$$P = Q\gamma E \qquad P_{lost} = (9000)(62.4)(18.19) = 10.22 \times 10^6\text{ ft-lb/s}$$

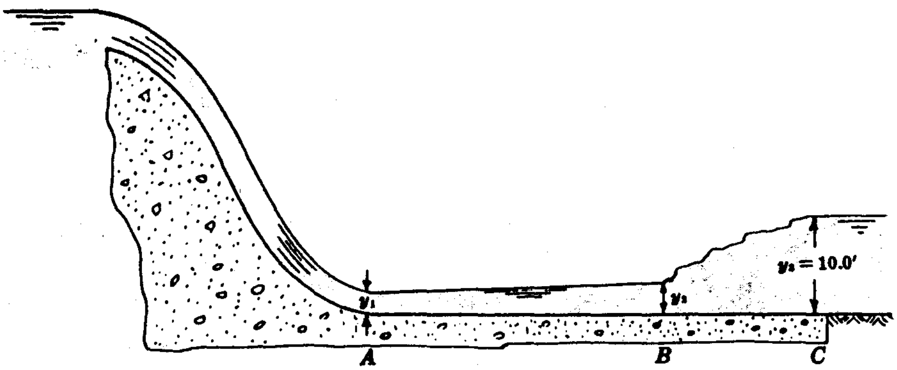


Fig. 14-101

14.396 Determine the elevation of the spillway apron if $q = 50$ cfs/ft, $h = 9$ ft, $D = 63$ ft, and the spillway crest is at elevation 200.0 ft.

$$(\pi)_1(\pi_3 - \pi_2)(\pi_3 + 1)^{1/2} + 0.353 = \sqrt{\tfrac{1}{8} + (2.828)(\pi_1)(\pi_3 + 1)^{3/2}}$$

$$\pi_1 = g^{1/2}h^{3/2}/q = (32.2)^{1/2}(9)^{3/2}/50 = 3.06$$

$$\pi_2 = D/h = \tfrac{63}{9} = 7.00 \qquad \pi_3 = d/h = d/9$$

$$(3.06)(d/9 - 7.00)(d/9 + 1)^{1/2} + 0.353 = \sqrt{\tfrac{1}{8} + (2.828)(3.06)(d/9 + 1)^{3/2}}$$

$d = 77.9$ ft (by trial and error) Elevation of spillway apron $= 200.0 - 77.9 = 122.1$ ft

14.397 Establish the equation for flow over a broad-crested weir assuming no lost head. See Fig. 14-102.

At the section where critical flow occurs, $q = V_c y_c$. But $y_c = V_c^2/g = \frac{2}{3}E$, and $V_c = \sqrt{g(\frac{2}{3}E_c)}$. Hence the theoretical value of flow q becomes $q = \sqrt{g(\frac{2}{3}E_c)} \times \frac{2}{3}E_c = 3.09E_c^{3/2}$. However, the value of E_c is difficult to measure accurately, because the critical depth is difficult to locate. The practical equation becomes $q = CH^{3/2} \approx 3H^{3/2}$. The weir should be calibrated in place to obtain accurate results.

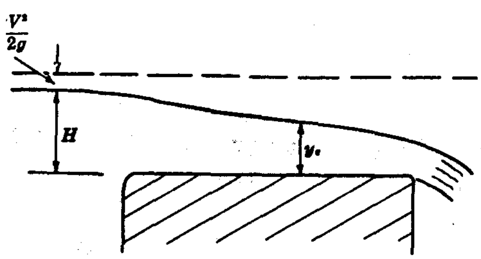


Fig. 14-102

14.398 Develop an expression for a critical-flow meter (Fig. 14-103).

An excellent method of measuring flow in open channels is by means of a constriction. The measurement of the critical depth is not required. The depth y_1 is measured a short distance upstream from the constriction. The raised floor should be about $3y_c$ long and of such height as to have the critical velocity occur on it.

For a rectangular channel of constant width, the Bernoulli equation is applied between sections 1 and 2, in which the lost head in accelerated flow is taken as one-tenth of the difference in velocity heads, i.e., $y_1 + (V_1^2/2g) - (\frac{1}{10})[(V_c^2/2g) - (V_1^2/2g)] = [y_c + (V_c^2/2g) + z]$, which neglects the slight drop in the channel bed between 1 and 2. Recognizing that $E_c = y_c + V_c^2/2g$, we rearrange as follows: $[y_1 + (1.10V_1^2/2g)] = z + 1.0E_c + (\frac{1}{10})(\frac{1}{3}E_c)$, $[y_1 - z + (1.10V_1^2/2g)] = 1.033E_c = (1.033)(\frac{3}{2}\sqrt[3]{q^2/g})$, or

$$q = (2.94)(y_1 - z + 1.10V_1^2/2g)^{3/2} \qquad (1)$$

Since $q = V_1 y_1$,

$$q = (2.94)(y_1 - z + 0.0171q^2/y_1^2)^{3/2} \qquad (2)$$

14.399 Consider a rectangular channel 10 ft wide with the critical-flow meter of Fig. 14-103 having dimension $z = 1.10$ ft. If the measured depth y_1 is 2.42 ft, what is the discharge?

$q = (2.94)(y_1 - z + 0.0171q^2/y_1^2)^{3/2} = (2.94)(2.42 - 1.10 + 0.0171q^2/2.42^2)^{3/2}$. As a first approximation, neglect the last term involving q. $q = (2.94)(2.42 - 1.10)^{3/2} = 4.46$ cfs/ft. Try $q = 4.80$ cfs/ft: $q = 2.94[2.42 - 1.10 + (0.0171)(4.80)^2/2.42^2]^{3/2} = 4.80$ cfs/ft (O.K.), $Q = (4.80)(10) = 48.0$ ft^3/s.

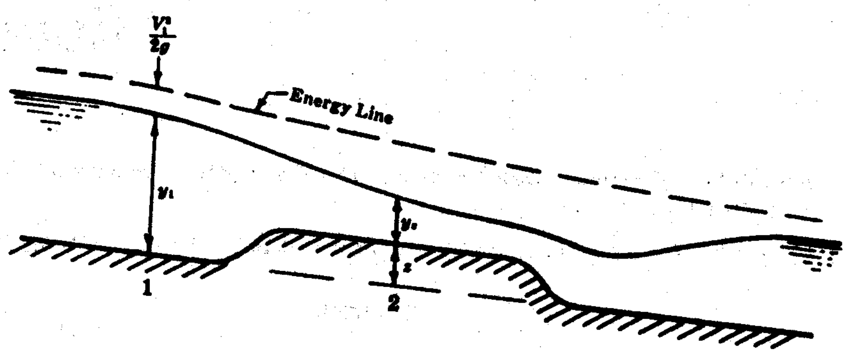


Fig. 14-103

14.400 In a 100-ft-wide rectangular channel, the depth upstream from a hydraulic jump is 4 ft. What flow rate is required in order for the downstream depth to be (*a*) twice the upstream depth and (*b*) ten times the upstream depth?

▮

$$q = \{[(d_1 + d_2)/2](gd_1d_2)\}^{1/2}$$

(*a*) $d_2 = (2)(4) = 8$ ft $\quad q = \{[(4+8)/2][(32.2)(4)(8)]\}^{1/2} = 78.63$ cfs/ft

$$Q = (100)(78.63) = 7863 \text{ ft}^3/\text{s}$$

(*b*) $d_2 = (10)(4) = 40$ ft $\quad q = \{[(4+40)/2][(32.2)(4)(40)]\}^{1/2} = 336.7$ cfs/ft

$$Q = (100)(336.7) = 33\,670 \text{ ft}^3/\text{s}$$